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That Was Then…This is Now: Utilizing the History of Mathematics and Dynamic Geometry Software

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That Was Then…This is Now: Utilizing the History of Mathematics and Dynamic Geometry Software

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Abstract

Pre-service mathematics teacher (PST) education often addresses within Geometry Classes how to utilize Dynamic Geometric Software (DGS). Other classes may also incorporate teaching pre-service teachers about the history of mathematics. Although research has documented the use of Dynamic Geometric Software (DGS) in teaching the history of mathematics (HoM) (Zengin, 2018), the focus of this research specifically targets the development of proof for pre-service teachers by utilizing DGS to revisit historical proofs with a modern lens. The findings concur with Fujita et.al. (2010), Zengin (2018), and Conners (2007) work on proof. The novelty of this article was the combination of incorporating the history of mathematics (HoM), dynamic geometry software (DGS), and Toulmin's model of argumentation. A pedagogical approach appeared to emerge: DGS's dynamic nature allowed PSTs to see several examples of a method to provide them with an illustration that may be used in proofs.

Introduction

An essential goal of mathematics pre-service teacher education is to increase their abilities to support future students' understanding of the methods of formal proofs and how to navigate between conjecture and proof. Although research has documented the use of Dynamic Geometric Software (DGS) in teaching the history of mathematics (HoM) (Zengin, 2018), the focus of this research specifically targets the development of proof for pre-service teachers by utilizing DGS to revisit historical proofs with a modern lens. In particular, this article examines whether and to what extent, geometrical constructions using Dynamic Geometry Software (DGS) along with incorporating the history of mathematics (HoM) encourage the understanding of pre-service teacher's understanding of proofs. This article begins with a description of the role of HoM in a pre-service mathematics teacher (PST) education program and then examines a framework used by Toulmin (2003) to analyze pre-service teachers' thought processes as they move from constructions to proofs.

Use of History in the Preparation of Teachers

Incorporating HoM into the curriculum for prospective teachers has many benefits which include limited to showing that mathematics is a human enterprise by presenting real life applications of the mathematics (Gulikers & Blom, 2001; Council of Chief State School Officers and National Governors Association, 2010; Common Core State Standards for Mathematics, 2010; Gönülates, 2004), helping students develop mathematical thinking motivating students to learn, analysing connections to other subjects, and identifying the impact mathematics has on society (Ozdermir, Goktepe, & Kepceoglu, 2012; Gönülates, 2004, Gulikers & Blom, 2001). The history of mathematics can be useful and relevant in the classroom dependent upon how it is integrated into the curriculum (Arcavi, et al., 1982). Teachers can introduce the historical significance behind mathematical topics throughout the course by naming cultural contexts (Barbin, et al., 1995).

Despite positive outcomes of teaching HoM, there are difficulties in fully integrating historical resources into mathematics curriculum. Panasuk and Horton's (2013) large study cites reasons that explain teachers' lack of enthusiasm of including HoM in their secondary classrooms: teachers limited knowledge of HoM, a lack of resources, high stakes testing, and negative dispositions of mathematics. The following section delineates four such challenges. The authors examined how incorporating the history of mathematics (HoM) in a preservice teacher (PST) setting encourages the unifying of students'conjectures with DGS proofs. Each of these challenges will be examined by including literature on both preservice and in-service teachers and its relationship to this study.

Limited Knowledge

Before teachers can develop lessons that include teaching HoM, they need to understand the concepts themselves and see how it can be meaningful to implement this practice into their classroom (Liu, 2003). A lack of HoM knowledge may relate to how teachers were taught mathematics (K-12), the courses they took as a pre-service teacher, and their confidence level of teaching this topic (Gulikers & Blom, 2011). Due to the lack of HoM, teachers often separate the history from their traditional instruction as they may not find it beneficial to their students' learning of mathematics (Panasuk & Horton, 2013). More recent studies conducted with pre-service teachers reveal HoM training plays a major part in changing teacher attitudes towards mathematics (Arcavi et al., 2015; Barbin et al., 1995; Baki and Gürsoyk, 2018; Gulikers & Blom, 2001; Ozdermir et al., 2012). According to Barbin et al. (1995), if teachers understand the history behind the mathematics this will influence how they teach and how students will perceive and understand mathematics. Suggestions on how to increase teachers' knowledge of HoM include: instructing pre-service teachers on how to use HoM in mathematics lessons, studying the mathematics structure and not the lives of the individuals, analyzing pre-service teacher's cognitive and affective levels while working on HoM problems (Baki & Gürsoyk, 2018), and having pre-service teachers examine original resources using a variety of tools to solve problems (Ozdermir et al., 2018). Finding grade appropriate, easy to use, and readily available resources are limited when it comes to the topic of HoM (Panasuk et al., 2013; Barbin et al., 1995). In addition, teachers lack the supporting detailed guidelines on how to use historical resources within their lessons (Gulikers & Blom, 2001). Within this study, preservice teachers were introduced to the HoM concepts by conducting proofs through analyzing the historical perspectives behind those proofs.

Limited Time

Teachers must address a significant number of standards and topics in a short amount of time, causing many to feel they do not have adequate time to add historical references (Avital, 1995; Fried, 2001; Gulikers & Blom, 2001; Panasuk & Horton, 2013). According to Fried (2001), teachers may not only feel that teaching the history of mathematics is time consuming, but also may question its relevance. The focus on modern mathematics and career readiness creates the argument for the use of history in the current curriculum. According to Panasuk and Horton (2013) “Until HoM is included as a practice standard in the mandated curriculum, many teachers would not include it” (p.43). However, the history of mathematics can be used to help “justify, enhance, explain, and encourage distinctly modern subjects and practices” (Fried, 2001, p. 395). Therefore, “if one is a mathematics educator, one must take a genuinely historical approach to the history of mathematics and risk spending time on things irrelevant to the mathematics one has to teach” (Fried, 2001, pp 397-398). According to the MAA (2018, p. 33), “The seeking out and employing of historical mathematical problems in classroom instruction is a rewarding and enriching experience in which mathematics teachers should partake.” Potential ways to include HoM are to require reading about the history of math outside of class or to interweave the history of mathematics within problems and in-class investigations (Avital, 1995). Therefore, teachers should seek out HoM materials to use in the classroom and once they spend time creating lessons, they can reuse and modify these lessons in the future (Gulikers & Blom, 2001). Within this study, preservice teachers learned how they can incorporate HoM within proofs that they could use in future lessons.

Disposition of Mathematics

Data analysis by Panasuk and Horton (2013), uncovered a relationship between a teacher’s enjoyment of teaching and their student’s enjoyment of learning when it came to the topic of HoM. Mathematics educators can make their curriculum more interesting by humanizing the subject through incorporating historical stories of individuals who have changed mathematics (Fried, 2001; Gulikers & Blom, 2001; Panasuk & Horton, 2013). Looking at mathematics through the lens of mathematicians helps students understand the problem-solving process and become more reflective about mathematics (Liu, 2003). By incorporating HoM, Panasuk and Horton (2013) found teachers may influence their students’ enjoyment, increase their self-confidence, and foster greater learning through exploration of the subject matter to provide a perspective that lays a foundation for learning. The HOM may be viewed as a window into the theory of the subject and is likely to provide a non-threatening opportunity for entry learning of mathematics” (p. 38). The following section will describe a model that incorporates HoM with an emphasis on geometric constructions and their proofs. Within this study preservice teachers reviewed mathematicians’ historical proofs to remake them using digital geometric software.

Key Terms Defined

This section defines key terms that are used throughout the remainder of this article.

- Dynamic Geometry Software (DGS)- Interactive software in which the user can create and manipulate geometric constructions, commonly used for geometry content. The affordances of this technology enables users to construct various geometrical figures, develop conjectures and assess the validity of these conjectures by dragging them and maintaining their properties of construction (IGI Global, 2021).

- Examples of DGS software include Desmos or Geogebra. Desmos is a free online kit of software tools which includes a graphing calculator, scientific calculator, and digital classroom activities designed by teachers. Geogebra brings together geometry, algebra, spreadsheets, graphing, statistics and calculus in one free online package.
- Geometric Constructions- a construction of a geometric figure using only straightedge and compass, as originally studied by the ancient Greeks (Wolfram, 2021).
- Preservice Teachers (PSTs)- A student in a period of guided, supervised teaching. The college student is gradually introduced into the teaching role for a particular class by a mentor or cooperating teacher (WVU, n.d.)

A Study of Constructions through Toulmin's Model of Cognitive Unity

To document preservice teachers' movement from conjecture to proof, the authors used Toulmin's (2003) model of cognitive unity. The concept of cognitive unity is defined as the continuity between the process of conjecture production and proof construction (Boreo et al., 2007). To further explore the concept, Pedemonte (2007) provided evidence of cognitive unity in a series of teaching experiments. He termed this notion as *structural continuity* to give more precision and detail to the concepts of argumentation and mathematical proof.

The authors tried to give evidence of when PSTs were able to unite (or not unite) their conjectures with their proof constructions. Research suggests that geometry students are more apt to have a conceptual understanding of proof if they could engage in argumentation exercises when being led to the forming of conjectures rather than merely reading and following pre-prepared proofs (Fujita et al., 2010; Mariotti, 2000). Although, not always clear to students, geometrical constructions may provide them with opportunities to form conjectures and consider why their constructions work. The authors used a theoretical model devised by Toulmin (2003), found in Figure 1 that has been used numerous times to successfully analyze students' mathematical argumentations and writing proofs (Hoyles & Kuchermann, 2001; Pedemonte, 2007).

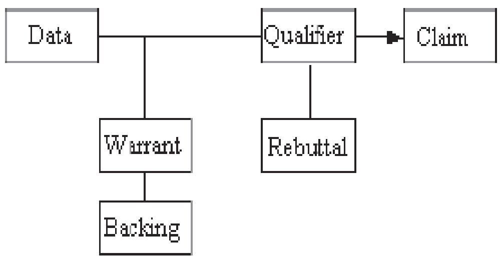


Figure 1. Toulmin's Model of Argumentation

The Toulmin (2003) model breaks an argument down into six main parts:

- *Claim*: assertion one wishes to prove.
- *Data*: evidence presented in support or rationale for the claim.

- *Warrant:* the underlying connection between the claim and evidence, or why the evidence supports the claim.
- *Backing:* tells audience why the warrant is a rational one. In scholarly essays, the warrant and backing would be the areas most supported by factual evidence to support the legitimacy of their assertion. In causal arguments, the warrant and backing are often taken for granted.
- *Counterargument/ Rebuttal:* addresses potential objections to the claim.
- *Qualifier*: additions to the claim that add nuance and specificity to its assumption, helping to counter rebuttals (Wright, 2012).

Similar to Fujita et al.'s (2010) teaching experiments with advanced secondary students, the authors utilized Toulmin's model to consider the cognitive unity by specifying in their constructions 'data', 'claims' or 'warrants.' Boero et al. (2007) proposed the construct of "cognitive unity" of theorems to explain possible reasons for the success or failure of students in writing deductive proofs stating that there is cognitive unity when the argumentations used to produce and validate a conjecture help to construct these proofs. Otherwise, there is a cognitive rupture.

Within this study, the authors explored pre-service teachers' views of how they themselves view geometric constructions and if the use of historical documents (i.e. Euclid's axioms, and an Archimedean proof) and the application of dynamic geometry software (Desmos, Geogebra, Geometer's Sketchpad) would assist them and their future students. The authors utilized this sequence, not to suggest that the elements must be in one and only one prescribed way, instead, it suggests that there may be multiple paths from conjectures to proofs.
The following research questions framed this study:

1. What are the views of pre-service mathematics teachers on the teaching of constructions?
2. What are the views of pre-service mathematics teachers of teaching constructions through historical documents?
3. Is Toulmin's framework a helpful device in preparing preservice teachers in the teaching of proof?
4. How does using dynamic geometry software with the history of math influence the connection between constructions and developing an understanding of proof?

Method

Participants

Participants within this study included 12 preservice teachers (4 males, 8 females) that had completed 40 semester hours of college mathematics courses and were currently enrolled in one of the author's secondary mathematics methods class. The University's Institutional Review Board granted permission for the author to collect data pertaining to this study from students who volunteered to serve as participants. Students were permitted to exit the study at any point during the course. Grades were not impacted by student participation. At this point in the program, students had completed two geometry courses: Advanced Concepts in Geometry and Euclidean Geometry.

Instruments

The main sources of data for the study included: an open-ended 4 question survey, artifacts of student constructions, and audio and visual transcripts of PST's diagrams while working on advanced proofs. Multiple pieces of evidence were collected from pre-service teachers over the semester course to triangulate data sources that represented their understanding of proofs using DGS. Collecting multiple pieces of evidence ensured transferability and validity (Creswell & Miller, 2000).

Open-Ended 4 Question Survey

Pre-service teachers responded to the four-question survey before and after participating in tasks that encouraged them to support secondary geometry student's writing of proofs. The following questions were asked of PSTs:

Before Construction Tasks:

1. What do you believe is the purpose of introducing geometric constructions in a high school geometry course?
2. What contributions can the history of mathematics bring to your classroom?

Following the Construction Tasks:

3. What was your experience in using DGS in constructions?
4. What method of teaching proofs do you consider most effective in teaching students to write proofs (DGS, straight edge and ruler, or the use of original historical sources)?

Artifacts of Student Constructions

The following description of the pre-service teacher assignment attempted to connect historical documents and modern technology. PSTs were asked to construct the following figures from the Common Core State Standards using both DGS and a compass and straightedge:

> Make geometric constructions: CCSS.MATH.CONTENT.HSG.CO.D.12: *copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line*.

Audio and Visual Transcripts of PST's Advanced Proof Diagrams

PSTs were then asked to supply information using Toulmin's graphic of argumentation. The constructions all used Euclid's first three axioms:

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To draw a circle with any center and radius.

PSTs were recorded while working on these proof diagrams for an audio transcription to document their thinking. After constructions occurred, PST's were asked to describe in writing how they constructed the figures in relation to how they were solved historically and reflected on their methods and tools. They then verbally discussed the value of their constructions with DGS or other methods used. Following student constructions and explanations, the author led students in a class discussion connecting DGS and straight edge and compass constructions. PSTs paper constructions from this activity became classroom artifacts.

Data Analysis

The authors triangulated the data by analyzing PST surveys, classroom artifacts, and proof diagrams searching for similarities in methods and descriptions used. These similarities helped create themes that existed among the PSTs. The process of horizontalization; or laying out the data giving it equal weight to help cluster the data into groups and themes, helped the authors organize the data leading to a narrower focus (Merriam, 2002). Inductive reasoning was used by searching for categories among the data and relationships between them, leading to relationships and patterns that either supported the data, or added to new conclusions to the data and research questions (Merriam, 2009).

In order to address research questions 1 and 2, the authors used content analysis to examine the qualitative data based on pre-service teachers' responses to post- activity surveys. First, the authors read the data multiple times resulting in a set of initial themes. Next, the authors conducted a process of unitizing or chunking text into units of analysis. The authors then entered the open coding stage which consisted of classifying the chunks of text. The coded data was then analyzed both quantitatively and qualitatively (Miles and Huberman, 1984).

In order to address research questions 3 and 4, the authors analyzed PSTs interpretation and implementation of Toulmin's model of argumentation, the authors adopted an interpretation from Krummheuer's work (Krummheuer, 2000). From the audio recording of the class, transcriptions were developed. Figures created by PSTs were then attached to the audio recording of each set of proofs. These transcripts were then augmented with diagrams of various parts of the proofs.

Validity and Reliability

Member checks occurred by sending participants (PSTs) a copy of their individual assignments with feedback and asking them to verify the accuracy of the content. Throughout this entire process, the authors discussed the progression of the study and emerging findings with tentative interpretations with one outside mathematics faculty member. The researches continued to be reflexive to ensure trustworthiness by engaging in critical self-reflection regarding my assumptions, biases, and the relationship to the study (which could have affected the results). The authors also tried to provide enough description of the data so readers would be able to determine the extent of the situation and research context (Merriam, 2002).

Results and Discussion

Survey Results

Within the survey, preservice teachers were asked their perspectives on the purpose of geometric constructions. The following themes emerged: constructions aid in visualization, increase engagement, require student to justify reasoning, and build on prior understanding. Some teachers believed that constructions serve to aid in visualization and building understanding within mathematics (n= 4). PSTs expressed that visually, constructions allow their students to see the problem in a different way, providing an extra layer of conceptual understanding to help make sense of the problem. PSTs felt that following a step-by-step process allowed their students to understand the steps of proof and the rationale for each step within the proof. Students are more apt to have a conceptual understanding of proof if they engage in argumentation exercises through constructions (Mariotti, 2000; Fujita & Kunimune, 2010). Preservice teachers felt that student understanding can be expanded upon with constructions by building a foundation to help make connections among theorems, geometric figures, and understand where the proof comes from. This supports research by Mariotti (2000) and Fujita and Kunimune (2010) which states geometrical constructions may provide students opportunities in which they can form conjectures and consider why their constructions work.

Preservice teachers felt that knowledge of theorems can help their students justify conclusions and constructions by understanding constructions in a historical context. Few students discussed how constructions can encourage engagement through hands-on activities, showing counter-examples and using tools to practice mathematical precision. Incorporating history into mathematics such as constructions and proofs can be used to help "justify, enhance, explain, and encourage distinctly modern subjects and practices" (Kragh, 1987). When the pre-service teachers in this study were asked what role the history of mathematics would play in their future classrooms, they had a variety of responses. PSTs expressed they would use history to teach mathematics by incorporating it as supplemental material, to help students better understand relationships, use historical tools, to see the evolution of mathematics, and to highlight other cultures.

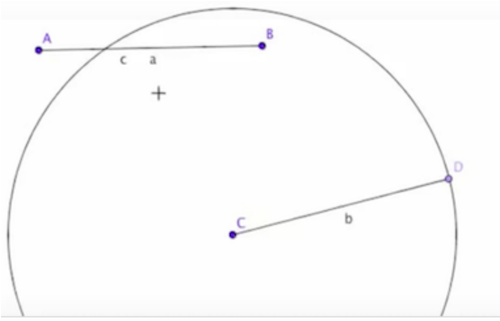
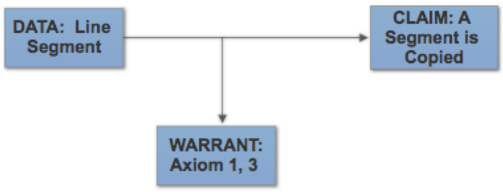
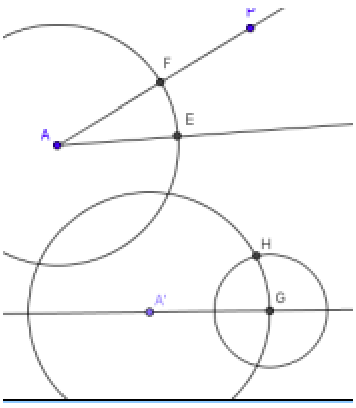
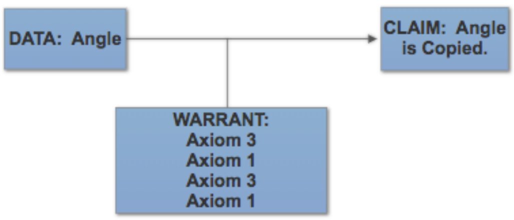
Some preservice teachers expressed that they would incorporate history when teaching mathematics to help students understand the evolution of mathematics and mathematical relationships (n= 3). Preservice teachers also stressed that they wanted to show how math has evolved over time by teaching how math was useful in the past and how it is also useful today (n=4). This corresponds with literature that states teaching HoM can show that mathematics is a human product by presenting real life applications of the mathematics (Gönülates, 2004; Gulikers & Blom, 2001). To help understand mathematical relationships, preservice teachers expressed they would share how formulas/proofs were derived by mathematicians and compare this to how we use them today as understanding mathematical relationships would help students develop mathematical thinking to make connections to other subjects and identify the impact mathematics has on society (Liu, 2003; Ozdermir et al., Gulikers & Blom 2001).

Few PSTs focused on teaching the tools that were used historically (i.e. compass and straightedge vs. technology) to help understand constructions, while one PST felt the history of math would only serve as a supplement for students who needed extra support or explanations to deepen their understanding. An interesting point made by one PST identified using the history of math as a way to highlight various cultures in the classroom and emphasize how mathematics has affected everyday life through this diverse perspective. Incorporating the cultural perspective when teaching mathematics is one that is recommended by Barbin et al. (1995) and Gulikers and Bloom (2001).

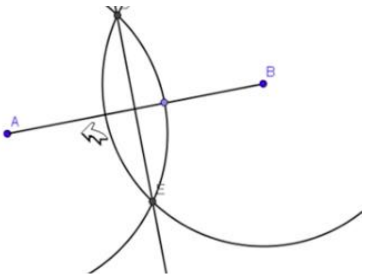
Artifact Results

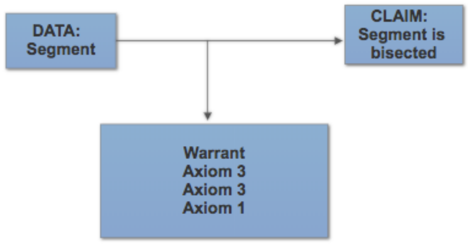
The constructions students were asked to prove using Geogebra or Desmos were found within the Common Core Geometry (GHSS 1) and were among Euclid's basic constructions. The stipulation that PSTs only use the line segment and circle tool related to Greek geometers' constraints of using a straightedge and collapsible compass. In addition to completing constructions, PSTs were asked to complete a diagram similar to Toulmin's in their in-class activities. Table 1 was developed as constructions by PSTs occurred in-class and connections were made to summarize their constructions. This table shows a side by side comparison between constructions and the explanation of arguments and serves as a bridge between constructions and proofs.

Table 1. Summary of Constructions and Diagrams

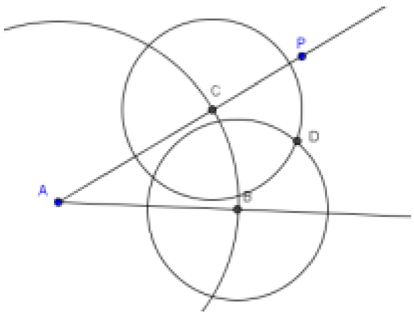
Classical Constructions in Geometry	Diagram of Arguments of Constructions by PSTs
Copying a segment 	
Copying an angle 	

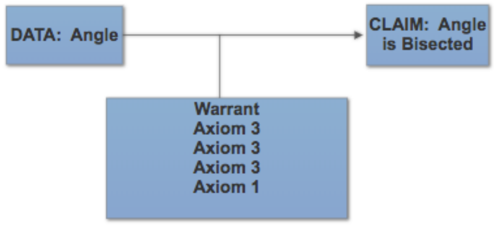
Bisecting a segment



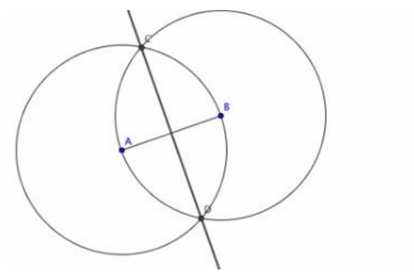


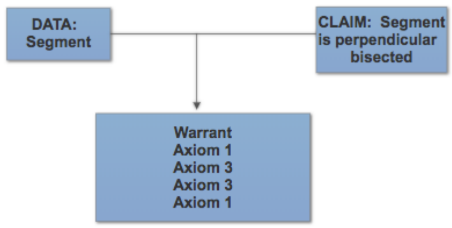
Bisecting an angle



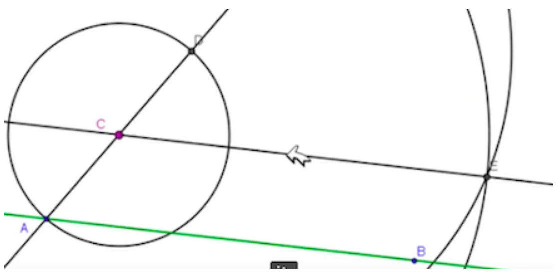


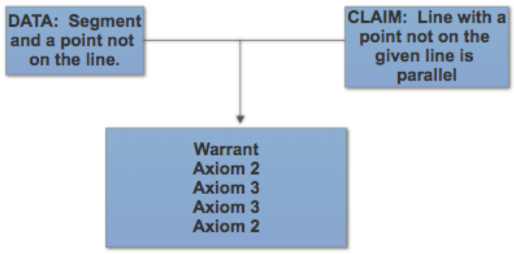
Perpendicular bisector of a line segment





A line parallel to a given line through a point not on the line.





Transcripts of students who have never diagrammed constructions before commented on the helpfulness of using Toulmin's model of argumentation.

PST 1: Wow...I like the flowchart method.

PST 3: It helps me to "see" the proof.

PST1: I do like this. But there were only three axioms. What do you do when there are so much more?

Author: Do you think you would use this method in your classroom and for what type of proofs would you use this?

PST 2: Definitely. I would use it when proving SAS, SSS, ASA theorems because they are usually the first proofs that you need to show to students.

From PST comments on this activity, authors found that cross-comparisons of constructions with Toulmin's method of argumentation were helpful in understanding formal proof.

Tracing the history of the Trisection of an Angle problem provided a task that integrated a historical document with dynamic geometry software (DGS). Through an analysis of preservice teachers' data, warrants, and claims throughout their proofs, the authors were provided with an understanding of PSTs conceptions of proof and how they may create learning opportunities for their future students. The diagram below (see Figure 2) was constructed through recordings of preservice teachers work following the use of applets found on the MAA's online publication, *Convergence,* (MAA, 2018). PSTs' arguments were videotaped and analyzed in terms of data, claims, and warrants/backings. The twelve PSTs, each with a similar background in content and pedagogical experience, used large posters to depict their statements and justifications, they then used Toulmin's graphical approach to show arguments through data, warrants/backing, and claims. PSTs used two versions of Archimedes' proof (MAA, 2018). The results of their arguments follow each diagram.

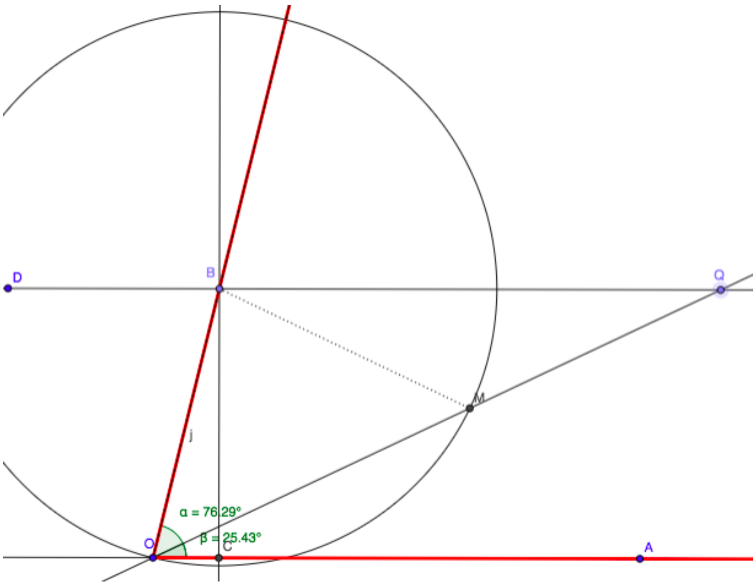


Figure 2. Applet Created by Keith Dreiling

Upon viewing the applet, PSTs were then asked to create a flowchart of a proof following the "data/warrant/claim" convention (See Figure 3). After demonstrating the proof using the Geogebra applet (See Figure 4), the author asked where in the demonstration the marked straightedge was used. PSTs were all able to identify where in the proof this occurred.

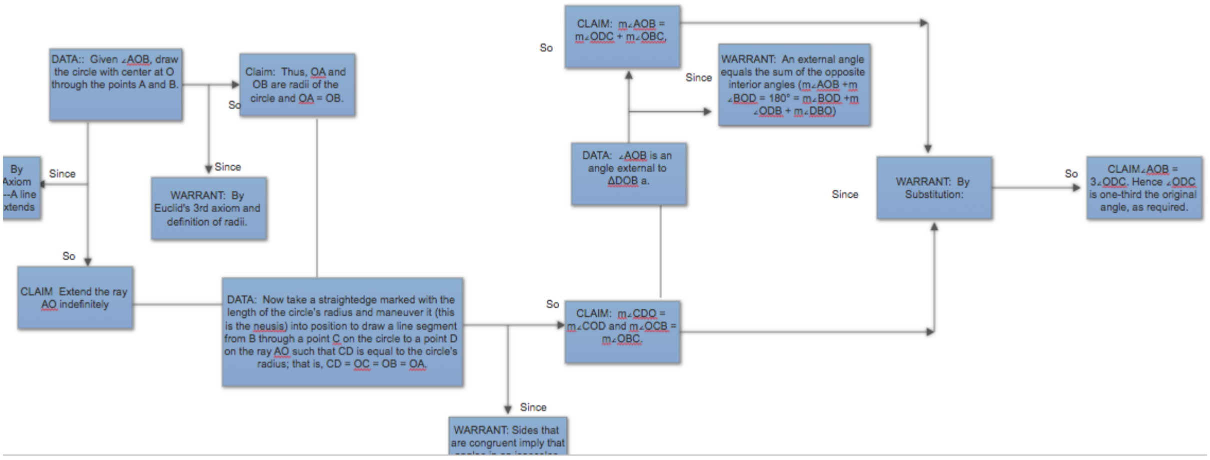


Figure 3. Argumentation in Trisection of Angle with a Marked Straight Edge

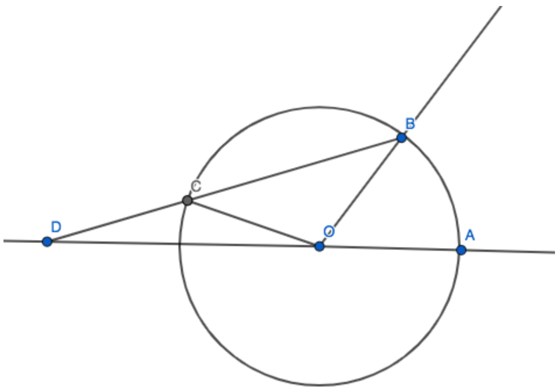


Figure 4. Archimedean Trisection of Angle Using Straightedge

A discussion followed with nine students claiming that the flowchart and applet helped them to not only understand the proof, but also felt that they understood how statements were connected "I can go from the flowchart to writing a proof about this in narrative form. Three students felt that flowcharting was "Way too confusing. I honestly don't understand it. At times it was hard to see how and where to place the lines because some claims then became data statements."

The following excerpt of student dialog demonstrated their confusion:

PST 3: But I don't see how a claim (something you "proved") can then turn out to be data and can help to make another statement. We wanted to separate the lines.

R: Think about a time in the history of mathematics where a lemma or another proof was then able to help prove another theorem. Do all lines need to imply that they flow from each other?

PST2: No, but it would look better.

R: It is not about looks…(students laugh).

PST3: So, then we can have different flowcharts.

R: Under what circumstances do you think that is possible?

PST1: Do all the lines have to be connected?

PST5: I would never have guessed to draw a parallel line.

A second team of students with similar background knowledge in geometry approached the angle trisection proof with a marked segment using an extended line. Without a diagram, students did not recognize the need for extending segment OA. They did recognize where in the proof that a marked segment was inserted and also realized the need for using the exterior angle theorem and substitution properties.

All PSTs mentioned that they could not extend a line segment nor create a parallel line. They all knew that these extensions were possible, yet they did not "think to do them" (PST1). Similar to PSTs who diagrammed Proof 1, PSTs who diagrammed Proof 2 also struggled when they needed to diagram CLAIM statements that then became DATA statements. As a group, students decided to use lines without arrows to connect data and claims that not directly connected. They used arrows to show that some claims led directly to data.

PST8: I wish that I learned this when I was in high school. I like the "so" and "since". I can write a two-column proof and narrative much easier. Yes, Geogebra helped. I like this way of integrating history. It makes it less daunting.

Data obtained from the preservice teachers within this study, revealed that Dynamic Geometry Software supported them in following the historical development of mathematical concepts and methods that can be applied to their future classrooms. For example, during the process of exploring the Euclidean constructions and the trisection of an angle using mechanical methods (Figures 3 and 4), the PSTs not only appreciated history but also examined how they would teach the content to their classes noting the restrictions of a collapsible compass and unmarked straightedges. These results confirm the work of Arbain and Shukor (2015) and Zengin's (2018) which stated that GeoGebra increases students' interest and visualization in mathematics.

In addition to the contribution of Dynamic Geometry Software to the learning of constructions, proofs, and the history of mathematics, the authors found that pre-service teachers appreciated the visual nature of Toulmin's model as a scaffolding device in teaching constructions and proofs. Like Fujita et.al., (2010) the authors also found that using flow-chart proofs with open-ended problems support students' development of a structural understanding of proofs by giving them a range of opportunities to connect proof assumptions with conclusions. The implication is that such scaffolds are useful to support PSTs understanding of introductory mathematical proofs.

Conclusion

The findings in this study concur with Fujita et.al. (2010), Zengin (2018), and Conners (2007) work on proof. The novelty of this article was the combination of incorporating the history of mathematics (HoM), dynamic geometry software (DGS), *and* Toulmin's model of argumentation. Within this study, a pedagogical approach appeared to emerge: DGS's dynamic nature allowed PSTs to see several examples of a method and to provide them with an illustration that may be used in proofs. PSTs not only used DGS, but also performed these procedures on paper with a marked straightedge and compass to experience the actual methods used in antiquity. Using Toulmin's model of argumentation served as a way to scaffold PSTs constructions. It may be interesting to investigate how in-service teachers and their students perceive the incorporation of HoM and DGS with Toulmin's model of argumentation in a high school setting.

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