Forty Years of Mathematics Education: 1980-2019

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Abstract

The purpose of this study is to establish the evolution and expose the trends of research in mathematics education between 1980 and 2019. The bibliometric analysis of the articles in Web of Science database indicated four-clustered structure. The first cluster covers the items related to the theoretical framework of mathematics education whereas the second cluster has the terms defining the methods for effective mathematics instruction. The third cluster includes the concepts interrelated to mathematics education while the fourth cluster encloses the studies about international mathematics assessments. The earlier studies look mathematics education mostly in students’ perspective and investigates generalization, restructuring, interiorization and representation. Between 1995 and 2010, curriculum and teacher-related factors were dominant in mathematics education studies. After 2010, the articles investigated specific topics and carried the traces from all stakeholders in mathematics education. The investigation on the trends of mathematics education would provide gain insight about the areas that need more research, contribute to the researchers, teachers, students and policy makers in this field and light the way for further studies.

Introduction

Mathematics has been important school subject for students that covers understanding basic concepts, becoming fluent in operations, practicing strategic knowledge, reasoning clearly and flexibly, and maintaining a positive outlook toward mathematics (National Research Council, 2001). With its fundamental nature, mathematics provides new glasses to the individuals through which they would question, see and imagine alternative possibilities (Ernest et al., 2016). In this manner, mathematics education has powerful effects in shaping the society. For this reason, it seems important to examine the studies on mathematics education, to determine the efforts for change, and to emphasize the studies that direct the school mathematics. In this study, we focused on the important developments in mathematics education after 1980.

In 1980, National Council of Teachers of Mathematics (NCTM) indicated recommendations in order to commence a decade of action in mathematics education and these recommendations focused on the importance of problem solving, the definitions of basic mathematical skills, use of technological tools such as calculators and computers, application of the effective and efficient standards to mathematics teaching, use of different
evaluation methods, the development of flexible curriculum accommodating the divers need of the learners, high level of professionalization, the emphasis on the importance of mathematical understanding for individuals and society (NCTM, 1980). These recommendations led to the emergence of new movements in which problem solving was emphasized in school mathematics curricula. At this time, new cognitive perspectives on mathematical concepts were developed and problem-solving processes were integrated in different mathematical contents. However, especially, the teachers faced with major difficulties during implementation. One of the main reasons is the lack of the linkage between the existing mathematics curricula and the expectations (Clements & Ellerton, 1996). In these years, “mathematics for all” turned out an essential principle and the goal of mathematics education with educational system due to the social and technological changes. The popularity of this principle has shown an increase since the 1980s and the idea of the equal opportunity for mathematics teaching for everybody was adopted in developing countries (Schubring 2015).

In 1990s, many studies were designed to integrate problem posing and problem solving approaches into mathematics curriculum (Fong, 1994; Foong, 1994). The strategy instruction in problem solving was emphasized and strategy instruction in solving word problems appeared in 1990s (Woodward, 2004). During these years, it is also possible to see the traces of constructivism on mathematics education so that the learners construct new knowledge actively by creating new schemas or reorganizing the existing schemas (Inglis & Foster, 2018). The researchers saw problem solving as one of the basis of constructivism that provided a number of key elements (Confrey & Kazak, 2006). Later, NCTM (1989) standards supported the attempts on excellence in mathematics education by defining curriculum and evaluation standards, emphasizing the conceptual understanding and focusing on problem solving. In late 1990s, reform-based mathematics methods and curricula were investigated (Schoenfeld, 2002). In these years, one of the main determinants of the studies in mathematics education was the NCTM principles and standards for school mathematics stressing the posing of questions, looking for different types of representations, and presenting different arguments during their interaction with mathematical tasks (NCTM, 2000). Hence, there was a shift in mathematical education studies from cognitive and information processing framework to constructivist orientation in 2000s (Woodward, 2004).

In 2010s, the studies focused on different perspectives such as technology, mathematics assessment and mathematical learning disability etc. in mathematics education. The use of the technological tools in mathematics education also provided diverse pathways for the students to construct mathematical knowledge (Drijvers et al., 2010). However, these tools do not ensure the development of collaborative approaches to teaching and learning because the role of teachers, design of the mathematical tasks, features of the learning environment and characteristics of students have crucial role (Geiger, Faragher, & Goos, 2010; Swan, 2007). Especially in the last decade, technology has also been a popular issue in mathematics education (Chuang, 2013; Cullen, Hertel, & Nickels, 2020; Drijvers, 2015; Garrett, 2014; Harrison & Lee, 2018; Kier, & Khalil, 2018; McCulloch et al., 2018; Thomas & Hong, 2013; Trouche & Drijvers, 2010). Many studies have been conducted especially in 1980s and 1990s to measure mathematical performance either by developing achievement tests or by administering tools for predicting mathematics achievement (Cummings & Hoover, 1985; Marks, 1990; Parmar, Frazita, & Cawley, 1996; Wilson, Suriyawongse, & Moore, 1988). However, recent studies thrilled from large-scale assessments and computer based tests (Shelley & Yildirim, 2013). The studies not only
spanned special issues such as diagnostic models, diagnostic tools and multilingual assessment but also investigated disadvantaged groups such as students with special needs and refugee students (Attar, Blom, & Le Pichon, 2020; Brendefur et al., 2018; Peltier et al., 2020; Wu, Wu, et al., 2020).

In 2010s, mathematics education studies also focused on different methods, approaches and trainings for different mathematical learning disabilities (Göransson, Hellblom-Thibblin, & Axdorph, 2016; Herold et al., 2019; Hinton, Strozier, & Flores, 2014; Wu, Shen, et al., 2020; Zhang et al., 2019). As emphasized in the topics of mathematics education related studies given so far, there is also no clear frontier among the related disciplines (Jiménez-Fanjul, Maz-Machado, & Bracho-López, 2013). In the literature review, although it is seen that the number of studies on mathematics education in recent years has increased in number and especially deals with different special issues, the main motivation of this study is to reveal the change of this development over the years. The purpose of this research is to establish the evolution and expose the trends of mathematics education related studies between 1980 and 2019. The investigation on the trends of mathematics education would provide gain insight about the areas that need more research, contribute to the researchers, teachers, students and policy makers in this field and light the way for future studies.

**Method**

The current study investigated the trends in mathematics education by focusing on the articles published during the past 40 years. In recent years, powerful methods have been developed to find and analyze research in literature and to establish a system for literature analysis (Drijvers, Grauwin & Trouche, 2020). The bibliometric analysis was used for the statistical evaluation of articles, figuring out the trends in mathematics education related studies by compromising the co-occurrence analysis of keywords (Cheng et al., 2014). Moreover, it encloses the publication outputs, cooperation among countries, cluster analysis, and evolution of research (Song & Wang, 2020). In short, the bibliometric analysis techniques help to apply the research, determine the interest and reveal the relationships.

**Research Design**

The research design of the study, as shown in Figure 1, covers the exploration, visualization, identification and verification phases that provides a path from searching the related articles to characterization of clusters created by the terms based on their occurrence.

![Research Design of the Current Study](image-url)
Data Collection

In this study, all the articles in Web of Science (WoS) core collection database published between 1980 and 2019 were investigated. The language of publication was set as English and the indexing of the journals was defined as social science citation index (SSCI). Finally, 1021 articles were obtained for further analysis. The details about the exploration phase is given in Table 1.

Table 1. Exploration Details

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data source</td>
<td>Web of Science</td>
</tr>
<tr>
<td>Search terms</td>
<td>“mathematics education”, “maths education” or “math education”</td>
</tr>
<tr>
<td>Citation index</td>
<td>SSCI</td>
</tr>
<tr>
<td>Publication period</td>
<td>January 1980 to December 2019</td>
</tr>
<tr>
<td>Document type</td>
<td>Article</td>
</tr>
<tr>
<td>Language</td>
<td>English</td>
</tr>
<tr>
<td>Number of articles</td>
<td>1021</td>
</tr>
</tbody>
</table>

For sake of simplicity, the publications were categorized in eight groups (each group in periods of five years) depending on their publications years. Based on the WoS records of the articles, keyword-, country- and cluster-based analysis were carried out to identify the general trend and provide evidences to reveal the structure of mathematics education related studies.

Data Analysis

The bibliometric analysis was conducted and VOSviewer software was used to provide statistical information about the most frequent items, links among items and the clusters of items (Van Eck & Waltman, 2010). This software is a tool for visualizing the distance-based bibliometric maps such as network map (covering the related terms, their occurrences and the clusters formed by these terms), overlay map (holding color bar to map scores with colors based on different criteria), and density map (illustrating the frequency of each item occurring within the corpus) as indicated in Van Eck and Waltman (2020). The outputs indicated the related terms, their occurrences and the clusters. In short, we first obtained the items, distinguished the links among items, created the clusters with items and links combinations, assigned the items into clusters and then created, visualized, explored and interpreted the bibliometric maps. This process characterizes the visualization phase of the study (see Figure 1).

In identification phase of the study, the general characteristics of the items were considered in order to entitle the networks. Afterwards, we interpreted the maps that were visualized based on the occurrence of the items in the descriptors of the article (title, keywords and abstract). In verification phase, we investigated clues and tried to support our findings of bibliometric analysis with the content of the articles. In this study, the outputs of the bibliometric analysis and the investigations on the related articles would be interpreted in relation to each other.
for establishing the evolution and exposing the trends of mathematics education studies between 1980 and 2019.

Results

This section presents the findings of bibliometric analysis including the identification of biometrical clusters and the characterization of the source documents such as publication years, publication numbers, keywords and countries.

Exploration

The exploration step refers to the investigation of articles related to mathematics education in WoS database. We searched the terms “mathematics education”, “maths education” or “math education” in the title and keyword section. In the first group of analysis, we focused on the trends in the number of mathematics education related articles. Figure 2 provides information regarding the change in the number of articles between 1980 and 2019.

![Figure 2. Trends in the Number of Mathematics Education related Articles between 1980 and 2019](image)

In Figure 2, the increase rates of publications between 1980 and 1999 were low but the number of publications was almost doubled in each of the five-years from 2000 to 2014. The above figure points out increasing trend in the number of articles over the years.

In the second group analysis, author keywords of the articles were investigated. The keyword analysis was conducted to find the highly occurring keywords and determine the most emphasized concepts. The results were presented in Table 2. The most occurring keyword had been mathematics education by far ahead but this leading word removed from top keywords since it was thought to have little determining effect. The first three highly occurring keywords are science education, reform in mathematics education and professional development. Moreover, the keywords relate to teachers such as teacher education, teacher knowledge, teacher beliefs and teacher learning are appeared in the list. These keywords reflect the studies between teacher and mathematics education.
On the other hand, the keywords such as curriculum, equity, assessment, educational policy mostly refer to education principles. The other keywords also give information about the focused research types, grade level, mathematical domain etc. in studies. This analysis provides to gain insight about the highly related terms regarding mathematics education. However, it is believed that a timeline analysis could be more informative to reflect the trend. In this analysis, the top 10 keywords were determined in each of the five years period and they were placed on the timeline. Since keywords did not exist within the articles published between 1980 and 1990, these words could not be shown for this time range. The result of timeline analysis is given in Figure 3.
concepts such as authenticity, conceptual change, generalization, schematic representation, pragmatic and semantic problem solving. However, the studies between 1995 and 2010 focused on more comprehensive terms such as reform in mathematics education, curriculum and professional development. Between 1990 and 1994, the concepts related to learning drew the attention of the researchers but in between 1995 and 2004 the researchers focused on teaching practice and classroom interactions. In these time periods, the focus of mathematics education related studies shifted from learning to teaching perspective. In addition, the terms such as motivation and beliefs which are related to affective aspects of skills appeared between 1995 and 1999 whereas the concept of achievement regarding cognition appeared in the period from 2000 to 2004 and achievement continued to be seen in all the following periods (2005-2019). On the other hand, even though they are not among the top 10 keywords in some periods, the concepts such as equality and motivation have come to prominence again in the last two periods (2010-2019). Teacher education was in the list between 2005 and 2014 and science education existed between 2005 and 2019 whereas technology entered the list in the last period (2015-2019). Moreover, the concept of professional development has come to prominence again in the last three periods (2005-2019). It can be said that the terms such as motivation, equity, professional development, reform in mathematics education, curriculum, achievement were among the top 10 keywords in the most of the periods and the concepts in the first period differed greatly from the trend in the following years. That is, the keywords in each period show that research in mathematics education underwent a major transformation after 1995.

In the third group of analysis, the top 10 countries published mathematics education studies in WoS database are investigated. The results are given in terms of the total number of publications and citations in Table 3.

Table 3. Top 10 Countries in Mathematics Education Research

<table>
<thead>
<tr>
<th>Rank</th>
<th>Country</th>
<th>Number of articles</th>
<th>Citations</th>
<th>Average citation per article</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>USA</td>
<td>468</td>
<td>11059</td>
<td>23.63</td>
</tr>
<tr>
<td>2</td>
<td>Turkey</td>
<td>88</td>
<td>373</td>
<td>4.24</td>
</tr>
<tr>
<td>3</td>
<td>England</td>
<td>64</td>
<td>792</td>
<td>12.38</td>
</tr>
<tr>
<td>4</td>
<td>Netherlands</td>
<td>54</td>
<td>665</td>
<td>12.31</td>
</tr>
<tr>
<td>5</td>
<td>Canada</td>
<td>46</td>
<td>874</td>
<td>19.00</td>
</tr>
<tr>
<td>6</td>
<td>Australia</td>
<td>43</td>
<td>572</td>
<td>13.30</td>
</tr>
<tr>
<td>7</td>
<td>Germany</td>
<td>32</td>
<td>311</td>
<td>9.72</td>
</tr>
<tr>
<td>8</td>
<td>South Africa</td>
<td>29</td>
<td>194</td>
<td>6.69</td>
</tr>
<tr>
<td>9</td>
<td>Spain</td>
<td>24</td>
<td>223</td>
<td>9.29</td>
</tr>
<tr>
<td>10</td>
<td>China</td>
<td>23</td>
<td>159</td>
<td>6.91</td>
</tr>
</tbody>
</table>

Table 3 shows that the USA led the list in terms of number of publications, citations and the average citation per publication in mathematics education research. Although Turkey was the runner up in terms of number of publications, its average citation per publication had the lowest value among the countries, surprisingly. The country which had the lowest number of publications and citations was China. On the other hand, the trends of countries in terms of their rank and number of publications were analyzed to obtain information on the state of mathematics education studies among countries. The results are given in Table 4.
Table 4. Trends of Countries in terms of Ranking and Number of Articles

<table>
<thead>
<tr>
<th>Country</th>
<th>Rank</th>
<th>Number of articles</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>1</td>
<td>3 4 4 15 36 89 118 199</td>
</tr>
<tr>
<td>Turkey</td>
<td>-</td>
<td>0 0 0 0 0 9 42 37</td>
</tr>
<tr>
<td>England</td>
<td>-</td>
<td>0 0 1 2 5 4 21 31</td>
</tr>
<tr>
<td>Netherlands</td>
<td>-</td>
<td>0 0 0 1 4 6 10 33</td>
</tr>
<tr>
<td>Canada</td>
<td>-</td>
<td>0 0 1 0 4 6 17 18</td>
</tr>
<tr>
<td>Australia</td>
<td>-</td>
<td>0 0 0 1 1 3 12 26</td>
</tr>
<tr>
<td>Germany</td>
<td>-</td>
<td>0 0 1 1 1 0 10 19</td>
</tr>
<tr>
<td>South Africa</td>
<td>-</td>
<td>0 0 0 0 1 8 10 10</td>
</tr>
<tr>
<td>Spain</td>
<td>-</td>
<td>0 0 0 0 1 2 7 14</td>
</tr>
<tr>
<td>China</td>
<td>-</td>
<td>0 0 0 0 1 6 16</td>
</tr>
</tbody>
</table>

Table 4 presents the contributions of top 10 countries to mathematics education research from 1980 to 2019. The number of articles has an increasing trend and the number of articles was highest in the USA during all periods.

**Visualization**

The visualization step represents the mapping of the terms encountered in mathematics education and the relationships among them. First, we obtained the overlay map of the publications between 2005 and 2019 since the number of the studies was low before 2005 to gain insight about the change in trend. The overlay map of VOSviewer is shown in Figure 4. The colors change depending on the publication years from blue to green and green to yellow as given in the bottom right corner. While the blue color represents the studies published since the early 2005s, the yellow color mostly corresponds to the publications after 2015.

Figure 4 shows that the terms such as theory, discourse, standard, principle, perspective, history, idea, policy, schooling, parent has a color between blue and green indicating that terms mostly occur in the publications between 2005 and 2010. These terms may also be interpreted like that the studies in this period seem to be
related to policy on mathematic education. On the other hand, the terms such as achievement, success, instructional practice, mathematics classroom, representation, construction, word problem, mathematics problems, solution, graph, addition, geometry also has a color between blue and green. These terms point to the prominent aspects in the research like the cognitive, instructional and subject domain in the period between 2005 and 2010.

The terms like motivation, emotion, attitude, self-efficacy, academic achievement, math, factor, association, gender, boy, girl, age, grade has a color between green and yellow. This shows that the articles between 2010 and 2019 have focused on these terms. It seems that the studies focused on various variables, cognitive and affective skills and explored the relationships in mathematics education. On the other hand, the terms such as experimental group, control group, test, pre, intervention, variance, significant difference, effectiveness also has a color between green and yellow. It can be said that the recent articles, especially after 2015, refer to the conduction of explanatory research. VOSviewer also provides the density map of the terms as shown in Figure 5.

In Figure 5, it is seen that the terms in overlay map are in the same position but this map represents the density of the terms differently from it. When the number of terms occurred in the neighborhood of a point becomes larger and the weights of the neighboring terms become higher, the color of the point becomes a tone about
yellow. On the left side of the map, it is seen that there is the dominant occurrence of the terms such as effect, item, group, success, achievement, scale, age, math, motivation, age, grade, difference and intervention whose colors are yellow.

On the right side of the map, the terms like idea, variety, theory, discourse, perspective, discipline, mathematics classroom have density and in yellow color. On the other hand, the density of the terms such as standard, contribution, field, policy, technology, survey and science is seen on the bottom part of the map in yellow color. The color of the remaining terms is in between green and blue. VOSviewer also forms a network map with the cluster-specific structure. This map is given in Figure 6.

In Figure 6, the network map shows the highly occurring terms and the association between them. Each circle is related to a term and the larger circles mean the more articles including this term. Besides, the more closeness between the circles indicates the more association between the terms. The program creates the clusters with different colors according to the occurrence and association of the terms. Here, the clusters are represented with green, blue, yellow and red colors.
Identification

In this step, we focused on the identification of the clusters and provided evidences to support our decisions. The information describing each cluster is shared in Table 5.

<table>
<thead>
<tr>
<th>Color</th>
<th>Cluster name</th>
<th>Highly occurring items</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>foundation</td>
<td>theory (112), perspective (93), issue (90), idea (86), field (80), example (64), community (53), standard (50), discourse (49), mathematics classroom (44)</td>
</tr>
<tr>
<td>green</td>
<td>implementation</td>
<td>effect (130), group (120), skill (99), difference (93), test (86), performance (85), participant (73), score (63), attitude (58), intervention (56)</td>
</tr>
<tr>
<td>blue</td>
<td>association</td>
<td>science (96), technology (87), grade (75), achievement (72), factor (70), sample (54), math (45), questionnaire (45), perception (43), gap (43)</td>
</tr>
<tr>
<td>yellow</td>
<td>evaluation</td>
<td>country (57), success (51), policy (46), programme (31), state (23), mathematics performance (13), nation (12), failure (12), international student assessment (11), PISA (11), TIMSS (11)</td>
</tr>
</tbody>
</table>

In Table 5, the cluster with red circles was identified as foundation since it included the terms such as theory.
perspective, standard and field. The highly occurring terms in this cluster point out the basis of mathematics education. The attempts seem to be on determining educational policy in mathematics, evaluating theory and perspectives, structuring mathematics classrooms, exploring the idea, community and discourse in mathematics instruction and creating standards for mathematics. The cluster with green circles was named as implementation because it reflected the terms like effect, group, difference, test, score and intervention. It is understood that the focus of the studies in this cluster was on determining the effectiveness of methods for development of mathematics education. The highly occurring terms in this cluster mostly refer to the explanatory research. The cluster with blue circles was identified as association since the terms such as science, technology, math, achievement, grade and factor. It recalls the studies which were interested in finding the relationships between variables regarding mathematics and determining the predictors in mathematics education. The cluster with yellow circles was called as evaluation because it represented the terms like country, success, international student assessment, nation, PISA and TIMSS. The highly occurring terms in this cluster refer to the assessment of mathematics education in international platform. It can be stated that the research in this cluster focused on the results of international exams in order to evaluate and compare the achievement in mathematics education on a country basis and to organize their own policies and programs in mathematics education.

Verification

The verification phase refers to the process of searching for evidence, which is carried out to determine the consistency of identification and includes the examination of the articles. The investigation of the articles that were associated with each cluster depending on the existence of items was given below.

Cluster 1: Foundation

This cluster includes items related to theoretical frameworks, perspectives, principles and standards of mathematics education and refers to the curriculum reform efforts. Based on the literature, it is seen that there have been some earlier attempts to identify the development of mathematics education based on theory change and reconstruction. In terms of trends in theoretical frameworks, at the beginning of the 1980s, constructivism was dominant as the theoretical framework in mathematics education. Since its structure was comprehensive, other theories were developed benefitting from constructivism such as action, process, object and schema [APOS] theory (Dubinsky& McDonald, 2001) and theory of didactical situations (Brousseau, 2006). New theoretical frameworks have not completely ignored the earlier frameworks, instead, they have contributed their richness by expanding the spectrum of research (Hannula, 2009). By the end of the 1980s, the radical version of the constructivism became in prominence and it was followed by social constructivism from the 1990s. Hanna and Sidoli (2002) conducted the keywords analysis of mathematics education research and pointed out the increase in the number of publications regarding social issues in the teaching and learning mathematics at the beginning. Similarly, Lerman, Xu and Tsatsaroni (2002) analyzed the publications in the 1990s and indicated a growth in the number of studies that focused on social theories from 1990 to 2001. With 1990s, social turn appeared in mathematics education research (Lerman, 2000). The author emphasized that the theories give importance to the social activities have become in prominence rather than individual attempt. It was found that
the extent of the social turn was not dominant after this period (Gates & Joegensen, 2015; Jablonka & Bergsten, 2010) and socio-cultural perspective started to place in the studies (Lerman, 2006). By the end of the decade, a number of studies were found helpful for reform-based mathematics methods and curricula (Cohen & Hill, 2001; Fuson, Carroll, & Drueck, 2000; McCaffrey et al., 2001; Schoenfeld, 2002). According to the changes in mathematical content, the interest in the addition and subtraction, problem solving has showed a peak during the 1980s. The topic of proof and argumentation have become more prominent since 2000.

**Cluster 2: Implementation**

Mainly, this cluster covers the items dedicated for effective mathematics instruction. When the related articles were analyzed, the topics involved the cultural, social and economic factors, school-related factors, teaching- and teacher-related factors and parental factors in mathematics education. For example, Desoete and Craene (2019) analyzed the contributions of metacognition for improving mathematics performance. In another study, Quintos, Civil and Bratton (2019) examined the role of parental engagement, Maričić and Stamatović (2017) investigated the effect of pre-schooling on mathematics education. Additionally, the effect of cooperative learning (Bekele & McPherson, 2011; Park & Nuntrakune, 2013), teacher training courses (Yaman, 2015) and preservice teachers’ attitudes (Kesicioğlu, 2015) were studied on mathematics education. The general characteristics of these articles refer to the attempts for improving mathematics performance.

**Cluster 3: Association**

This cluster encloses the factors that are interrelated to mathematics education. In the network map (see Figure 6), the clusters 2 and 3 were located very close to each other indicating that these clusters provide interrelated items such as grade vs. score, test vs. questionnaire and performance vs. achievement. However, the remaining items in each of these clusters provided evidence about the differentiation among them. The student factors such as self-efficacy (Arslan & Işıksal Bostan, 2016), motivation (Holmes & Hwang, 2016; Schukajlow, Rakoczy, & Pekrun, 2017) and gender (Halai, 2010; Sarouphim & Chartouny, 2017) were included in cluster 3. Also, technology and other related concepts such as science, technology, engineering and mathematics (STEM) existed in this cluster. For instance, the topics include but not limited to alternate approaches (White, 2019) and teacher characteristics (Brown, 2017) for extending insights into technology-supported mathematics teaching and learning, use of integrated virtual laboratory (Cheong & Koh, 2018), computer game-based environments (Vrugte et al., 2015, Vrugte et al., 2017), interactive whiteboards (Bourbour & Masoumi, 2017; Heemskerk, Kuiper & Meijer, 2014), computer manipulatives (Sarama & Clements, 2009) and the internet incorporation (Li, 2003). In cluster 3, the emphasis on science and technology is also important. Many articles investigated different issues of STEM. For example, Yıldırım and Sidekli (2018) analyzed the effect of STEM applications on mathematical literacy self-efficacy, technological pedagogical knowledge and mathematical thinking skills. Moreover, Chai et al. (2019) developed survey on technological pedagogical STEM knowledge to assess teachers’ self-efficacy. The studies also focused on the achievement gap (Jia, 2019) and cross-national differences (Langen & Dekkers, 2005) in STEM education. Related to the items in cluster 3, there are articles focusing on scale development in mathematical education values (Dede, 2011) and testing the effects of the
perception of mathematics education quality on anxiety and achievement with structural equation modeling (Ciftci, 2015).

Cluster 4: Evaluation

Mainly, this cluster consists of the terms related measurement, evaluation and progress in mathematics education. The articles investigated the large-scale assessment programs such as TIMSS (Klieme & Baumert, 2001) and PISA (Cantley, 2019; Nortvedt, 2018). However, another important term to be mentioned here is the policy. The articles explore the effects of large-scale studies on mathematics education policy (Lin, Wang & Chang, 2018; Nortvedt, 2018), the consequences of experimentalism in mathematics education policy and practice (Cobb & Jackson, 2008), policy implementation and responses to mathematics reforms (Spillane, 2000), implications for curriculum policy (Stacey & MacGregor, 1999) and instructional policy issues (Gall, 1984).

Discussion and Conclusion

This study aimed to investigate the bibliometric analysis of mathematics education research over the past 40 years and reveal the trend in this field. We focused on characterizing the documents based on publication years, publication numbers, keywords and countries, identifying biometrical clusters and visualizing the mathematics education related terms with maps. We provided a research framework covering exploration, visualization, identification and verification phases so that the results would be discussed within this manner. After investigating the articles related to mathematics education, a bibliometric analysis was conducted with VOSviewer software to determine the inter-related terms.

The mathematics education studies in WoS database were first published at early 1980s and despite the change in the rate of growth, the number of publications continued to increase in each period. According to keyword analysis, there were a great variety of studies in mathematics education to determine standards and principles of teaching and learning mathematics such as reform movements (Gravemeijer et al., 2016; Lundin, 2012; Sengupta-Irving, Redman, & Enyedy, 2013), curriculum (Fonger et al., 2018; Fouze & Amit, 2017; Pepin et al., 2017; Voigt, Fredriksen, & Rasmussen, 2020), educational policy (Dalby & Noyes, 2018; Lin, Wang, & Chang, 2018; Nortvedt & Buchholtz, 2018), equity (Jurdak, 2011, 2014; Nortvedt & Buchholtz, 2018; Tan & Thorius, 2019), assessment (Beumann & Wegner, 2018; Kim & Cho, 2015; Nortvedt & Buchholtz, 2018; Veldhuis & van den Heuvel-Panhuizen, 2014; Veldhuis et al., 2013), to evaluate the cognitive and affective skills such as problem solving (Boonen et al., 2016; Verschaffel et al., 2020), achievement (Ciftci, 2015; Veldhuis & van den Heuvel-Panhuizen, 2020), motivation (Schukajlow, Rakoczy, & Pekrun, 2017), to learn more about and support mathematics teachers such as professional development (Sztajn et al., 2007; Williams & Ryan, 2020), teacher education (Buchholtz, 2017; Healy & Ferreira dos Santos, 2014; Tattoo & Senk, 2011), teacher knowledge (Koponen et al., 2017; Olfos & Rodriguez, 2019; Scheiner et al., 2019), teacher beliefs (Kang & Kim, 2016; Cetinkaya & Erbas, 2011), to address the relationships with different aspects such as early childhood (Björklund, van den Heuvel-Panhuizen, & Kullberg, 2020; Ulusoy, 2020), algebra (Dougherty et al., 2015;
Simzar, Domina, & Tran, 2016), technology (Cullen, Hertel, & Nickels, 2020; Drijvers, 2015; McCulloch et al., 2018; Thomas & Hong, 2013; Trouche & Drizvers, 2010), science education (King et al., 2020; Maass & Engeln, 2019; Swanson & Coddington, 2016).

The timeline analysis of the keywords provides information about the terms related to mathematics education. To gain insight into the research trend, the keyword based (Li & Zhao, 2015; Cancino et al., 2017; Laengle et al., 2017; Muhuri et al., 2018) and country based (Ding & Yang, 2020; Fergnani, 2019; Kamdem et al., 2019, Song & Wang, 2020) analysis is mostly used within research. In the timeline analysis of keywords, there were no keywords of the studies the period of 1980-1990. According to the review of Schoenfeld (2016) in mathematics education, by the end of the 1980s, student thinking was an important topic and it necessitated to focus on the knowledge, the strategies of problem solving, metacognition and beliefs (Schoenfeld, 1985; Silver, 1985). By the late 1980s and early 1990s, multidisciplinary and interdisciplinary research in mathematics education were encountering such as mathematical epistemology (Greeno, 1988); problem solving in contexts (Schoenfeld, 1988); assessment (Marshall, 1988; Silver & Kilpatrick, 1988); teachers’ beliefs and conceptions (Thompson, 1988). In this research, at the beginning of 1990s, the results showed that the studies focused on more specific concepts such as authenticity, conceptual change, generalization, knowledge acquisition, learning, schematic representation, pragmatic and semantic problem solving. However, more comprehensive terms such as reform in mathematics education, curriculum, professional development, problem solving were included in the studies over the years. For example, there were changes in the trend like that pragmatic and semantic problem solving replaced with problem solving or teaching practice and classroom interactions draw attention more than learning later on. Lubiensky and Bowen (2000) examined mathematics education research available from the ERIC database between 1982 and 1998. They found that most of the studies were on gender, ethnic group, social class and the lack of opportunity. The level of research was mainly at the primary education whereas the number of the studies was less. The most studied mathematics subjects were integers, problem solving and geometry while the most focused topics were cognitive learning and teaching, student achievement, teacher behavior, curriculum, technology, and student characteristics. The findings of this study is parallel to our study in terms of the focus on problem solving and cognitive learning and teaching and emphasis on the lack of the studies in this period.

In early 2000s, the keywords such as achievement, conceptual knowledge and algebra were started to be used. This result pointed out that cognitive aspect of skills was prominent and the interest shifted to study on specific mathematical domain. The other new terms that are teaching and equity as well as the older keywords such as reform in mathematics, curriculum and teaching practice showed that the attempt to structure mathematics education in terms of standards and principles continued. Since 2005, different keywords were started to take place more in mathematics education research such as science education, teacher education, teacher learning. The expansion of the terms revealed that the focus of the researchers on the development of teachers for effective mathematics education and quality of teacher training programs increased and they started to associate mathematics with different disciplines. Hannula (2009) overviewed a handbook on PME activities 1976-2006 and PME conference proceedings 1997-2007 and indicated that the most popular topics showing rapid increase in mathematics education between 1997 and 2007 were teacher education and professional development.
Besides, algebra and algebraic thinking, affect, emotion, beliefs and attitudes, mathematical thinking were other popular terms in this period. Despite of the decline of the studies related to theories of learning and epistemology, language and mathematics, these topics were among the most popular research topics. It was also found that socio-cultural factors/studies and early mathematics showed an increasing trend during this period. Moreover, the topics such as problem solving, mathematical modeling, proof, proving and argumentation, cognitive science and cognitive models, assessment and evaluation drew attention as relevant research. Similar to the findings of our study, Hannula (2009) points to the popular topics as professional development, teacher education, algebra.

In 2010s, the new keywords were teacher knowledge, technology and educational policy whereas motivation rose to prominence again. It is seen that the other disciplines, affective skills and teacher education were focus in recent mathematics education research. It can be said that the terms such as motivation, equity, professional development, reform in mathematics education, curriculum, achievement were among the top 10 keywords in the most of the periods and the concepts in the first part of 1990s differed greatly from the trend in the following years. The earlier studies look mathematics education mostly in students’ perspective and investigate generalization, restructuring, interiorization and representation. Between 1995 and 2010, curriculum and teacher-related factors were dominant in mathematics education studies. After 2010, the keywords of the articles carry the traces from all stakeholders in mathematics education.

The overlay map of terms in mathematics education research revealed that the terms such as theory, discourse, standard, principle, perspective, history, idea, policy, schooling, parent refer to the terms mostly occur in the publications between 2005 and 2010. These terms may also be interpreted like that the studies in this period seem to be related to educational policy on mathematic education similar to the keyword analysis. The terms such as achievement, success, instructional practice, mathematics classroom, representation, construction, word problem, mathematics problems, solution, graph, addition, geometry point to the prominent aspects in the research like the cognitive, instructional and subject domain in the period between 2005 and 2010. The terms like motivation, emotion, attitude, self-efficacy, academic achievement, math, factor, association, gender, boy, girl, age, grade show that the articles between 2010 and 2019 have focused on these terms. It seems that the studies focused on various variables, cognitive and affective skills and explored the relationships in mathematics education. The terms such as experimental group, control group, test, intervention, variance, significant difference, and effectiveness also indicate that the recent articles, especially after 2015, refer to the conduction of explanatory research. According to the density map, theory, perspective, idea, discipline, achievement, attitude, effect, group, difference, technology and science were the highly occurring terms in the descriptive parts of the mathematics education related articles including title, keywords and abstract. It is seen that the keyword analyses both for general and each period, the overlay map and the density map give the results that are consistent substantially and contribute each other.

The network map visualized that the terms of mathematics education related studies were clustered in four clusters depending on the degree of the relationships among them. The closely related terms were located next to each other. The first cluster (foundation) covers the items related to the theoretical framework of mathematics
education whereas the second cluster (implementation) has the terms defining the methods for effective mathematics instruction. The third cluster (association) includes the concepts interrelated to mathematics achievement while the fourth cluster (evaluation) encloses the studies about international mathematics assessments.

In the country analysis, the USA had the highest number of publications, citations and the average citation per publication in mathematics education research. The USA had an increasing trend over the past four decades and it also retained the highest record during all periods. It was also found that although Turkey was the second country which has the highest number of publications after the USA, its average citation per publication had the lowest value surprisingly. The country which has the lowest number of publications and citations was China. During this process, the USA was the most productive and influential country with its contributions. The order of top ten countries contributed to mathematics education research in WoS database according to the number of publications were respectively the USA, Turkey, England, the Netherlands, Canada, Australia, Germany, South Africa, Spain and China. It is found that the number of publications was limited in the other countries in comparison to the USA over the past four decades and only the USA was in the list between 1980 and 1989.

**Limitations and Recommendations**

This research has some limitations depending on the data sources, indexation and language. The English articles published in the journals indexed in SSCI within WoS database were considered in this review. Future studies can expand the scope of the study by including different databases. Although the number of mathematics education studies has increased recently, more studies are needed to conduct for increasing the multi-/inter-disciplinary research as well as science and technology and the different field research such as special education, cognitive psychology. The existence of early childhood and algebra among the most repetitive terms in clusters and high-frequency keywords points to the need for more research at the advanced grade levels and on different mathematical domain. Moreover, the keywords related to teachers such as professional development, teacher knowledge, teacher beliefs, teacher learning, teacher education reveal that mathematics education is mainly studied in this group so it is suggested to conduct more studies with students.

**References**


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unsuccessful solvers. *Teaching and Learning, 14*(2), 61-72.


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