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Abstract

Professional organizations have provided recommendations for mathematics teaching and learning; however, few studies have investigated the practical integration of those recommendations into mathematics teacher education programs. In this study, we examine how the reported “big ideas” of courses in secondary mathematics teacher education programs emphasized the content and teaching practices necessary for future mathematics teachers, as recommended by policy documents. As part of a larger study, we conducted a series of interviews in secondary mathematics teacher education programs at four universities (names are descriptive pseudonyms): Great Lakes University (GLU), Midwestern Research University (MRU), Midwestern Urban University (MUU), and Southeastern Research University (SRU). We selected the institutions and programs based on their Carnegie Classification, the types of communities in which they were situated, the average number of graduates from a program, the departmental homes of their secondary mathematics education programs, and the demographics of their student populations. The analysis of data collected from 12 courses across four universities revealed specific ways in which big ideas in secondary mathematics teacher education programs emphasized areas related to mathematics learning, teaching, and issues of equity and access.

Introduction

Given the demands that future mathematics teachers face in supporting the learning of all students, including culturally and linguistically diverse students, prospective teachers need high-quality, equitable instruction in their teacher education programs (Association of Mathematics Teacher Educators [AMTE], 2017). Extant research has addressed aspects of knowledge related to mathematical content, pedagogy, and issues of equity needed by preservice teachers (e.g., Ball, 2017; Ball et al., 2008; Fuson et al., 2005; Leonard et al., 2010; Mintos et al., 2019; Turner et al., 2012; Zahner, 2015). Professional societies of mathematicians, mathematics educators, and teacher educators have drawn on this research to construct detailed recommendations for:

- (a) how mathematics should be taught generally to undergraduates, including future teachers (e.g., Mathematical Association of America [MAA], 2018);

(b) how secondary mathematics teachers should be prepared (e.g., AMTE, 2017; Conference Board of the Mathematical Sciences [CBMS], 2012; National Council of Teachers of Mathematics [NCTM], 2014); and

(c) how university instructors should communicate issues of mathematics equity and access to future teachers (e.g., AMTE, 2017; MAA, 2018; NCTM, 2000, 2014).

Little has been reported, however, about how such recommendations are incorporated into programs that prepare secondary mathematics teachers (National Research Council [NRC], 2010).

In presenting preliminary findings from this study at the 2019 North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA) Annual Conference, several of our secondary mathematics teacher educator colleagues described their ongoing efforts to talk with colleagues from other universities about program development and implementation—processes where considering the big ideas and goals of the program and courses is a critical first step and provides a common focal point for conversations. The discussion motivated us to write this paper. We examined big ideas from three required courses in each of four secondary teacher preparation programs (i.e., 12 courses total) using recommendations from professional organizations as our lenses (i.e., AMTE, 2017; CBMS, 2012; MAA, 2018; NCTM, 2000, 2014). The data we use here are from a larger study in which we conducted case studies to describe the opportunities that teacher preparation programs provided for mathematics preservice secondary teachers (M-PSTs) to learn about mathematical content, teaching practices, and issues of equity related to algebra. Our case study addresses the following question: How do instructors of required courses in secondary mathematics teacher education programs emphasize goals and big ideas related to content, pedagogy, and equity, as recommended by professional organizations for future mathematics teachers?

Conceptual Framework and Relevant Literature

To develop the conceptual and analytic framework for this study, we examined recommendations for mathematics teaching and learning from national professional organizations (i.e., AMTE, 2017; CBMS, 2012; MAA, 2018; NCTM, 2000, 2014). We provide a brief overview of each set of recommendations in the following sections, with attention to the documents' goals and structure, and to how the authors addressed content, pedagogy, and equity.

Association of Mathematics Teacher Educators (AMTE)

AMTE's (2017) *Standards for Preparing Teachers of Mathematics (SPTM)* was written to inform the preparation of mathematics teachers, including “clearly articulated expectations for what well-prepared beginning mathematics teachers need to know and be able to do upon completion of a certification or licensing program and the recommended characteristics for programs to support teachers' development” (p. xii). Focused on how programs can develop teacher candidates' knowledge, skills, and dispositions, and set them on a “career-long continuum of teacher development” (p. 3), the authors of *SPTM* built on existing research (e.g., Ball & Forzani, 2011; Darling-Hammond & Bransford, 2007; Gutiérrez, 2013) and policy documents of

professional organizations (e.g., CBMS, 2012; Council of Chief State School Officers [CCSSO], 2013; NCTM, 2014). They included a set of four “equally important and interrelated standards” that highlight key areas of teachers’ knowledge and skills:

- (C.1.) mathematics concepts, practices, and curriculum;
- (C.2.) pedagogical knowledge and practices for teaching mathematics;
- (C.3.) students as learners of mathematics; and
- (C.4.) social contexts of mathematics teaching and learning. (p. 6)

Each standard includes multiple measurable indicators and accompanying explanations. In addition, SPTM includes attention to how faculty working in teacher education programs can assess candidates and evaluate their programs, and it provides action steps toward achieving the vision outlined in the standards.

AMTE drew on recommendations from CBMS (2012) and MAA’s *Curriculum Guide to Majors in the Mathematical Sciences* (Schumacher & Siegel, 2015) to describe well-prepared beginning secondary teachers’ mathematical content knowledge (i.e., C.1). AMTE’s C.1 standard for secondary teachers emphasized the importance of beginning teachers’ knowledge of content, process, and practices and the need for partnerships between teacher educators and mathematics instructors. The recommendations for pedagogical knowledge for teaching mathematics (i.e., C.2, C.3, C.4) focused on preparation to support each and every student in learning meaningful mathematics and cultivating positive mathematical identities, stating that “well-prepared beginners give serious attention to students who are living in poverty, Latinx, Black, indigenous, and emergent multilinguals, students who have been historically excluded or marginalized in mathematics” (p. 127).

Conference Board of the Mathematical Sciences (CBMS)

Following the release of *Common Core State Standards for Mathematics* (CCSSM; CCSSO, 2010), CBMS published *Mathematical Education of Teachers (MET) II* in 2012, which built on *MET I* (CBMS, 2001), but was revised to ensure that mathematics teachers would be prepared to teach the mathematical content and practice standards included in CCSSM. The authors represented both the mathematics and mathematics education communities and emphasized this critical partnership to ensure that M-PSTs finished their programs with the knowledge, skills, and dispositions to effectively teach mathematics to their future students. *MET II* highlighted four major themes:

- (a) There is intellectual substance in school mathematics;
- (b) Proficiency in school mathematics is necessary but not sufficient mathematical knowledge for a teacher;
- (c) The mathematical knowledge needed for teaching differs from that of other mathematics professions; and
- (d) Mathematical knowledge for teaching can and should grow throughout a teacher’s career (p. xii).

Chapters 5 and 6 address the mathematical preparation of middle and high school teachers, respectively, highlighting essential experiences involving mathematical content, practices, and professional development (e.g., research). For all M-PSTs, the authors emphasized the need for mathematics courses, mathematics-specific methods courses, and courses focused on secondary school mathematics from an advanced viewpoint.

Mathematical Association of America (MAA)

In their 2018 *Instructional Practices Guide*, the MAA argued for radical changes in undergraduate mathematics education (including education for future teachers) to ensure mathematics accessibility to all students. These changes included incorporating complex tasks that support group work, discussion, and respect for students' voices, thereby building community and mathematical identities. Strategies for effecting change emerged through MAA's (2018) framework of three practices: classroom practices (e.g., fostering student engagement and selecting appropriate tasks), assessment practices (e.g., when, how, and why we use formative or summative assessments), and design practices (e.g., research-based components supporting student-centered learning).

MAA (2018) did not describe individual course content; instead, it focused on how undergraduate mathematics content should emerge by explicitly identifying and communicating big ideas and smaller learning goals. MAA recommended that instructors include learning goals that support productive beliefs about mathematics (i.e., support all students in believing they can use mathematics successfully and usefully in their personal and professional lives). They further recommended that instructors structure learning experiences (e.g., tasks and readings) and assessments around these goals and big ideas. Throughout the *Instructional Practices Guide*, MAA focused on communicating how instructional practices should be redesigned and enhanced to increase access to mathematics for students who have traditionally found barriers in undergraduate mathematics. Additionally, a section dedicated to "equity in practice" (MAA, p. 122) drew on Gutiérrez (2002, 2009) to describe questions that instructors should ask themselves about access, achievement, identity, and power; then, the instructors should design instructional practice more intentionally to achieve these goals.

National Council of Teachers of Mathematics (NCTM)

Over several decades (e.g., 1989, 1991, 2000, 2009, 2014, 2018), the NCTM has provided guidance for establishing big ideas in secondary mathematics teacher education programs. This guidance has sometimes appeared implicitly in their expectations for students (e.g., 1989, 2000, 2009, 2018) and at other times explicitly in their expectations for teachers (e.g., 1991, 2014). Focused on identifying principles, content, and processes important for mathematical learning and teaching, the authors of *Principles and Standards for School Mathematics* (NCTM, 2000) provided six principles (i.e., equity, curriculum, teaching, learning, assessment, and technology), five content areas (i.e., number and operations, algebra, geometry, measurement, data analysis and probability), and five process standards (i.e., problem solving, reasoning and proof, communication, connections, and representations).

Further, NCTM (2014) provided "a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics" (p. 9) and included eight Mathematics Teaching Practices (MTPs) developed from the five process standards (NCTM, 2000) and existing research (e.g., Charles & Carmel, 2005; Clements & Sarama, 2004; Hiebert et al., 2007; Seidel et al., 2005). Central to NCTM's recommendations is attention to equitable mathematics teaching practices. In fact, equity was included as the first principle, with an emphasis on high expectations and worthwhile mathematical opportunities for all, accommodations for

differences to help everyone learn, and access to resources and support for all students (NCTM, 2000, 2014).

Synthesis of Recommendations

In descriptions of valuable mathematical experiences for secondary students, university students, and future secondary mathematics teachers, researchers, and professional organizations consistently emphasize opening access to deep and critical mathematics to all students, especially those from traditionally underserved communities. Teaching for access and equity is emphasized by researchers and professional organizations, including AMTE (2017), CBMS (2012), MAA (2018), and NCTM (2000). AMTE, for example, highlighted the importance of equity as the first of five foundational assumptions about mathematics teacher preparation, stating, “Although equity, diversity, and social justice issues need to be specifically addressed as standards, they must also be embedded within all the standards” (p. 1). Similarly, NCTM (2000) emphasized equity as the first principle, highlighting the critical importance of ensuring that all students have access to quality mathematics instruction. MAA (2018) argued that undergraduate mathematics instructors must radically revise instruction to open access to underserved students.

Recommendations for instructional changes that support more equitable outcomes are remarkably similar, including the use of mathematical mindsets, classroom community, mathematical discourse, rich and authentic tasks, and formative assessment. Similarities are partly the result of new recommendations built on earlier ones. For example, AMTE (2017) confirmed, reiterated, and expanded on CBMS’s (2012) recommendations. Similarly, MAA (2018) drew on both NCTM’s (2000) and CCSSO’s (2010) recommendations for practices and processes in primary and secondary mathematics education in the development of the *Instructional Practices Guide*; the authors adapted the frameworks of groupworthy tasks and cognitive demand (Stein et al., 1996; Lotan, 2014) for university mathematics. CBMS (2012) and MAA (2018) specifically recommended that productive beliefs supporting high-quality mathematics learning for all learners (e.g., habits of mind and growth mindsets) must be learning goals in mathematics courses for all undergraduate students, including future teachers. While these standards provide recommendations for mathematics educators to utilize in their course design, we know little about how these recommendations are implemented in specific courses in secondary mathematics teacher education.

Methods

As part of a larger study, we conducted a series of interviews in secondary mathematics teacher education programs at four universities (names are descriptive pseudonyms): Great Lakes University (GLU), Midwestern Research University (MRU), Midwestern Urban University (MUU), and Southeastern Research University (SRU). We selected the institutions and programs based on their Carnegie Classification, the types of communities in which they were situated, the average number of graduates from a program, the departmental homes of their secondary mathematics education programs, and the demographics of their student populations. Table 1 summarizes the variation in institution and case study program characteristics, and Table 2 presents the university undergraduate population demographics.

Table 1. Characteristics of Secondary Mathematics Teacher Education Programs and Universities

University	Basic Carnegie Classification ^a	Context	Number of graduates ^b	Academic home
GLU	Master's: larger programs	Small city	34	Mathematics department
MRU	Doctoral: highest research	Mid-size city	22	College of education
MUU	Master's: larger programs	Large city	12	College of education
SRU	Doctoral: highest research	Mid-size city	39	College of education

Note. GLU: Great Lakes University, MRU: Midwestern Research University, MUU: Midwestern Urban University, SRU: Southeastern Research University

^aSee Carnegie Classifications for sources & definitions (<http://carnegieclassifications.iu.edu/>).

^bAverage annual number of graduates across the three reported academic years

The selected programs varied across these dimensions, including two with a high level of research activity, one program situated in a mathematics department in contrast to the others housed in colleges of education, and varying numbers of graduates.

Table 2. Degree-Seeking Undergraduate Race/Ethnicity Enrollment ^a

Characteristics	GLU	MRU	MUU	SRU
Degree-seeking undergraduate enrollment	21,137	32,694	8,827	26,120
Asian	2%	4%	10%	9%
Black	5%	4%	10%	7%
Latinx ^b	4%	5%	35%	5%
White	84%	71%	37%	73%
Multiracial	3%	4%	2%	3%

Note. GLU: Great Lakes University, MRU: Midwestern Research University, MUU: Midwestern Urban University, SRU: Southeastern Research University

^aEnrollment statistics were collected from the Common Data Set for each university. Only race/ethnicity categories with at least 2% in at least one of the universities were included in the table. Nonresident aliens and race/ethnicity unknown categories were not included.

^bWe used “x” to include all gender identifications.

The undergraduate enrollment of these institutions varied from approximately 9,000 to 30,000. The diversity of undergraduate students differed as well, ranging from GLU with 84% White students to MUU with non-White students representing 57% of the student population.

At each university, the research team and a university liaison selected approximately 10 required courses in the secondary mathematics teacher education program, including mathematics, mathematics education, mathematics for teachers, and general education courses. To explore how the recommendations of professional organizations

(i.e., AMTE, CBMS, MAA, NCTM) were enacted in a set of diverse courses, we examined one content course (i.e., Linear Algebra), one pedagogy course (i.e., Secondary Mathematics Methods), and one general education course focused on equity (i.e., Teaching in a Diverse Society), which are required at each university; course titles were standardized to protect anonymity. We selected these three courses based on the results of a national survey of U.S. secondary mathematics teacher education programs, which revealed that most of the programs required versions of these courses.

In Table 3, we present information about the courses at each university, including—at the time of the study—the rank of each participating instructor, the number of years they had taught at the current institution and their total number of years of teaching experience, as reported by the instructor in parentheses, and the terminal degree program for each (all had earned—or were currently earning in the case of the graduate assistant—a PhD).

Table 3. Focal Course University Instructors’ Background at Time of Study

	Rank ^a	Years at this university (Total years of experience)	PhD discipline
Linear Algebra			
GLU	Full	13 (22)	Mathematics
MRU	Post-Doc	2 (2)	Mathematics
MUU	Full	22 (34)	Mathematics
SRU	Instructor	20 (20)	Mathematics
Secondary Math Methods			
GLU	Assoc	12 (25)	Mathematics teacher ed
MRU	Asst	5 (17)	Mathematics education
MUU	Asst	8 (12)	Education
SRU	GA	2 (10)	Mathematics teacher ed
Teaching in a Diverse Society			
GLU	Assoc	2 (10)	Teaching and curriculum
MRU	Assoc	12 (22)	School administration
MUU	Asst	2 (10)	Educational policy
SRU	Full	2 (3)	Education

^aRank: GA = Graduate Assistant, Post-Doc = Post-Doctoral Fellow, Asst = Assistant Professor, Assoc = Associate Professor, Full = Full Professor

For each course, we collected the syllabus and interviewed the instructor who had most recently taught the course. We asked each instructor, “What are the goals or big ideas of this course?” Linear algebra is not necessarily offered only for M-PSTs, so we asked a follow-up question: “Do you do anything specific in this course to help prepare future mathematics teachers?” We analyzed course goals and big ideas, both as reported in the interview and as written in corresponding course syllabi under “Course Objectives” or “Course Goals.” To clarify statements from the big ideas question and course objectives, we examined other elements of the course syllabus or responses to follow-up questions as needed.

To answer our research question, (i.e., How do instructors of required courses in secondary mathematics teacher education programs emphasize goals and big ideas related to content, pedagogy, and equity, as recommended by professional organizations for future mathematics teachers?), we reviewed a set of mathematics and mathematics education policy documents (i.e., AMTE, 2017; CBMS, 2012; MAA, 2018; NCTM, 2000, 2014). In addition, we examined instructors' responses from interview transcripts and text from corresponding syllabi, noting emergent themes that were both common and unique across the courses (Creswell, 2007). We then focused our analysis on emergent themes related to recommendations in policy documents. We also explored connections between recommendations from professional organizations and the information collected in course syllabi and instructors' responses to interview questions. Finally, after writing summaries of the responses, we iteratively reviewed the themes and original responses to verify our summaries, while considering what instructors reported through the lens of selected policy documents.

Findings

In this section, we present findings from each set of courses across the universities: Linear Algebra, Secondary Mathematics Methods, and Teaching in a Diverse Society. We report similarities and differences between big ideas and course objectives across courses through the lens of recommendations from professional organizations. Given the different natures of the three courses under review and the instructors' responses to the interview questions, the structure of each summary was unique and was guided by an examination of the recommendations alongside the individual course data (i.e., interview transcripts, syllabi, and corresponding course materials).

Linear Algebra

In this section, we organize emergent themes from the reported intentions of Linear Algebra instructors using CBMS's (2012) recommendations about experiences in Linear Algebra that would benefit future secondary teachers and the framework of MAA's (2018) three instructional practices: intended classroom practices, assessment practices, and course (and task) design practices. The four instructors often described their big ideas and goals for Linear Algebra in similar ways as follows: (a) moving between concrete and abstract mathematics (GLU, MRU, SRU); (b) applying tools computationally (MUU, MRU, SRU); (c) transitioning to explaining and proving (MUU, SRU); and (d) developing an understanding of tools and concepts, including systems of linear equations, eigenvectors, eigenvalues, and eigenspaces, needed in other mathematics and disciplines (GLU, MUU, SRU).

When we asked instructors to state what they included specifically for future teachers, all four instructors reported that they did not integrate specific content for future teachers. However, instructors reported that future teachers would benefit from knowing mathematics (MRU, SRU) and experiencing certain activities through problem-solving in the class (GLU, MUU, MRU). For example, the GLU and MRU instructors described the way they organized the class and focused on the struggle and process of problem solving, rather than answer seeking, as being helpful for future teachers.

We next share instructors' responses to questions about students' collaborating or presenting work in class and on major assessments, describing emergent classroom practice themes based on instructors' reports. We explicitly asked the GLU, MRU, and SRU instructors about opportunities for collaboration in class. The GLU instructor described such opportunities in the interview and in his syllabus. He reported using group work extensively during class meetings and computer lab activities, explaining that his students worked in small groups on activities to foreshadow, discover, and consolidate ideas and concepts. The MUU, MRU, and SRU instructors described their teaching as mainly lecturing. The SRU instructor specifically reported that although he did not ask students to collaborate in class, he assumed that students might collaborate on homework.

The MUU and SRU instructors reported that they tried to make their lectures engaging and open to students' questions and participation. They described the shift to explaining and proving mathematical concepts as a big idea in their courses. The MUU instructor described his structure of class sessions as lively, with spontaneous discussions of student questions and struggles: "going through the challenges that [the students] also went through and showing how they have overcome that." The SRU instructor described pushing his students again and again to explain their reasoning by focusing on explanations, proofs, and counter-examples and by asking students to not just state facts but to ask, "Okay, how did you think about that? What are you doing? Why are you thinking that way? What's the reasoning?"

We asked each instructor explicitly about assessments, and each described (and their syllabi confirmed) a similar assessment pattern: three exams and a final exam (GLU, MRU, MUU, SRU), weekly homework (GLU, MRU, SRU), and weekly quizzes (GLU, MRU, MUU). In response to this question, two instructors described other activities: computer lab activities (GLU) and journaling (MUU). The GLU instructor reported that a third of the class time (i.e., 14 hours) was spent in small-group computer lab activities, of which seven were collected and graded. The MUU instructor reported encouraging students to write mathematical journals. In the beginning weeks of the course, although he did not collect them or assign points, he stopped to tell students to write certain ideas from their homework or class notes in their journals. After several weeks, he would stop pointing out these ideas, expecting students to take ownership of their journals. He stated that he tried to "encourage the students to think [of] journal writing as a strategy that helps them learn mathematics."

Although we did not ask explicitly about design practices, two instructors reported directly on their use of course learning goals or the development of tasks. In his syllabus, the MUU instructor listed clear learning goals for students that included attention to content and process. For example, one goal was: "Be able to apply some technology such as calculators to facilitate problem solving in Linear Algebra." Although he did not explicitly discuss his task design, the GLU instructor reported that he modeled valuing mathematical processes and encouraged students "to recognize that there's many alternative solutions to problems or ways to approach problems, all of which . . . can be equally valid."

CBMS (2012) described Linear Algebra as, after Calculus, "the most powerful, comprehensive theory that teachers will encounter" (p. 57). Moving between concrete and abstract concepts was a common big idea for

three instructors, as were developing an understanding of matrices and eigenvalues, and solving systems of equations. CBMS described Linear Algebra as an important transitional course before learning proofs because of the natural flow from computational examples to abstract properties and mathematical objects. Beyond the movement from concrete and abstract, CBMS also described big connections, including (a) generalizing number algebra to matrix algebra and reflecting on resultant properties, (b) interpreting matrices geometrically to connect “solving equations and finding inverse functions” (p. 58), and (c) classifying solution sets of systems of linear equations.

To encourage change in undergraduate mathematics teaching, MAA (2018) described several classroom practices aimed at supporting access to mathematics for all students. MAA stressed the importance of instructors distancing themselves from traditional lectures, using active learning, and building class communities to support learners from diverse backgrounds. The instructors we interviewed reported classroom practices of straight lecturing (MRU), a more interactive lecturing style (MUU, SRU), and regular use of group work and active participation in classwork (GLU).

MAA (2018) further described classroom practices to support productive struggle and class interaction by “creating a safe space for incorrect answers” and “focusing on reasoning” (pp. 5-6). MAA drew on Stein et al. (1996) and Boaler (2015) to recommend integrating high-level and open tasks that allow multiple solution strategies—tasks for which “there is not a predictable, well-researched approach or pathway explicitly suggested by task instructions” (p. 31). Three of the four instructors explicitly referenced similar ideas when discussing their classroom practices, pointing out the importance of students seeing more than one valid strategy (GLU, MUU) or adapting strategies to new situations and explaining how and why they work (GLU, SRU).

MAA (2018) recommended assessment practices, including using formative assessment cycles and summative assessment when appropriate. The Linear Algebra courses we explored used similar summative assessments (e.g., quizzes, exams) in addition to assessments that have the potential to be part of a formative assessment cycle (e.g., homework and computer lab activities). One instructor (SRU) mentioned ongoing struggles to grade and return assessments quickly, and he suggested that he might be open to using formative assessments if he had better strategies for providing immediate feedback. MAA (2018) also recommended providing clear learning goals to students as a course design practice. One instructor (MUU) provided learning goals in his syllabus, but none of the Linear Algebra instructors described using learning goals to support transparency in their teaching.

In summary, CBMS (2012) described certain big ideas of Linear Algebra as critical for future teachers and MAA (2018) described instructional practices to support meaningful learning. Instructors reported big mathematical ideas, similar to those recommended by CBMS, although no instructors had integrated specific examples for future teachers. The GLU instructor reported using group work (i.e., computer lab activities) regularly in class. Based on MAA recommendations, Linear Algebra courses can support deeper learning by all students, including those from communities traditionally denied access, using groupworthy tasks. All instructors reported a common set of assessments, including three exams and a final exam with weekly homework or quizzes.

Secondary Mathematics Methods

Similarities and differences emerged among the big ideas reported by the four Secondary Mathematics Methods course instructors. The first common theme that appeared was the opportunity for M-PSTs to learn about ways in which students conceptualize mathematical ideas. The M-PSTs had experiences in which to develop their understanding of student thinking while designing, implementing, and reflecting on their lessons. For example, the MUU instructor provided M-PSTs with the opportunity to interview a student to ascertain the student's beliefs about mathematics and knowledge of a particular mathematical topic. M-PSTs developed questions and performance-based tasks that they would pose to the student during the interview. In a written report, M-PSTs described the student's mathematical knowledge and beliefs based on their analysis of data gathered during the interview. Emphasizing the importance of M-PSTs' reflection after teaching, the GLU instructor shared questions that she asked M-PSTs in an effort to encourage them to investigate the student's learning: "As you are talking to the student [during the implementation of your lesson], what information did you gather, what did it tell you about, and then what support did you need to provide as a result of that?" The instructor used such questions to support M-PSTs' development of mathematics lessons tailored to the students' individual and collective needs.

The second big idea highlighted across all Secondary Mathematics Methods courses was the exploration of a variety of instructional materials that are supported by current research and policy documents. The instructors described the goals and activities that they used to engage M-PSTs in reviewing the curriculum and resources. For example, the GLU instructor included a goal that addressed this learning opportunity: "to acquaint the teacher assistant with available instructional resource materials, such as curricula, professional journals, and relevant research." The MUU instructor described a specific activity in which M-PSTs explored and critiqued textbooks. She said:

I have them look at materials [the traditional course sequence versus integrated math courses] and evaluate them, and that's the subject of again, usually a class discussion about what they think the opportunities are that are afforded by these textbooks, versus traditional textbooks, the challenges of teaching math in this way.

M-PSTs in her course then observed and evaluated the implementation of integrated mathematics curricula. The instructors reported that engaging M-PSTs in discussions about reform-based mathematics curricula and relevant research studies was likely to support M-PSTs in developing new perspectives on mathematical learning and teaching, as their prior mathematics learning experiences were typically associated with traditional textbooks.

The Methods course at three universities (i.e., GLU, MRU, and SRU) included an iterative cycle of designing and implementing lesson plans that were integrated into the required field experience components, thereby connecting standards and research with practice. The GLU instructor, for example, stated, "We spend a great deal of the semester focusing on this [teaching-learning] cycle . . . you start developing plans . . . then you implement the plans, and then the cycle goes around." Similarly, the SRU instructor emphasized planning and implementing mathematical lessons; M-PSTs in her course had the opportunity to learn about facilitating

classroom discourse and using appropriate instructional strategies in the field, and they concurrently took a methods course in which they discussed and reflected on related readings. Such practices reflected the instructors' efforts to enable M-PSTs to connect the instructional practices they had learned about across multiple courses. The opportunity to teach lessons in field experience classrooms while taking related courses also supported M-PSTs' learning about instructional strategies (e.g., facilitating discourse and designing coherent lesson plans) and reflection on their learning in practice.

While all four courses emphasized M-PSTs' development of pedagogical content knowledge, the course offered by MRU emphasized M-PSTs' development of specific mathematical content for teaching. The MRU course was the only one among the four in which developing school mathematical concepts was an explicit course objective. The syllabus highlighted the objective as follows: "The mathematical topics that we will examine are ratios and proportional reasoning . . . and quadratic relationships and factoring. These are BIG ideas in middle school and early high school mathematics, and they are important for reasoning algebraically." M-PSTs in this course were expected to keep a three-ring binder of problems exemplifying these topics. The instructor stated in her syllabus, "One of your greatest assets in understanding students' mathematical thinking is understanding and deepening your own mathematical thinking." M-PSTs generated mathematical conversations with each other, reflected on their own mathematical knowledge around these mathematical concepts, and used their mathematical knowledge to design problem sequences for students. They submitted the binder of problems to receive feedback and were awarded points for thoroughness, organization, explanations, analysis of targeted problems, quality of problem sequence and discussion, and mathematical correctness. This instructor's focus on a few key secondary mathematical topics and her assessment strategies for providing feedback on M-PSTs' analysis of targeted problems on these topics are worth noting, as most other courses were organized around pedagogical foci rather than connections between mathematical content and the design of problem sequences.

Attention to the specific goals and activities described above is well supported by mathematics education research and recommendations from professional organizations. For example, AMTE (2017) and NCTM (2014) recommended that mathematics teachers use evidence of student thinking to develop and revise their lessons. Anticipating students' thoughts about mathematics is relevant to knowledge of content and students, an important component of the mathematical knowledge needed for teaching (Ball & Forzani, 2011; Ball et al., 2008; Shulman, 1986). Instructors of the Methods courses in this study emphasized this knowledge domain, offering M-PSTs opportunities to interview students, learn about students' mathematical beliefs and knowledge, and develop lessons based on these analyses of student learning.

AMTE (2017) recommended providing M-PSTs with multiple opportunities to learn to teach through clinical experiences with coherent, developmentally appropriate content. Learning about teaching through instructional design addresses the knowledge of content and teaching which teachers need to develop prior to and during their careers (Ball et al., 2008). In most of these Methods courses, the M-PSTs experienced learning cycles in which they planned, implemented, and reflected on their lessons as they utilized specific instructional strategies (e.g., orchestrating classroom discourse). The M-PSTs also learned about the importance of posing purposeful questions to probe students' mathematical ideas and make mathematical structures visible, which is a critical

component of mathematics teaching practices (NCTM, 2014).

Knowledge of appropriate instructional materials and their characteristics is another essential domain of teachers' knowledge development (e.g., AMTE, 2017; Ball et al., 2008; Hill et al., 2005). Instructors of the Methods courses attended to this need by describing goals and activities that were aimed at supporting M-PSTs' learning of reform-based curricula and resources. In their courses, M-PSTs learned about these resources as they critiqued the content of textbooks and observed how students learned mathematics through the books.

Finally, learning to teach mathematics requires a primary focus on mathematics and flexible knowledge of school mathematics (AMTE, 2017; CBMS, 2012; NCTM, 2000). The unique layout of the MRU Methods course was closely aligned with this recommendation; the central focus of the course was constructing mathematical ideas and their connections. M-PSTs in this course had multiple opportunities to solve mathematical problems prior to designing a lesson plan based on specific mathematical topics. The attention to learning about school mathematics and its connections was possible in this course because the mathematical concepts were purposefully selected around big ideas, similar to the pedagogical focus selected in the other Methods courses. The instructor emphasized that to fully understand students' mathematical thinking, M-PSTs should deepen their own mathematical understanding. Instead of grading all the mathematical problems that M-PSTs had solved, the instructor assessed a few random problems for thoroughness, organization, explanations, analysis, discussion, and mathematical correctness. This selectivity enabled a balanced focus between mathematical content and pedagogy.

In summary, the focus of the Methods instructors on developing M-PSTs' knowledge of students' mathematical thinking and reform-based curricular materials, as well as engaging them in mathematical teaching practices through an iterative cycle of lesson development, implementation, and revision, is well supported by AMTE (2017) and NCTM (2000, 2014). These two organizations highlight the importance of high-leverage mathematical practices that M-PSTs can develop to promote all students' mathematical learning through a coherent series of tasks that promote reasoning and sense-making.

Teaching in a Diverse Society

Each of the four secondary mathematics teacher education programs required students to complete a course related to teaching diverse students. The title of this course varied across programs; however, to protect anonymity, we gave all courses the generic title, Teaching in a Diverse Society (TDS). These courses were all general education courses; therefore, the curriculum was not subject-specific, and students from multiple educational disciplines took these courses together. Several themes emerged from the big ideas and objectives provided by the TDS instructors.

First, all TDS instructors highlighted multiple aspects of diversity present in the United States. For example, when the SRU instructor was asked about the big ideas of the course, she said:

We talk about race and ethnicity; we talk about class, gender, and sexual identity, exceptionality, like

special needs students. We talk about language, geography, religion. And really, my goal at the end is that students would . . . be ready to teach in a diverse environment.

No common aspect of diversity was explicitly mentioned by all four TDS instructors; however, race, culture, ethnicity, religion, sexual orientation, ability, and language were each mentioned by three of the instructors.

The second theme emphasized across the TDS courses was that schools are situated in historical, sociopolitical, and geographic contexts. For example, the MUU instructor emphasized the local context and included “understand the impact of family and community in the learning experiences of English language learners in the classroom” as a course outcome. Taking a national approach to context, the GLU instructor pointed out the importance of M-PSTs’ “understanding how their work in the classroom and in the schools is a part of democratic practice in the United States.”

The third and most prevalent theme acknowledged across all four TDS courses was the impact of the first two themes (i.e., Theme 1: diversity; Theme 2: historical, sociopolitical, and geographic contexts) on educational opportunities in particular schools and for certain learners. The TDS instructors highlighted strategies that they used with M-PSTs to raise their awareness of these issues and potentially mitigate these effects. For example, the MUU instructor mentioned opportunities that she offered for M-PSTs to engage in investigating students’ school experiences, including reading and discussing articles such as “Nothing to Do: The Impact of Poverty on Pupil’s Learning Identities” (Muschamp et al., 2009) and “Barbie Against Superman: Gender Stereotypes and Gender Equity in the Classroom” (Aksu, 2005). In addition, the M-PSTs at MUU analyzed U.S. federal laws developed to ensure all students’ access to education (e.g., Individuals with Disabilities Education Act [IDEA], McKinney-Vento Act [protects the rights of homeless children]).

The TDS instructors also mentioned critical reflection as an important activity for M-PSTs in developing dispositions and skills for teaching diverse learners. The MRU instructor noted this focus on reflection in her syllabus: “We will explore various realms of diversity . . . [A]s part of that exploration we will engage in significant reflection, written and oral, personal and collective, challenging our assumptions, and questioning our beliefs.” Similarly, the GLU TDS course was described as “grounded in the idea that an essential aspect of good teaching is having the time and space to reflect upon the kinds of issues that impact your pedagogy and instruction.”

To summarize, the TDS instructors highlighted the following themes: (a) Student diversity comes in many forms; (b) Schools are situated in historical, sociopolitical, and geographic contexts; and (c) Student diversity and school context often result in producing inequitable opportunities for particular learners. These focal points, related to the impact of diversity and contextual factors on educational opportunities reported by TDS instructors, are supported by recommendations from professional mathematics (e.g., MAA) and mathematics education organizations (e.g., AMTE, NCTM). In fact, attention to historical inequities in mathematical learning opportunities was highlighted front and center in NCTM (2000) as the first principle, in NCTM (2014) as the first essential element, in AMTE (2017) as Assumption #1, and in the Manifesto of MAA (2018).

NCTM (2000) expressed concerns about pervasive low expectations, tracking practices, and a less challenging mathematics curriculum for “students who live in poverty, students who are not native speakers of English, students with disabilities, females, and many nonwhite students” (p. 13). They highlighted engaging curriculum, the use of technology, enhanced assessment practices, and increased attention to mathematics processes (beyond memorization and symbolic manipulation) as possible mitigation strategies to increase equity in mathematics classrooms. NCTM (2014) extended these claims, providing a set of “productive beliefs” that “influence the access that students have to effective instruction, high-quality curriculum, and differentiated learning supports” (p. 63), highlighting the importance of providing equitable (differentiated as needed) experiences that lead to positive learning outcomes for all students, especially those who have been traditionally underserved by educational systems.

AMTE (2017) took a strong parallel stance toward the need for teacher education programs’ commitment to preparing teachers who have the skills and dispositions to teach all learners: “Assumption #1: Ensuring the success of each and every learner requires a deep, integrated focus on equity in every program that prepares teachers of mathematics” (p. 1). The authors repeatedly emphasized the disparate opportunities resulting from historic discrimination and sociopolitical factors, and stressed the importance of preparing teachers who are advocates for their students with these disparities in mind:

Well-prepared beginning teachers embrace and build on students’ current mathematical ideas and on students’ ways of knowing and learning . . . They also attend to developing students’ identities and agency so that students can see mathematics as components of their cultures and see themselves in the mathematics. (p. 13)

They recommended opportunities for M-PSTs to critically analyze current mathematics education systems, challenge deficit views about student learning, recognize the key roles that identity and power play in mathematics education, and spend time in community settings to learn from and about students, families, and communities.

MAA (2018) continued this call for both recognition that these systemic inequities exist, and action to change the status quo:

Inequity exists in many facets of our society, including within the teaching and learning of mathematics. . . . We owe it to our discipline, to ourselves, and to society to disseminate mathematical knowledge in ways that increase individuals’ access to the opportunities that come with mathematical understanding. (p. vii)

The authors described the statistical disparities of underrepresented populations among both mathematicians and university students in mathematics departments and encouraged instructors to be aware of implicit and explicit messages that are sent to students about who “belongs” in mathematics.

In a nutshell, the focus of the TDS instructors on multiple aspects of student diversity, the multiple layers of context in which this diversity resides, and the resulting inequitable learning opportunities for some students is well supported by the recommendations of professional mathematics and mathematics education organizations, including NCTM (e.g., 2000, 2014), AMTE (2017), and MAA (2018). All three organizations highlight the

importance of equitable opportunities and outcomes for all students, encouraging attention to all aspects of mathematics teaching and learning (e.g., assessment, curriculum, and technology).

Throughout the Findings section, we have reported on our exploration of the “big ideas” of three required courses (i.e., Linear Algebra, Secondary Mathematics Methods, and Teaching in a Diverse Society) in secondary mathematics teacher education programs at four universities. We have highlighted emergent themes across the courses and examined them through the lenses of recommendations from mathematics and mathematics education professional organizations (i.e., AMTE, CBMS, MAA, and NCTM). In the final section, we discuss what we learned from this analysis and the implications for research and practice in mathematics education.

Discussion

Although professional mathematics and mathematics education organizations have provided recommendations indicating that secondary mathematics teachers need opportunities to learn about mathematics, mathematics teaching, and equity issues in mathematics teaching and learning (AMTE, 2017; CBMS, 2012; MAA, 2018; NCTM, 2000, 2014), little is known about how these recommendations are integrated into secondary mathematics teacher education programs (NRC, 2010). To begin to investigate this question and promote dialogue about the integration, this preliminary study highlighted ways in which course goals and big ideas in three courses (i.e., Linear Algebra, Secondary Mathematics Methods, Teaching in a Diverse Society) across four secondary mathematics teacher education programs emphasized areas related to mathematics learning, teaching, and issues of equity and access, as recommended by policy documents.

Across all courses, we noticed that instructors addressed many policy and research recommendations, both explicitly and implicitly. In fact, course instructors reported many big ideas that were closely related to these recommendations. For example, the GLU Linear Algebra instructor pointed to the strong community of mathematicians and mathematics educators in his department as leading him to experiment with multiple teaching strategies that he hoped would align with his students’ future teaching needs. Additionally, the MRU methods course centered on the “reconstruction” of school mathematics (e.g., proportional reasoning and integers) to address M-PSTs’ development of big mathematical ideas for teaching. Finally, the MUU TDS instructor provided opportunities for M-PSTs to engage with their students’ families and communities to demonstrate their valuing of the important knowledge located in these contexts.

Our investigation led us to note similarities in the courses across universities: (a) All Linear Algebra instructors addressed mathematical big ideas, such as moving between the abstract and computational, studying aspects of matrices and systems of linear equations, and developing computational tools; (b) All Secondary Mathematics Methods instructors focused on the essential teaching practices that are supported by AMTE (2017) and NCTM (2000, 2014), such as designing, implementing, and reflecting on high-quality mathematics lessons based on the evidence of students’ mathematical thinking; and (c) All instructors of Teaching in a Diverse Society addressed aspects of diversity, historical and sociopolitical contexts of learning, and the inequitable access to meaningful

mathematics that often results from these factors.

Although the universities required similar versions of these three courses, each course was also unique. Experiences of M-PSTs across these programs will differ due to many factors, including geographic context, program emphases, and the priorities of the course instructor. In addition, we noted important differences in content and practices across courses. The Linear Algebra instructors used different types of instructional strategies; for example, the GLU instructor uniquely described a set of hands-on computer lab tasks that students completed as group assessments. In Methods, a distinctive big idea was reported by the MRU instructor, who structured course content around key school mathematical ideas, of which M-PSTs must deepen their understanding, instead of a core set of mathematics teaching practices. Different areas of emphasis emerged across the TDS courses. For example, the SRU instructor emphasized the importance for M-PSTs to “develop a personal vocabulary and voice for discussing and writing about difference,” and the MRU instructor suggested that teachers seek out “culturally relevant materials” for their classrooms.

Conclusion

This study was intended to build on both existing research (e.g., Ball et al., 2008; Fuson et al., 2005; Hill et al., 2005; Turner et al., 2009) and professional recommendations for mathematics teacher preparation (e.g., AMTE, 2017; CBMS, 2012; MAA, 2018; NCTM, 2000, 2014), and to encourage researchers to explore areas that are less well investigated. For example, we know little as a field about which aspects of secondary teacher education programs resonate with M-PSTs, how particular program characteristics impact the learning of M-PSTs’ future students, and which learning trajectories are productive for M-PST learning across the program. We acknowledge that, although recommended by AMTE and CBMS, creating coherent programs across multiple departments and disciplines is a time-consuming challenge that is often unrecognized and unrewarded by universities. We wonder what creating a culture of communication among the dozens of faculty who support future teachers at each university would look like. We also wonder how such a culture could naturally build coherence by asking: What are the big ideas of our program? What are the fundamental ideas we want threaded throughout the program? How do we ensure that students have multiple opportunities to encounter these cumulative ideas through the program?

From the findings of this study, next steps could include an investigation of how “big ideas” play out in written curriculum (e.g., textbooks, course materials, and other resources) and implemented curriculum (e.g., classroom instruction) as well as an investigation of how M-PSTs perceive the opportunities provided throughout their secondary mathematics teacher education programs. What would be the benefits of an in-depth exploration of each of our programs? How would such a conversation get started? How would it be sustained? For example, from an equity stance, how might TDS instructors interact with mathematics and methods instructors to stimulate conversations about opportunities for discipline-specific, equity-related experiences? Are there institutions in which these connections are being productively implemented, and—if so—how can other programs learn from them?

The integration of course goals and policy documents promotes dialogue related to the preparation of secondary mathematics teachers. The study reported here, in which we connected big ideas from multiple courses with recommendations from recent policy documents, promises to inform teacher educators, especially new mathematics teacher educators in the field. Future work will provide insights into how other recommendations from policy documents play out in practice; this current work is an initial step to illustrate big ideas of required courses commonly offered in secondary mathematics teacher education programs.

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