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Adult Students' Open Interactions with Technology: Mediating Higher-Level Thinking

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Abstract

Examining the actions taken during a teaching experiment can provide insight into practices applicable to the use of mathematics technology to assist adult learners. A case study in the form of a teaching experiment was conducted with a small number of subjects to allow for detailed examination of the influence of technology on student thinking. This report examines the type of questions offered by the interviewer during the experiment's sessions and the associated choices made by the study's subjects. Indications are that with higher-level prompts in the form of both requests for observation and requests for justification, the tool in use can become a means by which new understanding is internalized.

Key words: Mathematics, technology, higher-level thinking, mediation

Introduction

Kaput (1994) believed that technology has the potential to revolutionize the study of mathematics as greatly as the invention of Arabic numerals or the invention of calculus. He noted that mathematical and technological advances can allow problems previously solvable only by the elite to become routine. The realization of high hopes for technology's impact on mathematics learning is affected by the rate at which understanding of its use for learning progresses. Access to technology is easier than the far greater challenge of determining how to use technological tools effectively (Fey, Hollenbeck, & Wray, 2010).

Students do not automatically construct knowledge merely because technological representations are presented to them (Burrill et al., 2002). They may perform poorly both with and without technology. Teachers may need substantial professional development in order to implement technology effectively in the classroom (Burrill et al., 2002). A study that examined the effect of technological representations on adult developmental mathematics students' understanding of representations associated with functions will be built upon to suggest relationships between the choices a teacher makes and the effect of those choices on student thinking. The original research questions examined the subjects' *internal representations*, those representations existing inside the mind of the learner (Goldin, 2003), that students possessed following technological explorations. This follow up report will examine the moves on the part of the interviewer. The general research question guiding this further examination of the data is "What facilitating moves support the construction of knowledge during student engagement with open technological tasks?" Qualitative analysis was used to examine the types of facilitating questions and statements made by the interviewer during the original case study, and suggestions as to the theoretical relationships between questioning and thinking in a technological setting will be presented. A look at literature regarding the use of mediation will be followed by a brief view at the research methodology, and an examination of the results.

Supporting the Use of Technology

Technology has the potential to empower students, to aid them in understanding and using standard representations, and to aid them in mathematical reasoning and communication (Garrett, 2010). It can help students build a better understanding of mathematics (Blair, 2006). One of the ways it does this is by providing students with the opportunity for exploration (Fey et al., 2010). Campe (2011) suggested that teachers

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implementing technology in the classroom guide students in discovering connections by "pretend[ing] that [they] do not know the answer, giv[ing] hints sparingly, and ask[ing] questions that support the students' reasoning and sense making” (p. 623). She also suggested that teachers mediate students' learning efforts by providing some previously prepared technological representations and by using careful questioning to help them to "explain, find patterns, and make predictions" (Campe, 2011, p. 622). A look at mediation will highlight its potential as a tool for orchestrating the use of technology in the classroom.

To mediate is to arbitrate or to intercede (mediate, 2012). Mediation in teaching and learning refers to classroom actions that support students in the construction of their own knowledge and help them understand their own ways of thinking (Grosser & de Waal, 2008). A mediational approach seeks to provide enough support to build independent problem solvers. A teacher's mediation can serve to structure a discussion so that it becomes a productive conversation between students (Zolkower & Shreyar, 2007). Appropriate mediation can serve as an aid to cognitive development and a tool for fostering higher-level thinking (Falcade, Laborde, & Mariotti, 2007; Henningsen & Stein, 1997).

Students who might benefit from mediation – those who are having difficulty constructing knowledge - may be experiencing what Piaget described as disequilibrium. He saw cognitive development as consisting of alternating stages of equilibrium and disequilibrium (Vander Zander, 1989). Disequilibrium occurs when learners are presented with new information they must incorporate into their existing ways of understanding. Equilibrium is reestablished when the new information becomes part of what they understand. As described by Vander Zander (1989) assimilation is "the fitting of new experiences to old ones; accommodation is the fitting of old experiences to new ones” (p. 49). Students assimilating data will produce a new scheme that integrates new knowledge with prior understanding. That understanding will suffice until it is challenged by more new information, and disequilibrium occurs again. Mediation can scaffold the assimilation process by helping students structure their construction of knowledge. In this way, mediation becomes an aid to cognitive development.

One way of promoting cognitive development is through the use of open tasks. Mueller, Yankelewitz, and Maher (2010) noted that the use of challenging open-ended tasks and the use of concrete materials with which students could build models of their ideas fostered the development of mathematical understanding, a form of cognitive development. They also noted the importance of the teacher's role in the use of such tasks. Mediation included inviting, requiring justification, orchestrating small group collaboration, structuring whole-class student sharing, and using strategic questioning (Mueller, Yankelwitz, and Maher 2010). Teachers implementing open-ended tasks can also mediate students' efforts by reminding them of earlier work they did that connect to the new task (McKnight & Mulligan, 2010). They may assist students in choosing appropriate representations for genuine mathematical understanding. A student may be able to verbally express genuine knowledge, for example, but he or she may have difficulty putting that knowledge into writing (McKnight & Mulligan, 2010). Such a challenge is an opportunity for mediation.

If technology is to empower students and build mathematical understanding, it must be incorporated into teaching and learning in a way that supports those goals. Providing students with the opportunity to engage in open-ended exploration is one way of providing such empowerment (Mueller, Yankelwitz, and Maher, 2010). Teacher mediation will help students build understanding from those explorations. It will also provide scaffolding as students learn what it means to explore and build their own knowledge throught the use of technology (Campe, 2011). A look at the moves made in a research study connecting student technological explorations with changes in their thinking will help illustrate specific ways that such mediation might take place.

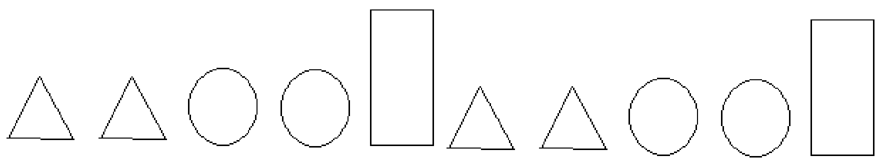
Methodology

A case study in the form of a teaching experiment was conducted with subjects recruited from a mid-sized university in the southern United States that served several hundred developmental mathematics students. Names used to refer the subjects are pseudonyms. Open recruitment was conducted on campus through the use of flyers and a brief introduction of the study in developmental mathematics classrooms. Marlon, a 53-year-old African American male, was enrolled in the second level of developmental mathematics, Math 99, having taken the first level twice, passing it the second time. The first level, Math 98, was a review of basic mathematics and included an introduction to algebra and linear functions. Math 99 included a review of those topics along with a look at quadratic functions. Marjorie, a 36-year-old African American female, was enrolled in the highest level of developmental mathematics, a bridging course for those who were not prepared for college algebra but were not required to take the two basic developmental mathematics courses. She could not remember a previous mathematics course, indicating that it had been 14 years since she had been in high school. Seven sessions conducted by the author were held with Marlon, following which six sessions were held with Marjorie. The original data analysis focused on the effect of technology use on the subjects' understanding of representations associated with functions. The data was re-examined for this report in order to examine the facilitating moves and mediation conducted by the interviewer during the subject's open ended explorations.

Three patterns are shown in Figure 1. Each pattern was shown to the subjects separately as a way to introduce them to the mathematics of functional relationships and learn something about their mathematical thinking. Early in the experiment, the subjects were introduced to the first two patterns found in Figure 1, "Looking at patterns" and "Looking at dot patterns. The other pattern found in Figure 1, "Another dot pattern" was also introduced to Marlon. The patterns were chosen to provide the subjects with a conceptual foundation for the examination of representations of functions. Once the subjects had examined these patterns, found some mathematical ideas in the representations, and created tabular representations, they explored that tabular data using the dynamic geometry and algebra software Geometer's Sketchpad v. 4.07s (Key Curriculum Press, 2006). They graphed the coordinate points associated with the tabular data and sought to find the equation of a function the graph of which on the xy-plane passed through those data points.

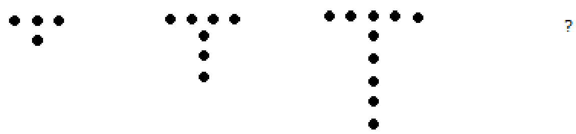
Looking at patterns

Study the pattern below and tell me everything you notice about it.



Looking at dot patterns

Study the pattern below and tell me everything you notice about it.



Another dot pattern

Figure 1. Patterns used for the examination of function representations

Data Analysis

For purposes of analyzing facilitating moves and mediation related to adult students' free explorations, earlier qualitative data analysis was re-examined and code patterns related to the new research question were emphasized. Codes relating to student thinking were searched. Quotations which had previously been coded with one of the three headings *problem solving*, *reasoning*, or *sense-making* were now coded together under the name *higher-level thinking.* Problem solving referred to occasions when the subject was building knowledge by making conjectures. Reasoning referred to occasions when the subject was drawing logical conclusions. Sense making referred to occasions when the subject was connecting new knowledge with existing understanding. A hierarchical diagram showing a description of the codes and the relationship between them is given in Figure 2.

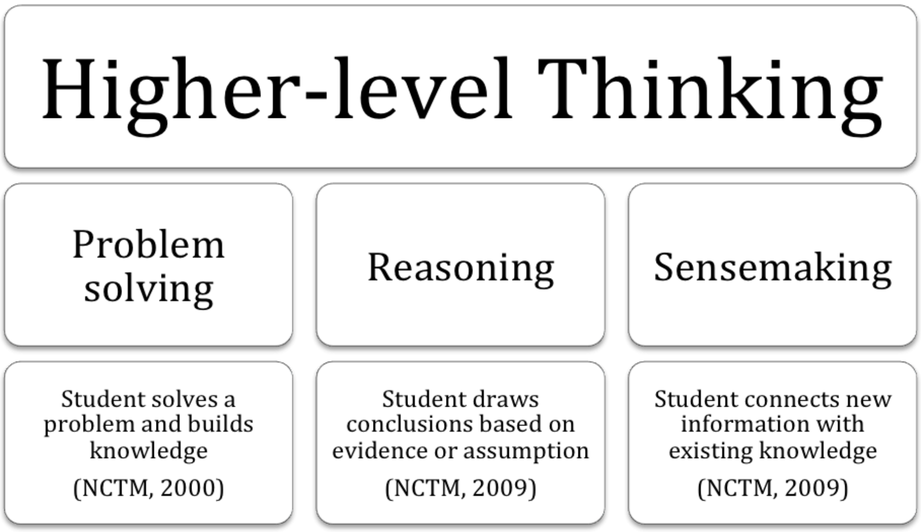


Figure 2. Ideas related to higher-level thinking

Problem solving was used to note those incidences in which the subject was working to solve a problem presented to him or her and in the process was building knowledge, applying strategies, or monitoring and reflecting on the process (NCTM, 2000). Reasoning was used to refer to evidence that the subject was drawing conclusions based on evidence or assumption (NCTM, 2009)**.** Sense making was used to show events during which the subject gave indication of how he or she was making sense of things, particularly events that showed connections with existing knowledge (NCTM, 2009). Thus problem solving placed an emphasis on the process of building and applying knowledge, reasoning placed an emphasis on drawing conclusions from evidence, and sense making placed an emphasis on making connections.

Interviewer moves coded as *What do you see* and *What happened* were related to fostering the subject's observation of the mathematics represented by the technology. *What do you see* indicates mediation intended to foster a closer observation of a given situation. Such prompts provided the subject with a choice as to what elements of the representations he or she would study. *What happened* is similar, but was directed toward an event or occurrence rather than a static representation. This type of mediation brings out the dynamic qualities of the technology and the evidence the dynamic representations give of mathematical ideas. Both involve prompting the subject to make observations. Likewise the two codes *explain your meaning or choice* and *why* are related, both indicating the promotion of justification on the part of the subject. *Explain your meaning or choice* was directed particularly toward the justification of actions chosen or statements made by the subject. *Why* was focused more generally on mathematical events that occurred during the explorations. Since both the observation requests and justification requests were intended to promote problem solving, reasoning, and sense-making on the part of the subject, they were coded as *higher-level prompts*. The hierarchy of codes related to higher-level prompts is seen in Figure 3.

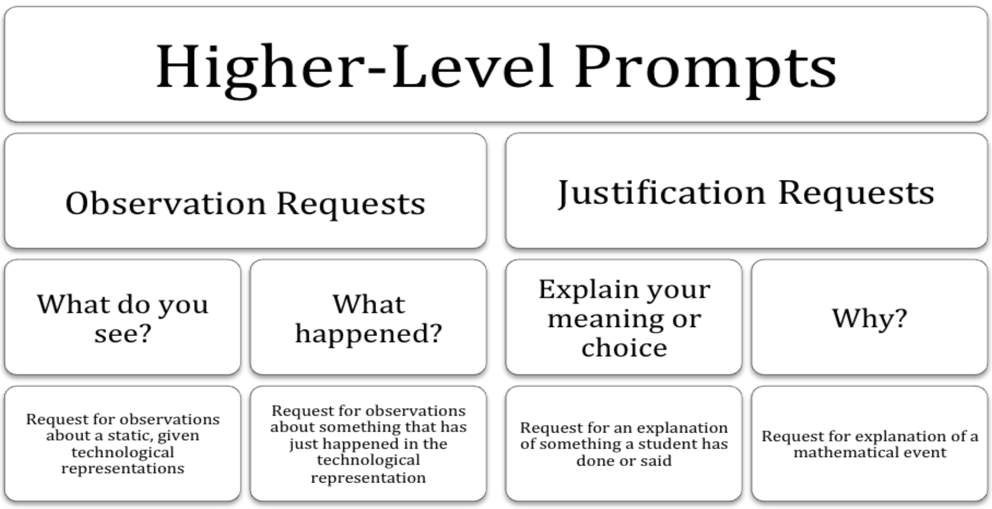


Figure 3: Ideas related to higher-level prompts

Other interviewer moves were also noted. *Timing challenges* indicate those incidences during which the interviewer either asked the subject to make too big of a conceptual leap in understanding or hurried the subject along by telling them more than they seemed to be getting on their own. *Direct instruction* included occasions when the interviewer told the subject directly what to do or how to use the technology, including clarifications of technology procedures or tool functionality. *Prompting memory* occurred when the interviewer asked the subject to remember something he or she had previously experienced, such as the behavior of a mathematical tool or insight into a mathematical concept. *Promoting action* was used to describe mediation that encouraged the subject to try something in particular, to consider how something in particular might be done with the technology, or to write something down about what he or she was thinking or observing. When the interviewer asked a mathematically specific question designed to promote mathematical thinking, these incidences were coded as *promoting mathematical thinking.*

Results

In seeking out possible relationships suggestions between the interviewer's actions and the subjects' thinking and learning, co-occurring codes were examined for the interviewer moves coded as timing challenges, higher-level prompts, prompting memory, promoting action, promoting mathematical thinking, or direct instruction. Analysis showed that higher-level prompts had the closest association with higher-level thinking, occurring 29 times, as opposed to 15 for the next highest co-occurrence, which was promoting mathematical thinking. Following are descriptions of particular incidences during which higher-level prompts and higher-level thinking occurred together. An investigation conducted by Marjorie will be described, followed by two episodes with Marlon.

Interactions with Marjorie

During the fourth session, Marjorie graphed the coordinate points representing the sequence of dot patterns in "Looking at dot patterns." She had already created a table of values on paper and accurately described the relationship between the step number and the number of dots by noting that you multiply the step number by three and add one to get the number of dots. In the fifth session she was asked to consider what function would result in the graph of a line that would pass through those coordinate points. She noted that in an algebraic expression "the letters represent numbers" and that "if we already know . . . what the numbers are that add up to the value . . . we would just go and just replace . . . maybe one of the values." So she decided to "replace . . . one of . . . the numbers in the equation." We had recently been discussing the number of dots in the 50th pattern.
As she considered what to replace with a variable, she said "Probably the 3. Maybe like have . . . 50 times x equals 151? Or 150?" After being encouraged to try it, she used the new function menu to create q(x) = 50x. After some discussion, she graphed it and said "It did something, but it didn't . . . that don't look right. That doesn't look right at all." She briefly described some of what she was seeing, noting that "it is at a slight angle" and moved the cursor into the 2nd quadrant and then down into the 4th quadrant near (-7, -39), close to the y-axis. At this point, I prompted her with the question "So what did you learn from . . . doing that?" which is one way of asking the student to explain what happened. She noted that it did plot a line and speculated that "maybe if I tried . . . adding more." She tried r(x) = 70x, after which I prompted her to talk about what she was doing. She then tried s(x) = 30x and continued to try different things and talk about what was happening. She observed that "the lower the number is, the more it moves away from the y-axis." She then tried t(x) = 10x. She said, "Yes it does!" She moved the cursor back and forth from algebraic to graphical representations as she explained what she had observed. Once she saw that a slope of 1 took her line beyond where she wanted it to be, she concluded that "I've got to . . . keep it between one and 10." Eventually she tried 3 times x. She had reasoned that the coefficient had to be between 1 and 5 and because of what she had seen in the pattern, she tried 3. Because of the scale of the graph at the time, the graph of the function appeared to land on the graphed points. As she discussed what had happened she mentioned that "It was a difference of three when [she] did the step and dots", referring to the rate of change in the pattern.

Prompts to think and talk about what she was learning encouraged the running conversation she kept up, which helped provide insight into her thinking. This opportunity gave evidence that she was engaging in reasoning and problem solving with the software. She was noticeably excited when it behaved the way she expected it to behave, and technology was an aid to her as she reasoned her way toward her conclusions. Encouraging students to talk aloud about their thinking as they work is sometimes referred to as a *think-aloud protocol* (Koichu &

Harel, 2007*)*. In this incidence, encouraging Marjorie to think aloud appears to have empowered her technological exploration. When she saw that f(x) = 3x did not exactly match her data points, the following exchange occurred. The information in italics describe her technological actions.

> MARJORIE: It's not necessarily on the step and the dot, it's . . . right past it . . . . It's right past it a little but just maybe like, it looks like maybe by one [*cursor to second graphed point*] so I wonder maybe if I was to try 2 times x and see if that would put it right on it [*cursor to graph menu*]
>
> INTERVIEWER: What made you decide to try the three?
>
> MARJORIE: Because I had just thought about the um [*opens plot new function*] the difference of three between the step and the dots
>
> INTERVIEWER: Okay
>
> MARJORIE: It just occurred to me - and I know that 5, um, is, it brings it closer, so I know that I have to look between 5 and - well 1 is way too far um - I haven't tried 4 and I haven't tried 2 yet. 3 Brings it a little bit further - so I think I'm going to try 4 instead, instead of 2 [*types 4 in white area of new function menu*]- but then when I try 4 times x lets' see what that does - equals [*she now has entered and plotted the function 4x , then moves cursor over to the new graph as she continues to speak*]- And 4 times x does not put it on there either. The three and the x was better. So let me try 2 and x and see. Ha-gotta get off that and plot new function [*has gone up to graph menu, then down to deselect algebraic representation and back up to graph menu and clicks plot new function*]- you know I'm really not quite sure why I'm going to try 2 and x because the one and x was so far out, 2 and x will probably be right next to it. [*she is entering 2x as she speaks - having first started, then deselected, then started again*]
>
> INTERVIEWER: Before you hit okay, where do you think the 2 x, two times x is going to be - show me with your cursor where you think it's going to land.
>
> MARJORIE: I think 2x might be right here [*cursor near (4, 6)*] Because the one and x was right there [*cursor at about (6,6) on the graph of v(x) = 1*x*] that's the one and x
>
> INTERVIEWER: Okay
>
> MARJORIE: And that's 4 and x [*cursor at about (2.5, 10) on the graph of* $g_1(x) = 4x$] and that's 3 and x [*cursor just below (3, 10) on a segment passing between plotted points near* $f_1(x) = 3x$] so I think that 2 and x will probably be right there [*moves cursor in area of first quadrant between graphs of v(x) = 1x and* $f_1(x)=3x$ *along a short linear path near where the graph would land*]
>
> INTERVIEWER: Okay
>
> MARJORIE: Really because that's 5, 4, 3,[*cursor moves from one graph to the next as she speaks, hitting the segment when she gets to 3*] I think 2 will be right there between 3 and 1 [*cursor waving around in the space in the first quadrant between the functions x and 3x along a short linear path near where the graph would land*] because I think no matter what I try or if I do it going this way [*cursor in new function window*] it's not going to put it directly on the dot, the step dot coordinates [*cursor near the graph of 3x and then back to new function window*]. Yeah. And I don't think so. So really 2 (is) probably a waste of time but I'll put 2 out there anyway just to see [*she clicks okay to graph it*]
>
> INTERVIEWER: Okay
>
> MARJORIE: Yeah, I was right. [*cursor at about (6, 12)*]

Two higher-level prompts were offered during this exchange: "What made you decide to try the three?" and "Before you hit okay . . . show me with your cursor where you think it's going to land." Marjorie reasoned about the mathematics in which she was engaged, making an accurate prediction about the graph of a function. There is empowerment in her statement "So really 2 (is) probably a waste of time but I'll put 2 out there anyway just to see" followed by "Yeah, I was right." The technology allowed her to confirm her thinking which is a quality associated with problem solving and reasoning, and thus with higher-level thinking. Without the higher-level prompts, her reasoning may not have emerged. Notice at the beginning of the exchange she said "I wonder maybe if I was to try 2 times x and see if that would put it right on it." Without the higher level prompts, she may have just tried 2 times x without stating her reasoning and perhaps without experiencing as great a degree of empowerment or advancing to as great a degree of awareness of her own thinking.

Interactions with Marlon

Marlon brought misconceptions to the study, including confusion about the representations of coordinate points and functions which prompted him to use f(x) = a + b to try to graph a function which went through the point (a, b). As he worked with the software he was able to make some progress in clearing up these misconceptions. For example, during the fifth session, I asked him to recall the function he had previously found, the graph of which passed through the data points he had plotted representing the relationship shown in “Another dot pattern.” He could not remember what that function was, and in his efforts to remember, his confusion over the representations for coordinate points and functions interfered. He opened the function menu. He knew that the function had to pass through the point (1, 2) and so he graphed f(x) = 1 + 2. Since the graph did not travel in a diagonal line through the points, he tried again. He looked at the currently graphed point which was farthest to the top and right of the graph at that time, (8,9), and graphed g(x) = 8 + 9. This graph turned out to be out of the viewing window, and he had to change the scale of graph to see where it was. He did this himself with no facilitation after he had done some additional exploration which had produced h(x) = 8 + 9 and the equation 1 +2 = 3. When he finally saw the graph of g(x) he said “I’m getting a straight edge again here” and indicated 1+2 = 3 and then the graph of g(x) = 8 + 9 with his mouse. When I asked him where the graph of g(x) had come from, he at first replied “this last one I just put in” and indicated 1 + 2 = 3, then said “as you were” which was a phrase he commonly used when he realized something was wrong. He then counted to see that g(x) crossed the y-axis at 17. I asked him “Where might 17 come from?” This was genuinely puzzling to him, and he wondered aloud “How did I get 17 in there?” I then made an observation request and asked him if there was anything on the screen that might give him 17. The situation at the time he responded is illustrated through the image presented in figure 4, which is followed by his statements.

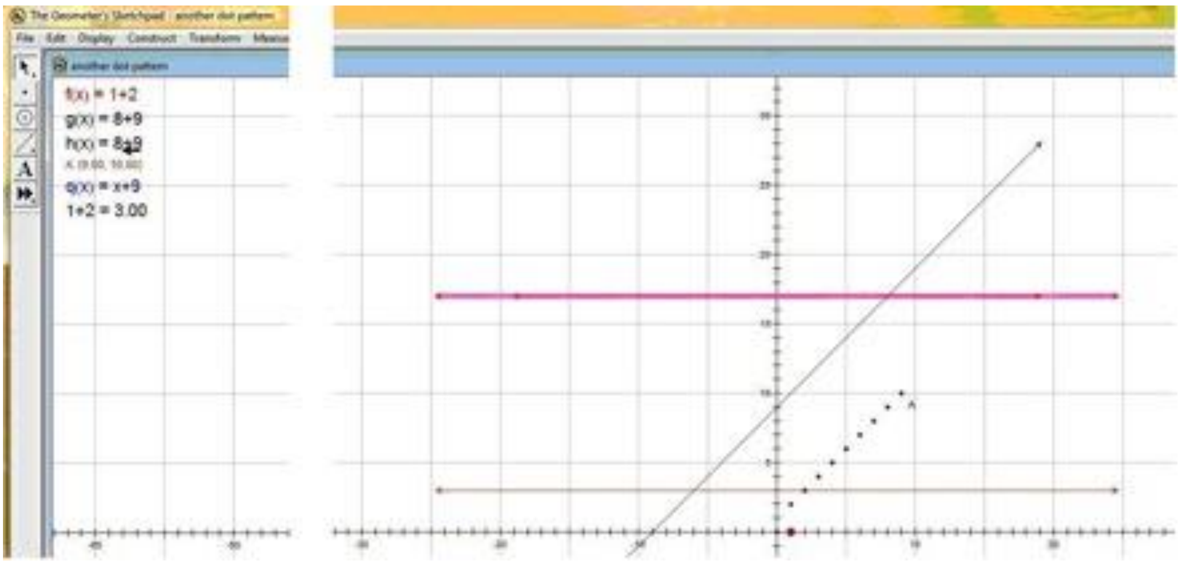


Figure 4: Technological situation when Marlon made the statements below.

> MARLON: [*he put the cursor at h(x) = 8+9 then moved it down toward the bottom of the list between q(x)=x+9 and 1+2=3*] ,ummmm, [*He moved the cursor back up the list and then to 1 + 2 = 3*] I don’t see, I mean, that would give me 17. How’d I get that one there? [*cursor near 1+2=3 and q(x) = x + 9*] And this is the, this is the same -similar to the one that I gave down here [*cursor at (0,3)*] through 3. Okay [*cursor to 1 +2 =3, cursor up to h(x) = 8 + 9*]. Oh, not unless these added together.

Looking at the multiple representations presented by the technology, he realized that in the functional notation f(x) = a + b, those two numbers were in fact added together to determine where the graph would be located, and were not representative of the two numbers describing a coordinate point, which was the way he had been trying to use them to graph the function. The location of the graph of h(x) = 8 + 9 in a different place than he expected it to be revealed the misconception and his examination of the different representations present following an observation request helped clear up the confusion. Even though the representation 1 + 2 = 3 was not the

functional representation which matched the graph which passed through (0, 3), the presence of that representation may have been important to his construction of the understanding that "these added together."
In another episode that took place during session four, I had asked him to enter the variable x into the functions he was graphing. He eventually graphed f(x) = x + 9 and g(x) = x - 9 and noticed that they crossed the y-axis at (0, 9) and (0, -9) respectively. He was constructing understanding that functions of the form f(x) = x + b cross the y-axis at (0, b). When I questioned him further, he was able to deduce and demonstrate that f(x) = x + 1 went through his graphed data points for "Another dot pattern." During session five, after some discussion, I asked him to try something that he hadn't done before. He entered v(x) = x – 9 and then said "Could I add more?" and I told him he could try that. The software automatically added parentheses to what he entered to give the function v(x) = (x – 9) + 6. After giving the graph and equation matching colors, he moved the equation close to the graph, studied it and said, "Now how did I get that one?" The following exchange occurred.

> INTERVIEWER: How did you get that one? That's a good question.
>
> MARLON: Okay
>
> INTERVIEWER: Think about what's happening with that one. Why is it going the way it's going?
>
> MARLON: Okay. x negative 9 - so - x negative 9 here - x negative 9 [*cursor at algebraic representation*] — I'm coming across here to a 3, [*cursor at (0,-3)*] Ah! What's happening is that it's subtracting [*during the following, the cursor moves back and forth along the algebraic representation until "negative 3" when he moves it back to (0, -3)*] the negative 9 from the 6 and it's giving me a negative 3 and that's the reason why it's intersecting here [*indicates (0,-3)*] because . . . its subtracting the -9 from a positive 6 which gives me actually negative 3 and again it is diagonal.

His cry of "Ah!" seemed to indicate confidence, as did his clear explanation. Although there was most likely much he still did not understand, he understood a correct mathematical idea which made sense to him and was built on his own demonstrated prior knowledge. He had chosen the exploration himself, he had put his plan into action using the technology, and he had drawn a correct conclusion about the connection between the algebraic and graphical representations. Such an exploration may have been difficult for him to do without the use of technology, and in this way he was empowered by his use of technology. Part of this empowerment involved a higher-level prompt, which he himself initiated, indicating his growing ability to reason. He asked himself "Now how did I get that one?" I turned that question back over to him and asked him to think and reason about why the graph was behaving the way it was. The higher-level prompts appear to have helped clarify the expectation for reasoning and sense-making.

Discussion

Open technological tasks can be supported by teachers' mediating moves that include both observation requests and justification requests. Student construction of knowledge during such mediation may be evidenced by student communications that indicate higher-level thinking in the form of building and applying knowledge (problem solving), drawing conclusions from evidence (reasoning), and making connections (sense-making). Although the present study is limited in scope, presenting a suggestion of relationships for further investigation, evidence of such occurrences can be seen in the cases of Marjorie and Marlon.

While Marjorie was examining the effect of k in functions of the form $f(x) = kx$, prompts to describe what she was learning, what she was doing, and what happened after she did it were associated with her eventual discovery of the correct slope for a function arising from a growing pattern of dots. Prompts asking her to explain her decision to try that slope and to predict where the graph of a function with a particular slope would be located were associated with connections that she made between multiple representations. She described the connection of the slope 3 to the increase from step to step in the number of dots. She also correctly predicted the location of the graph of $f(x) = 2x$. The discussion involved showed evidence of problem solving and reasoning, and thus higher-level thinking.

An observation request prompted Marlon to carefully consider what he was observing in the screen shot shown in figure 4 and as a result helped him progress in his understanding of representations of functions. He made connections between algebraic and graphical representations in a way that helped clear up his misconception about the relationship between the representations of coordinate points and functions. He knew that $h(x) = 8 + 9$ was not graphed where he expected it to be, he observed that it was passing through the y-axis at 17 and he was able to deduce that 8 + 9 represented a sum and not the coordinates of a point. In another episode, when he tried $v(x) = (x - 9) + 6$, a prompt to consider why the graph was behaving the way it was, which followed his own self-prompt “Now how did I get that one?” promoted the connection of multiple representations and his correct conclusion that the graph did indeed connect with the algebraic representation in that -9 + 6 were combined to obtain -3, which was the y-intercept.

Although higher level prompts were most closely associated with higher level thinking, other types of prompts also affected the subjects’ learning. Prompts that promoted mathematical thinking and prompted memory were associated with higher level thinking. Such prompts may help scaffold higher level thinking by encouraging students to consider mathematical ideas they may not have otherwise considered and to recall past learning. Teachers should take care to notice incidences in which the difficulties students are having engaging in higher level thinking are the result of timing challenges – asking students to make too big of a conceptual leap or hurrying students along by providing them with more information than they can assimilate. Such prompts were not associated with higher level thinking. Direct instruction was not associated with higher level thinking either, but the necessary technological instruction laid the groundwork for later higher level student interactions with technology. If proper groundwork is laid, then at the time students are asked to use technology for open tasks that require higher level thinking, no such direct instruction is needed because the student is fluent with the technology. In this situation, the teacher can focus more easily on higher level prompts.

Conclusion

Higher level prompts during Marjorie and Marlon’s interactions with open technological tasks promoted higher-level thinking. New internalized meanings developed through the use of a tool during interactions with the interviewer. Students’ disequilibrium in the setting of an open technological task can be a spark for teacher intervention that supports student thinking rather than a cause for teachers to replace it with their own thinking. Asking students to stop and take a moment to observe what has just happened and what is now present on the screen can lead to further requests for students to make connections, reason logically, and draw conclusions. Students can use the technology to then test those conclusions. The power of such interactions, as seen in this study, can allow teachers to support higher-level thinking. The present study gives supporting evidence for the implementation of open technological tasks accompanied by higher-level prompts in the form of requests for justification and the promotion of observation. Such mediated open-ended technological tasks bring together the best of teaching and learning related to open-ended tasks and appropriately supported work with technology (Campe, 2011; McKnight & Mulligan, 2010; Mueller et al., 2010). More research is needed regarding forms of mediation that provide the best support for the implementation of open technological tasks, and the practical application of such techniques when time in the computer laboratory is limited. Such research and implementations of this kind can aid students of all levels in using technology to reach the high level of mathematical understanding that Kaput (1994) envisioned.

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