Prospective Teachers' Knowledge on Middle School Students' Possible Descriptions of Integers

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Article Info	Abstract
	This study aims to reveal prospective middle school mathematics teachers' knowledge regarding the possible misconceptions and/or errors of middle school students in describing integers. In addition to this, it aims to reveal the possible reasons underlying these from the perspective of prospective teachers. Data was qualitatively collected from 38 Turkish prospective teachers by an open-ended questionnaire. The results show that although prospective teachers are familiar with possible misconceptions and/or errors of middle school students in describing integers, each prospective teacher does not have a rich repertoire regarding students' thinking process even though it seems that there is a rich repertoire in total. They did not give sufficient details about the content and pedagogical aspects of the misconceptions and/or errors. They proposed quite general ideas about the misconceptions and/or errors and the reasons for them.

Introduction

The concept of integers about which middle school students and prospective mathematics teachers have problems in conceptualizing (Ercan, 2010; İşgüden, 2008; Köroğlu & Yeşildere, 2004; Smith, 2002; Spang, 2009; Steiner, 2009) is one of the main topics in the middle school mathematics curriculum. It is used in real life while making interpretations regarding temperature, altitude of settlements, position of an elevator among floors in a building, or net value of money etc. (Bennett & Nelson, 2001; Hubbard & Robinson, 1996; Musser, Burger, & Peterson, 2003; NCTM, 2000; O'Daffer, Charles, Cooney, Dossey, & Schielack, 2008) and helps students to understand the construction of the real number system.It is in close relationship with number systems in which it is an extended number system of whole numbers. Levenson (2012) argues that while students learn the number system in school, there are several key transitions which they need to pay attention to "how the 'new' numbers behave differently from previously recognized numbers" (p.211). She further argues that while students learn a new set of numbers, they also need to work on how the previously studied operations are defined with these numbers. In this regard, the topic of integers is required to be constructed giving the importance to the relation of whole numbers, counting numbers, and fractions with integers. Moreover, in addition to making basic operations in integers, other operations such as making transformation among different representation of numbers or making ordering between numbers are essential (MoNE, 2013). In order to help students learn meaningfully, teachers need to pay in-depth attention to student thinking, thus teachers need to become aware of students' existing knowledge and capabilities (Franke & Kazemi, 2001; Levin, Hammer, & Coffey, 2009). As a result, they can design their lessons by taking these into consideration. Research studies show that middle school students' misconceptions and/or errors regarding integers are varied (Ercan, 2010; İşgüden, 2008; Köroğlu & Yeşildere, 2004; Melezoğlu, 2005; Spang, 2009), thus classroom activities should be supported with models, manipulatives (Akyüz, Stephan, & Dixon, 2012; Körükçü, 2008), or technology (Thompson & Dreyfus, 1988).

In available literature, studies conducted with teachers represent examples of activities which support more comprehensible teaching considering students' thinking. For example, Spang's (2009) study shows that elementary students were not able to distinguish between the subtraction sign and the sign of a negative number nor could they distinguish between the plus sign and the sign of a positive number. The study suggested using the "pebbles in a bag" activity to enable students to understand the meaning of integers and the meaning of the signs of operations. By means of the "pebbles in a bag" activity, students give meaning to positive integers having *more* pebbles in the bag, and negative integers having *fewer* pebbles in the bag. Körükçü (2008) suggested that teaching with concrete models help elementary students comprehend integers. Prepared activities using visual manipulatives (counter, number line, and thermometer) helped 6th-grade students to do addition or

subtraction of integers, and to compare integers with each other. Research results show that the "zero principle" also helped them to learn making operations with plus and minus signs. The results report that there was a positive effect of teaching integers with visual materials on elementary students' mathematics achievement in integers, together with operations of integers, and their level of recalling them. According to Akyüz et. al. (2012), teachers' instructional activities have an efficient role in supporting elementary students' understanding of integers. Their research result shows that students have three main images about which they argue for ideas about integers supported by an experienced mathematics teacher. The images are (1) 'situation specific imagery: assets, debts and net worth' which are the informal knowledge of students that need meanings related to integers to be constructed by teachers, (2) 'notational imagery: flexibly structuring space vertically' which supports students' notation imagery using a vertical number line on which assets, debts and net worth are shown; and (3) 'situation-specific to notational imagery: good and bad decisions' which require interpreting the notations of +(+), −(-), +(-), −(+) as good or bad situations. These images are revealed with the teacher's help, and therefore teacher's role is so crucial when designing such a learning trajectory, which is created by supportive contexts.

For prospective teachers, there are very few studies on the thought processes of them regarding integers (Smith, 2002; Steiner, 2009). For example, Smith (2002) prepared a guide for pre-service teachers to teach integers using manipulatives. He points out that counters can be used in integers operations which enable students to learn the concept of integers more tangible. The researcher emphasized that students need to understand "zero principle" before operating integers. According to Steiner (2009), most elementary students just memorize rules about change in signs while operating integers. To make integers more comprehensible for elementary students, the "novel model" which requires to design appropriate examples with integers operations using 'money own', and 'money debt' concepts instead of using positive and negative integers, and to show the examples on a number line, was created by the researcher and it was used with pre-service teachers. Steiner's (2009) research results show that the model has revealed that pre-service teachers have difficulty with set of integers, deciding whether decimals are integers or not, which sets are included by integers, whether a zero is positive or negative, how to model addition of integers with different signs, or how to model subtraction with two negative numbers. In addition, prospective teachers have some misconceptions such as negative integers have the same meaning with zero, zero means nothing existed, or the operation of adding a positive integer to a negative integer when either has a greater absolute value than the other is made subtracting the smaller one from the big one (e.g.: -8+6 is operated like 8-6). Although there are such problems mentioned, the novel model helped pre-service teachers to conceptually understand integers. These studies conducted with teachers represent examples of activities which support more comprehensible teaching considering students' thinking. In addition to studies developing prospective teachers' conceptual understanding of integers as mentioned (Smith, 2002; Steiner, 2009), it is necessary to understand to what extent prospective teachers are able to use this kind of conceptual understanding within student thinking on the basis of students' misconceptions and/or errors and the reasons underlying them.

One of the important aspects of teacher knowledge is pedagogical content knowledge which enables to understand student thinking including an awareness of not only students' misconceptions and/or errors of content but also the leading factors that cause students to have those misconceptions and/or errors (Ball, Thames, & Phelps, 2008). Knowledge of students regarding misconceptions and/or errors suggested by prospective teachers enables them to gain insight into their knowledge of student thinking (Ryan and McCrae, 2006). Moreover, teacher educators can gain in-depth insight into prospective teachers' knowledge regarding how prospective teachers might think when they are asked to suggest probable student thinking patterns. In this regard, examining prospective teachers' knowledge of integers with respect to possible misconceptions or errors of students and reasons underlying the misconceptions or errors help to reveal the strengths and weaknesses in prospective teachers' knowledge of integers and of middle school students' thinking processes of integers. It was examined by means of the following research questions:

1 What do prospective mathematics teachers know about middle school students' misconceptions or errors regarding the definition of integers?
2 What do prospective mathematics teachers know about the possible reasons for misconceptions or errors of students regarding the definition of integers?

In this study, related terms for student thinking are misconception and error. Misconception is "a student conception that produces a systematic pattern of errors" (Smith, diSessa, & Roschelle, 1993, p.119). It includes students' confusions over mathematical concepts. Additionally, "an error is a mistake, slip, blunder, or inaccuracy, and a deviation from accuracy" (Luneta & Makonye, 2010, p.36). In this study, prospective teachers were asked to determine misconceptions and errors as it is not very clear whether the represented confusion is a misconception or not without observing a real classroom environment. In this study, pedagogical content

knowledge specifically referred to the knowledge of pre-service teachers about what misconceptions and/or errors middle grade students might have and why those students have those misconceptions and/or errors when defining integers.

Theoretical Framework

In teacher education programs, one of the goals is to develop prospective teachers for this kind of competence in knowledge and conceptual understanding in mathematics to teach effectively (Kwoen, 2000). In preparation programs, pre-service teachers are educated to teach effectively. Effective teaching needs to be ensured by being competent in knowledge of mathematics and is shaped by conceptual understanding of that content (Kwoen, 2000). Those programs are designed for pre-service teachers to practice the necessary knowledge and learn through content courses, method courses, or training sections (Ball et. al., 2008; Inoue, 2009; Jegede & Taplin, 2000; Nilsson, 2008; Zakaria & Zaini, 2009). Pre-service teachers learn how to teach their future students and acquire the necessary skills to learn how to teach in a systematic way (Morris, Hiebert, & Spitzer, 2009). Accordingly, mathematics teacher education programs seek to provide prospective teachers with knowledge and conceptual understanding in mathematics (Kwoen, 2000).

Understanding what makes learning of a subject easy or difficult in the teaching process is easier if teachers have pedagogical content knowledge (PCK). Pedagogical content knowledge is 'a special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding' (Shulman, 1987, p.8). That includes useful forms of representation, and powerful analogies, illustrations, examples to be comprehended of a subject by students (Shulman, 1986; Shulman, 1987). In other words, an inevitable interaction of teacher and students requires that teachers need knowledge of content and students (KCS); that is "knowledge that combines knowing about students and knowing about mathematics. Teachers must anticipate what students are likely to think and what they will find confusing." (Ball et. al., 2008, p. 401). Misconceptions and/or errors of a mathematical concept make students confused while learning a mathematical content in a meaningful way. Moreover, teachers' knowledge of possible reasons for students' misconceptions and/or errors help them overcome the difficulties that students may experience in organizing their understandings (Ball et. al., 2008; Chick, Pham, Baker, & Cheng, 2006; Shulman, 1986), let teachers implement solutions, (Ball et. al., 2008) be careful for student's misconceptions and/or errors and design their lessons considering the reflections of situations (Shulman, 1986). Specifically, to describe a mathematical concept, teachers' pedagogical content knowledge requires being aware of mathematics classrooms' current situation including cognitive, social, and affective aspects, and monitoring students' improvements on mathematical concepts supported by the knowledge of schemas of the currently learned concepts and concepts beyond learning of mathematics in students' minds (Leikin & Zazkis, 2010). In other words, such knowledge requires teachers to get to know classroom dynamics, know the concept schema of students and understand extended mathematical structure in their minds (Leikin & Zazkis, 2010). In this regard, when teachers are familiar with or anticipate their students' misconceptions and/or errors in describing mathematical concepts, they enable students to construct the concept and make necessary improvements in students' concept image (Leikin & Zazkis, 2010). In addition, providing reasons for misconceptions shows the awareness of teachers about the cognitive, social, and affective aspects of classroom dynamics and knowing the possible reasons for misconceptions and/or errors helps teachers organize student understanding and introduce ways to overcome problematic situations through teaching strategies (Shulman, 1986).

Misconceptions or Errors and their Reasons/Resources for them

Contrary to popular belief, which views that students have no idea about concepts they have not learned in schools, students have explanations created in their minds for the concepts, even if the explanations are not an accepted form in mathematics (Smith, et.al., 1993). It can be said that students have ideas before learning from their teachers because of the fact that students create the concepts themselves (Küçük & Demir, 2009). Considering the previous concepts in students' mind, the role of teachers in teaching concepts may be wondered. According to Küçük and Demir (2009), the role of teachers in this process is to help students in imaging, correcting, and recognizing the relationship between concepts in mathematics taught in an appropriately designed classroom environment. If students are unwilling to make connections between previous concepts in their minds and newly attained knowledge, they tend to do the same with their prior knowledge (Barke, Hazari, & Yibarek, 2009). How a concept which is interpreted and shaped in students' mind with teachers' help transforms into a misconception or error may be a source of curiosity for teachers.

Misconceptions are derived from many sources including having a lack of correct information, understanding the information in a wrong way, not being careful while listening to new information, having distracted attention, not being capable of observing the learning process or creating new personal information (Newton, 2000). In addition, Tekkaya, Çapa, and Yılmaz (2000) conclude that misconceptions result from the following: teachers' lack of knowledge in certain topics, inefficient prior knowledge and having incorrect prejudgment, teaching methods based on rote learning and teacher-centered models, topics in curriculum which are disconnected to each other and to real life, incorrect information in textbooks and lack of revision of the books. Moreover, misconceptions can be hidden in correct answers of careless teachers (Luneta & Makonye, 2010; Smith, et. al., 1993). Students make errors deriving from different sources. According to Drews (2011), errors could be derived from being careless or having lack of awareness in what is done in answers, misinterpretation and lack of knowledge about what a text or symbol expresses. Students may forget the requirements to answer questions or they may really not know how to answer (Drews, 2011; Ryan & William, 2007). Errors of students could also be derived from misconceptions (Drews, 2011; Ryan & William, 2007). Errors can be recognized in students' exam papers, or their discourses used in mathematics lessons (Luneta & Makonye, 2010; Smith, et. al., 1993). Drews (2011) lists the reasons for making errors. According to the researcher, an error may be derived from "lack of knowledge about topics, misconceptions, being careless or not being aware of giving and checking answers, or misinterpretation of symbols or text". Furthermore, the errors might originate from teachers' inappropriate task selection or teachers' assuming that students understand all he/she teaches as it is supposed to be. Although errors seem to be easy to make because of many reasons mentioned above, they might be corrected carrying out the right application at the right time. In this regard, teachers should be capable of recognizing misconceptions and/or errors and their reasons for them in their area of expertise. This study enables to reveal prospective mathematics teachers' knowledge of misconceptions and/or errors related to describing integers and suggesting reasons specific to a mathematical objective of describing integers.

Method

In this study, qualitative research design was used to describe prospective middle grade mathematics teachers' knowledge of students' misconceptions and/or errors of integers. In this study, the case study design was used. "Case study research is a form of qualitative research that endeavors to discover meaning to investigate processes, and to gain insights into and in-depth understanding of an individual, group, or situation" (Lodico, Spaulding, & Voegtle, 2006, p. 269). Case studies are preferred to be used in questions which are searching for the reasons for why and how an event occurs under the least control of the researcher on that event (Yin, 2003). In research studies, cases in which an individual, classroom, or school takes part might be central to the study. Also, an event, an activity or an ongoing process might be identified as cases (Fraenkel & Wallen, 2011). In this study, senior prospective middle school mathematics teachers studying in the department of teacher education program in a public university in Turkey are the case of the study. The researchers aimed to gain in-depth insight into prospective teachers' perceptions regarding how students would think when describing a mathematical concept, specifically integers. Thus, to this end, the study resorted to qualitative research design. Senior prospective middle school mathematics teachers who were studying in the department of middle school mathematics teacher education program in the 2010-2011 spring semester in a public university in Turkey constituted participants of the study. Convenience sampling was used for selecting participants. It enables the researcher to select sample based on time, money, energy, location, or accessibility of respondents, etc. (Merriam, 1998; Fraenkel & Wallen, 2011). The participants were found to be a valuable source of data by the researcher since they were senior students who had taken all their required courses to graduate from middle school mathematics teacher education program.

Context of the Study

The context of the study is the undergraduate middle school mathematics teacher education program, which is a four-year undergraduate program. This is one of the major teacher education programs in Turkey, where the means of instruction is English. This program focuses on higher-order thinking skills in the development of prospective teachers. Graduates of this program are qualified to teach mathematics in middle schools, grades from 5 to 8 in Turkey. In the first two years of this program, prospective teachers complete courses which focus mostly on mathematics and science, and then they complete courses about education.

Prospective teachers learn how to design the teaching and learning process of mathematics during their mathematics teaching methodology courses and practice teaching. The mathematics teaching methods courses are offered in their third year. Related with the topic of this study, number sets, specifically integers for middle

school grade levels are one of the mathematical contents these courses entail. Specifically, using integers in daily life, modeling operations of integers through multiple representations, what middle school students' common difficulties might be, and how to overcome the difficulties middle school students experience in learning integers are discussed in those methodology courses. Each of the mathematics topics is covered in 5 class hours. The course book by Van de Walle, Karp, Karp, & Bay-William (2010) is used in these methods courses to guide prospective teachers in their thinking process. When dealing with each topic, including integers, course instructor supports prospective teachers within the context of discussing content knowledge and pedagogical content knowledge aspects of the mathematics topics covered by the course book. Furthermore, students take two practice teaching courses in which they are given a chance to observe or practice teaching in real classrooms for 14 weeks, observe students in natural lesson environments, use teaching strategies, or learn how school systems function in their last year.

Data Collection

In this study, participants were given a questionnaire and asked to write down their responses for the following open-ended question:

While explaining integers, teacher asked the students: "How can you define integers?"
a) What misconceptions and/or errors may students have about the definition of integers?
b) What might be the reasons for the students' possible misconceptions and/or errors related to the definition of integers?

The questions were prepared to identify prospective teachers' perceptions of middle school students' possible misconceptions regarding the description of integers and their perceptions of the possible reasons underlying these misconceptions of middle school students' definition of integers. Data were collected from 38 prospective teachers in the practice teaching course. Participants were informed that the results of the questions would not be graded and nor would they be reflected to their letter grades received from their courses.

Data Analysis

In this study, codes and themes were prepared based on the available literature (Ercan, 2010; Köroğlu & Yeşildere, 2004; Steiner, 2009; Newton, 2000; Tekkaya et. al., 2000; Işıksal & Çakıroğlu, 2011) and participants' responses. Subsequently, they were reread, re-examined, and categorized by the authors. As mentioned in 'misconceptions or errors and their reasons for/resources of them' part, many researchers explained the sources of misconceptions and/or errors. Because this study specifically focuses on the possible sources of students' misconceptions and/or errors about integers, categories were created with the help of research studies which discuss misconceptions and/or errors in mathematics or in science. In addition to this, a more similar study which focuses on obtaining the categories of possible source of the students' misconceptions and/or errors about fractions (Işıksal & Çakıroğlu, 2011) was guided on the basis of the actual data of the study.

For the categories of prospective teachers' knowledge of students' possible misconceptions and/or errors, studies which focus specifically on integers were used. Research studies show that middle school students experience difficulties in establishing the relationship between the number sets (Ercan, 2010; Köroğlu & Yeşildere, 2004; Steiner, 2009; Işıksal & Çakıroğlu, 2011). Studies revealing students' difficulties of integers via examples and non-examples of integers (Ercan, 2010), students' misconceptions and/or errors of integers during teacher instruction (Köroğlu & Yeşildere, 2004); and studies which reveal prospective teachers' misconceptions and/or errors with integers (Steiner, 2009) guided the categorization. This result reflected the 'misconceptions and/errors regarding number sets' category for this study. This category was elaborated based on middle school students' specific difficulties stated in literature. For example, 'there is no relationship between number sets' (Ercan, 2010) was used as a category when prospective teachers suggested that describing integers is related to the lack of relationship among whole numbers, rational numbers or counting numbers and integers. as Moreover, 'integers are composed of only positive integers' and 'integers are whole numbers' categories are used considering middle school students' thinking (Ercan, 2010; Köroğlu & Yeşildere, 2004; Steiner, 2009). Additionally, when zero is considered to cause problems in learning numbers for middle grade students (Steiner, 2009), the 'misconceptions and/or errors regarding zero' is used in this study. Considering the details of the misconceptions and errors regarding zero based on available literature, zero is the smallest number, zero means nothing, zero is a positive integer, and zero is not an integer categories are used. For the answer of the second research question, a more similar study which focuses on obtaining the categories of the

possible source of the students' misconceptions and/or errors about fractions (Işıksal & Çakıroğlu, 2011) was guided considering the actual data of the study which is related to describing integers. Student related factors (students' comprehension of negative numbers, lack of students' mathematical knowledge about numbers, lack of students' conceptual understanding of integers, students' schema of zero) and teacher related factors (teachers' inaccurate explanations and teachers' instructional approaches) are organized based on related literature.

Researcher's Position

One of the characteristics for all qualitative studies is that researchers are the primary instrument for gathering and analyzing data (Merriam, 1998). All observations and analyses are determined by researchers' worldview, values, and perspectives. The researchers can make arrangements to gather and produce meaningful information from their data. However, they may overlook some particular situations, make mistakes, or biases may stand out. Explanation of the biases and assumptions reveals researchers' expectations and values which influence research studies conducted (Fraenkel & Wallen, 2011). In this study, the researcher made data analysis based on related literature (Ercan, 2010; Köroğlu & Yeşildere, 2004; Steiner, 2009; Newton, 2000; Tekkaya et. al., 2000; Işıksal & Çakıroğlu, 2011) and consulting other researchers. What's more, she described in detail how the research setting was created and what the research findings were.

Validity and Reliability

In this study, internal validity and reliability were measured through peer examination. Peer examination means making critic on research findings together with colleagues (Merriam, 1998). "But such an examination or review can also be conducted by a colleague either familiar with the research or one new to the topic" (MoNE, 2009, p.220). In this study, in order to ensure validity and reliability of data, the researcher reexamined findings with the help of three graduate students from the mathematics education department. The researcher studied with graduate students and the supervisor to scan the data and assess whether the findings were consistent with the data.

Results

The results are presented in two parts: prospective middle school mathematics teachers' views about middle school students' probable misconceptions and/or errors in describing integers and their views about the probable reasons why middle school students have those probable misconceptions   and/or errors in describing integers.

Prospective Middle School Mathematics Teachers' Views about Middle School Students' Misconceptions and/or Errors in Describing Integers

As it is seen in Table 1, some participants suggested more than one possible misconceptions and/or errors. When we consider all the participants, a majority of respondents (61% of the participants) gave an answer for the question. The majority of participants (50% of participants) attributed the possible misconceptions and/or errors that middle school students experienced when describing integers to the misconceptions and/errors regarding number sets. In the light of the result, it can be said that prospective teachers assume that middle school students establish their description of integers on a relationship between other number sets.

Table 1. Prospective teachers' knowledge regarding misconceptions and/or errors of middle grade students in describing integers

	Number of participants who answer the question	% of participants	Number of responses	% of responses
Misconceptions and/or errors regarding zero	10	26	10	29
Misconceptions and/errors regarding number sets	19	50	24	71
Blank responses	13	34	0	-
Total	25	61	34	100

A smaller part of the participants takes into account the misconceptions and/or errors middle school students experienced with zero. It suggests that the number set, rational number, prevents describing integers correctly. These two results indicate that middle school students also tend to describe integers as using number sets where zero is not needed to be specified (e.g.: rational numbers). In sum, zero does not play a crucial role for describing integers of middle grade students from the perspective of prospective teachers; however, the relationship among number sets is essential for describing integers.

Misconceptions and/or Errors regarding Zero in Describing Integers

Participants suggested misconceptions and/or errors of students regarding zero that are accompanied with some examples of participants' views presented in Table 2. According to the prospective teachers, there are four misconceptions and/or errors related to zero: zero is the smallest number, zero means nothing, zero is a positive integer, and zero is not an integer. These suggestions showed that middle school students need to be clarified in the misconceptions and/or errors concerning the zero. Ten prospective teachers (26 % of participants) think that middle school students may have problems in describing integers since they need to have further subject matter knowledge on the concept of zero.

One of the misconceptions and/or errors regarding zero was that students would consider *zero as the smallest number*: Only one participant suggested the idea that "it is thought that the smallest number is zero. Naturally, they find it hard to change this schema." Although the misconception and/or errors not recognized by most of the participants, it is one of the fundamental knowledge to make a link between integers and whole numbers. This suggestion clearly shows that the prospective teacher emphasizes that integers are described based on whole number concept. In this regard, zero is one of the essential components which connects whole numbers to integers. In this regard, zero is one of the essential components which connects whole numbers to integers.

The second misconception and/or error mentioned by one participant was; *zero means nothing*. According to the participant, "If they [middle school students] think that zero is nullity, they may hold the idea that there is nothing smaller than nullity" in the set of integers. Similar to the previous misconception and/or errors, this suggestion draws attention to the zero's critical role between two number sets. Different from the previous one, the participant emphasizes the relationship between counting numbers and integers. That the meaning of zero is highlighted is an important point. While zero is considered as a number in the first misconception and/or error, zero is a number corresponding to an object in the second one. This suggestion indicates that the prospective teacher is aware of the subject-object duality.

Another misconception and/or error which is suggested by only one of the prospective teachers is that middle school students would consider *zero as a positive integer*. This misconception and/or error was suggested by the participant as follows: "While trying to determine the sign of zero, they may place it into the positive category." The sign of zero might be considered as positive, and therefore the definition remains incomplete. It indicates that the definition is made via counting numbers; otherwise there is no need to take attention to the sign of zero while the definition is made via whole numbers. It is seen that the prospective teacher relates integers with the description of counting numbers, negatives of counting numbers and zero.

Finally, the other possible misconception and/or error suggested is *zero is not an integer*. According to the participants (18% of participants), zero might be excluded from the integer set. Seven of the participants suggested the misconceptions and/or errors stating that "they may not include zero." The misconceptions and/or errors of "*zero is not an integer*" was the most frequently suggested category by these participants (7 of participants). Similar to the previous one, this misconception and/or error is concerned with the incomplete definition but it cannot be said clearly what number set integers are built on.

Prospective teachers' first and second proposals of zero category indicated that middle grade students have a tendency to describe integers as whole numbers. Besides, the third proposal of zero categories indicated that integers might be seen as counting numbers. The last misconception and/or error (zero is not an integer) suggested by the participants shows that middle grade students might not have an awareness of the fact that zero is an integer. That misconception and/or error was more significant for prospective teachers as most of the participants suggested it.

Though a variety of answers has been suggested regarding zero including the magnitude of the number, the meaning of the zero, the sign of the zero and the involvement of the number of zero in integer set, prospective teachers' responses indicated that each possible misconceptions and/or errors -except for the last one- was

proposed by only one participant. It is indicated that teachers have knowledge of quite a variety of probable misconceptions and/or errors about zero, however they do not have this kind of knowledge individually as quite few numbers of participants suggested them. As it is seen in Table 1, the number zero does not play a crucial role in describing integers by middle grade students from the perspective of prospective teachers.

Table 2. Prospective teachers' knowledge regarding misconceptions and/or errors regarding zero

Misconceptions and/or errors		Examples of participants' views	# of participants	% of participants
Misconceptions and/or errors regarding zero	zero is the smallest number	*It is thought that the smallest number is zero. Naturally, they have difficulty changing this schema.*	1	2.5
	zero means nothing	*If they think that zero is nullity, they may have an idea that there is nothing smaller than nullity.*	1	2.5
	zero is a positive integer	*While placing zero into the positive or negative category, they may place it in the positive category.*	1	2.5
	zero is not an integer	*They may not include zero.*	7	18
		Total	*10*	*26*

Prospective teachers' reasons for having such kind of thinking might stem from students' descriptions of integers as a combination of whole numbers and the negative of whole numbers and zero. In this regard, when middle school students think that zero is the smallest number, their understanding of negative numbers which is a part of that combination would be limited. Similarly, for the idea that zero is not an integer might indicate that the integer set might be incomplete when this type of consideration occurs. Moreover, when middle school students think that zero is nothing, prospective teachers may think that middle school students connect zero with the meaning of nullity considering counting numbers and they would define integer as counting numbers. Although those kinds of connections are possible for proposing the statements in Table 1, prospective teachers do not give details for how those kinds of misconceptions and/or errors connect with describing integers.

Misconceptions and/or Errors regarding Number Sets

Participants also considered that middle school students might have difficulties in describing integers regarding the relationship between number sets. Half of the participants paid attention to this kind of misconceptions and/or errors (61% of participants). The first misconception and/or error under this category is that according to participants, middle school students would think that *there is no relationship among number sets*. In other words, students would think that number sets are separate from each other. For example, one of the participants suggested that "students may not realize that numbers are a single set. In this category, prospective teachers gave importance to the placement of integers among number sets and relating the number sets each other. When their responses are analyzed, prospective teachers consider that each of the number sets is separate. Therefore, according to the prospective teachers, the failure of the relationship of integers with other number sets is an obstacle for middle grade students when describing integers.

Another misconception and/or error mentioned by the participants was that middle school students would think that *integers are composed of only positive integers*. For example, eleven of the participants (29% of participants) expressed that "they may take into account only the positive integers." This suggestion draws attention to the sign of integers. However, integers also include number zero that has no sign. In this regard, prospective teachers consider that middle school students tend to describe integers as whole numbers or counting numbers.

Finally, another misconception and/or error suggested by the participants was that students would refer *integers to the whole number sets.* For example, one of the participants stated that "*They may define integers as a group of numbers consisting of only 0 and positive integer numbers.*" This suggestion indicates that middle school students think integers are whole numbers.

Table 3. Prospective teachers' knowledge regarding misconceptions and/errors regarding number sets

	Misconceptions and/or errors	Examples of participants' views	# of participants	% of participants
Misconceptions and/errors regarding number sets	There is no relationship between number sets	*They may not regard numbers as separate sets.*	10	26
	Integers are composed of only positive integers	*They may take into account only the positive integers.*	11	29
	Integers are whole numbers	*They may define integers as a group of numbers consisting of only 0 and positive integer numbers.*	9	24
	Blank responses		13	34

Data revealed that prospective teachers could identify seven misconceptions and/or errors (see Table 2 and Table 3) that middle school students might have in describing integers.

Blank Answers

Apart from all of these proposals of participants, about thirty-four percent of the participants cannot suggest any possible misconceptions and/or errors of middle school students. In this regard, prospective teachers have difficulty in student thinking of describing of integers.

Prospective Middle School Mathematics Teachers' Views on the Reasons underlying Middle School Students' Misconceptions and/or Errors in Describing Integers

Participants were asked about the reasons for middle school students' possible misconceptions and/or errors in describing integers. The collected data showed that prospective teachers' views on the reasons underlying middle school students' misconceptions and/or errors in describing integers are classified into two as: student-related factors and teacher-related factors. When we consider all the participants, a majority of respondents (92% of the participants) gave an answer for the question.

Student-related Factors

As seen in Table 4, prospective teachers proposed that if middle school students have problems in understanding negative numbers, in students' mathematical knowledge about numbers, in their schema of zero, and in their conceptual understanding of integers, they will probably experience difficulty while describing integers. More than one-third of prospective teachers (%34 of participants) suggested that when students' prior experiences of negative numbers were problematic, they could not provide accurate descriptions of integers. For example, one of the participants stated that "*It's difficult seeing negative numbers in daily life or it is difficult to use*". Prospective teachers assume that middle school students avoid thinking negative numbers and that affects their descriptions of integers. On one hand, this kind of thinking points out that middle school students tend to think positive numbers which they have been familiar with when they have incomplete understanding of negativity. On the other hand, it points out that describing a mathematical concept is affected by problems of conceptualizing negativity. In this regard, negativity is seen as the major factor which prevents students from describing integers according to most of prospective teachers. Interestingly, a misconception and/or error about negativity is not suggested by the participants but negativity is proposed as a reason by the majority of the participants. Thus, it might be said that prospective teachers' knowledge about misconceptions and/or errors for describing integers is inadequate in terms of negativity.

In addition to this, when students are confused with number sets, or they do not suggest appropriate connection between the sets, %29 of participants suggested that middle school students may have difficulty in describing integers. For example, one of the participants stated that "*If students know the exact connections between integers and other numbers, I think they can answer 'what an integer is'*". Parallel with misconceptions and/errors regarding number sets that are suggested by prospective teachers, the same issue within students' mathematical knowledge about numbers is essential as a reason as well. In this regard, it is clearly concluded

that according to prospective teachers, one of the other main factor is about the relationship between numbers in describing integers.

In addition to these, prospective teachers (13% of participants) suggested that integers may not be correctly defined if students do not develop meaningful understanding of the concept. For example, one of the participants said that "*students do not give meaning to [integers] since it is abstract for them*". Why a concept is not described due to being abstract in the minds of middle grade students may indicate that there is a problem with making sense of the concept. This reason is similar to the firstly suggested reason which states "*It's difficult to see negative numbers in daily life or it is difficult to use them*". However, the reason is not sufficiently clear.

The other problematic point is students' schema of zero. 13% of the participants suggested that an appropriate schema of zero is required to describe integers. They argued that if zero is not known as integers by middle school students, integers may be described without zero. For example, the participant stated that "positive integers are mentioned as 1 and further and negative integers are -1 and further. [In this regard] zero is a separate place in their minds". As similar to the misconceptions and/or errors regarding zero, zero is suggested as a reason for not describing integers within student-related factors. In this regard, consistent in both issues, prospective teachers suggested zero as an example of misconceptions and/or errors and students' schema of zero as a reason for describing integers. In this regard, zero is the other main factor which prevents students from describing integers accurately according to prospective teachers.

In total, 71% of participants suggested student-related factors as the main reason for having misconceptions or errors in describing integers. Prospective teachers' answers for the student-related factors indicated that students' inability to construct related concepts of integers including negative numbers, numbers, integers, and zero inhibits their accurate descriptions of integers. In this regard, most of the prospective teachers considered that as long as concepts related to numbers are comprehended by middle grade students, their integer description will be successfully accomplished.

Teacher-related Factors

The other reasons suggested by the participants as of misconceptions and/or errors by students when describing integers are teacher-related factors including teachers' instructional approaches and inaccurate explanations. According to 16% of the participants, as teachers' way of instruction is related to memorization of definition, middle grade students have misconceptions and/or errors. For example, the participant stated that "The students are forced to memorize the definition [by their teachers]. That also causes a misconception." Even though how memorizing a definition ends up with misconceptions and/or errors is not clear, a possible explanation for this is that key points in definitions might create potential confusions in the absence of conceptual understanding. In this regard, prospective teachers regard memorization as an obstacle to making accurate description of integers.

For the inaccurate explanations, 11% of the participants thought that middle school students may encounter teacher explanations which cause students to misunderstand. The reason was mentioned in, for example, by the participant: "the teacher could have had such an explanation that direct students' improper results". 'the lowest number is zero and there are no numbers less than zero', 'numbers start from zero', or 'you cannot subtract 3 from 2' are given examples for inaccurate explanations of teachers. Similar to the previous suggestion despite the details of the relationship between describing integers and such teacher explanations, it is likely that inaccurate definition statements or instruction on the concept of integers, or examples of number representations are the suggested reasons for not describing integers properly. In this regard, besides the teachers' way of instruction during the course, what teachers say during instruction is also brought to the forefront.

26% of participants suggested teacher-related factors as the other reason for having misconceptions or errors in describing integers. For the majority of these students (16%) suggested the misconceptions stem from instructional approaches. In this section, it is seen that prospective teachers proposed quite general descriptions and give less elaborate answers. While teacher-related factors are suggested as a source of misconceptions, it has not been exemplified by most teachers. Moreover, none of students suggested more than one misconception in this category. Therefore, it suggests that such superficial and general information given by prospective teachers might show their lack of pedagogical content knowledge in the context of describing integers.

Table 4. Possible reasons for having misconceptions and/or errors

Reasons/sources		Examples of participants' views	# of responses	% of responses
Student- related factors	Students' comprehension of negative numbers	*It's difficult seeing negative numbers daily life or it is difficult to use.*	13	34%
	Lack of students' mathematical knowledge about numbers	*If students know the exact connections between integers and other numbers, I think they can answer 'what an integer is'.*	11	29%
	Lack of students' conceptual understanding of integers	*Students do no give meaning to [integers] since it is abstract for students"*	5	13%
	Students' schema of zero	*Positive integers are mentioned as 1 and further and negative integers are -1 and further. [In this regard] zero is a separate place in their minds".*	5	13%
Total # of participants who answer			*27*	*71%*
Teacher-related factors	Teachers' instructional approaches	*The students force making memorize the definition [by their teachers]. That also causes a misconception.*	6	16%
	Teachers' inaccurate explanations	*The teacher could have had such an explanation that [direct] students improper results.*	4	11%
Total # of participants who answer			*10*	*26%*
General (not specific) answers		*Students' mathematical knowledge is important*	6	16%
Blank answer(s)		-	3	8%
# of participants who answer the question		-	35	92%
	Total number of participants		38	100%

General (not specific) answers

16 percent of all student participants put forward fairly common reasons stating as "*Students' mathematical knowledge is important"* and a few of the participants (8% of the participants) did not propose any reason. Similar to the previous category of student-related factors, prospective teachers avoid giving details of their answers which might be related to their PCK.

As shown in the Table 1 to Table 4, prospective teachers made quite a variety of suggestions on misconceptions and/or errors of describing integers and their causes of them. As shown in Table 1 and Table 4 in a parallel way, students may have misconceptions and/or errors about the concept of zero and it is derived from students' schema of zero. Similar to this, prospective teachers considered that middle grade students might have difficulty with number sets because of the lack of students' mathematical knowledge about number sets.

Discussion

Prospective Middle School Mathematics Teachers' Views about Middle School Students' Possible Misconceptions and/or Errors in Describing Integers

Misconceptions and/or Errors regarding the Zero in Describing Integers

One of the misconceptions and/or errors participants suggested are related to the concept of zero. As similar to various research studies that middle grade students tend to have difficulty in zero (Ercan, 2010; Köroğlu & Yeşildere, 2004; Smith, 2002; Steiner, 2009), in this study, prospective teachers supported the idea that zero may be misinterpreted by middle grade students. Although the participants seem to be familiar with the difficulties of "zero is the least number", "zero means nothing", "zero is a positive integer", and "zero is not an integer", each of them is not a commonly proposed misconception and/or error in the current study. A small portion of prospective teachers proposed that the number zero could create problems in describing integers (26 percent of prospective teachers).

As İşgüden (2008) stated, students have difficulty in deciding whether zero is an integer or not. What's more, middle school students were not able to determine the elements of integers, and tended to define zero as a non-integer or included only negative numbers in the integers set. (Köroğlu & Yeşildere, 2004). As Smith's (2002) study mentioned, zero principle help middle school students learn making operations through concrete materials that might also let students understand zero does not always mean there is nothing. For example, counter model provides students seeing zero is a representation of a combination of equal positive and negative numbers. In contrast to Steiner's study in which one of the misconceptions of prospective teachers is related to whether the number zero is positive or negative, it is not found that teachers have such a misconception. Therefore, there is not a problem in those proposed misconceptions   and/or errors in terms of mathematical content knowledge of prospective teachers about zero integer.

Misconceptions and/or Errors regarding Number Sets in Describing Integers

The other suggested misconception is, according to fifty percent of participants, misconceptions and/errors regarding number sets in describing integers. Middle school students might consider that there is no relationship between number sets, integers are whole numbers, and integers are composed of only positive integers. Prospective teachers suggested that when they have misunderstandings of the position of integers among numbers, middle school students would have difficulty to describe it. Although this kind of misconceptions and/or errors is suggested, any example of what kind of a relationship between number sets should exist is not given. Even though available literature supports that middle school students could not interpret integer in a decimal or rational format (Ercan, 2010), such kind of details of the relationship between number sets are not discussed in this study. Which number sets are implied by the prospective teachers are not clear in their responses. In this sense, the content of the proposals of the teachers has been deemed inadequate. In this regard, prospective teachers need to be supported using mathematical communication giving sufficient details to their peers in mathematics teaching method courses.

Moreover, Steiner's (2009) research results showed that prospective teachers have also problems about the connection between numbers such as decimal and integers and which sets are included by integers. As one of the participants stated "*students may not realize that numbers are a single set. They may think that integers, rational numbers, natural numbers, etc. are separate sets.",* it might be said that the relationship between number sets is clear for prospective teachers. However, in given examples, it is also seen that some participants think differently and according to them numbers are separate sets: *"they may not regard numbers as separate sets."* The arguments offered by those participants show that to what extent these numbers have separate structures is considered differently by each prospective teacher. To provide the commonality in content knowledge of the relationship among number sets, prospective teachers need to be supported by rich content of mathematics teaching courses including the relationship among number sets, not only for the middle grades but also for further grades to better understand the whole picture of number sets.

Blank Answers

Accordingly, to some degree, prospective teachers have knowledge about students' misconceptions and/or errors regarding describing integers. However, thirty-nine percent of the teachers had left the question completely empty. The percentage is quite high, which may indicate that teachers need pedagogical content knowledge regarding misconceptions of students. In fact, the possible difficulties of students were suggested by only few participants in almost every category. On the other hand, some of the participants suggested more than one misconception and/or error. In this regard, it might be said that each prospective teacher does not have a rich repertoire regarding students' thinking process even though it seems that there is a rich repertoire in total. As prospective teachers are inexperienced in students' cognitive process and they need to make practice in real

classroom environments to be more experienced (Morris, et. al., 2009; Stylianides & Stylianides, 2010; Inoue, 2009), it would be valuable to have such knowledge before starting the teaching profession. In this regard, what role methods for mathematics teaching courses have for prospective teachers in understanding middle grade students' mistakes, or how prospective teachers can gain experience, or overcome suggested difficulties might be the foci of other studies. Thinking and reflecting about possible difficulties of their future students is very important for prospective teachers in the sense that they can design their lessons considering such background knowledge. The research was primarily focused on describing integers. Prospective teachers' instructional designs regarding difficulties of operations, negative numbers, or comprehending real-life problems in different contexts in relation to integers might be a useful extension for further studies.

Prospective Middle School Mathematics Teachers' Views on the Reasons underlying Middle School Students' Possible Misconceptions and/or Errors in Describing Integers

This study has shown that prospective teachers' knowledge on middle school students' misconceptions and/or errors stem from two reasons: student-related factors and teacher-related factors. In general, the categorization helps us to understand probable reasons for students' difficulties in describing integers. In available literature, it is not possible to reach a specific categorization of integers. In this study, with the categorization, middle grade students' possible reactions to the topic of integers are depicted with the help of teacher candidates' pedagogical concerns.

Student-related Factors

According to prospective teachers, students' comprehension of negative numbers, lack of students' mathematical knowledge about numbers, lack of students' conceptual understanding of integers, and students' schema of zero might be the reason which leads to a failure in describing integers.

Students' Comprehension of Negative Numbers

It is well-known that middle grade students have difficulty in conceptualizing integers (Fischbein, 1987; Ball, 1990). Considering that the acceptance of negative numbers was in the 19th century, difficulties in comprehending negative numbers is rather acceptable for middle grade students. However, teachers' suggestions for the reasons for students' failure in comprehending negative numbers is inefficient. In other words, what kind of comprehension is problematic or what kind of understanding is expected from middle grade students are not clearly explained. Teachers have preferred the concept of "negative numbers" instead of saying "negative integer". This concept is broader than the concept of negative integers. In this context, it is said that prospective teachers need to pay attention to the use of mathematical terminology.

Lack of students' Mathematical Knowledge about Numbers

Specific to describing integers, prospective teachers suggested that middle school students might have difficulty due to lack of mathematical knowledge about numbers. Those prospective teachers who consider that lack of mathematical knowledge about numbers creates difficulties in describing integers might think that integers are described based on other number sets. In other words, those participants pay attention to transforming integers to other numbers. Approximately one-third of the teachers highlight this relationship. In this regard, integers can be defined based on previously learned number sets such as counting numbers, fractions, or whole numbers. However, for middle grade students the obstacle to describing integers was not expressed clearly. Although lack of students' mathematical knowledge about numbers was suggested by prospective teachers for a reason for middle school students' difficulty in describing integers, expressing the relationship between rational numbers and integers is particularly problematic for prospective teachers as well. For example, one of the participants shared his/her experiences about a dialogue with a student during which the student defined integers as "integers are numbers with a denominator which is equal to one". The participant's opinion for this review was that "(students) may think that some of the rational numbers can be expressed as integers". Supporting this view, P15 thinks that "(students) may think that fractions can be considered as integers". Although students in those dialogues think in a partly accurate way, prospective teachers interpreted them as misconceptions and/or errors. In this regard, prospective teachers need to review their knowledge on the relationship between different representations of decimals, rational numbers, or whole numbers. This kind of ideas of prospective teachers may

give insight about their own misconceptions and/or errors (Bal, 2006). Parallel with Işıksal and Çakıroğlu's (2011) study, lack of mathematical knowledge is a suggested source for middle school students' division and multiplication of fractions for the perspective of prospective teachers.

Lack of Students' Conceptual Understanding of Integers

According to prospective teachers, middle school students might also have difficulty in describing integers because of not conceptually understanding integers while they are learning. It has remained as a fairly general statement that has to be cautiously interpreted. Similar to the previous categories, prospective teachers do not give sufficient details about the necessary conceptual understanding of integers for describing integers. Similar to the students' comprehension of negative numbers, this category seems like a general advice for those who have difficulty in accomplishing integers rather than specifically describing integers. Because describing a mathematical concept is not created using the concept to be defined more than once and using the newly learned concept (Zazkis & Leikin, 2008).

Students' Schema of Zero

With regard to students' schema of zero, prospective teachers consider that middle school students' schema of zero creates difficulty in describing integers. Ercan's (2010) study supports this study which suggests that middle school students have confusions as they are not sure about whether zero is an integer or not.

Teacher-related Factors

In addition to the previous categories, prospective teachers considered that students might have difficulties in describing integers because of teachers on the basis of teachers' instructional approaches and teachers' inaccurate explanations. These reasons are important in terms of reflecting teachers' knowledge as a source of students' difficulties. This aspect of the answers may require pedagogical content knowledge of teachers. Specifically, teachers' inaccurate explanations such as 'the lowest number is zero and there are no numbers smaller than zero', 'numbers start from zero', or 'you cannot subtract 3 from 2' indicate insufficient pedagogical content knowledge. Those statements of prospective teachers support that misconceptions can be hidden in correct answers of careless teachers (Luneta & Makonye, 2010; Smith, et. al., 1993). In this study, instructional way of teachers specifically rote memorization was seen as a way of preventing the proper settlement of integers for teachers. This instructional way has been found to be suggested as the only one way. Those statements were not supported by examples. In Işıksal and Çakıroğlu's (2011) study, rote memorization was a source of algorithmically based mistakes. In this regard, it might be said that the function of rote memorization might be a source depending on contents or contexts where it is students' own way of learning strategy or teachers' imposition to students.

Delimitation of the Study

We should note that the findings of this study are limited to the context being studied. Prospective middle school mathematics teachers in one of the public universities in Ankara might not be representative of students who are studying at other universities in Turkey. The data should be interpreted with caution because the data was collected through written responses.

Note

This study is originally derived from the first author's master thesis.

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