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Promoting Prospective Mathematics Teachers' Professional Vision on a Whole-class Reflective Discussion: Contributions of Digital Resources

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Abstract

The present study aims to understand how the professional vision of prospective teachers (PTs) regarding whole-class reflective discussion develops as they explore a multimedia case (MC) in a mathematics teacher education course. The data come from the written productions of seven groups of PTs (N=15) related to whole-class discussion of one 6th grade lesson in two sections of the MC: first, they have focused on the teacher's lesson plan and on her intentions for the whole-class discussion aiming at the students' algebraic thinking; and in the second, they have analysed two video episodes of that lesson phase. The results show an integrated development of two processes of PTs' professional vision – noticed events and the nature of the reasoning about them – with significant evolution from one section to another as they explore the MC. The study shows PTs' alternative ways of accessing and making sense of the complexity of both ambitious and authentic teaching practices that can also be employed in at distance teacher education and mitigate the consequences of the impossibility to access the classrooms, such as that resulting from the recent COVID-19 pandemic.

Introduction

Teachers' professional vision has been considered a key element in the learning of the prospective teachers (PTs), since it is associated with decision-making in the practice of their future profession. It involves important dimensions, such as attending to relevant aspects of teaching and learning in the classroom and reasoning about them based on their knowledge and understanding (Seidel & Stürmer, 2014; van Es & Sherin, 2002). However, preparing PTs to learn to notice involves complex work (van Es & Sherin, 2017), especially in what concerns an “ambitious mathematical pedagogy” that values collaboration between students to solve demanding tasks, and the communication of mathematical ideas, such as in inquiry-based mathematics teaching (Menezes et al., 2015; van Es et al., 2017).

Multiple studies have highlighted the potential of using videos to enhance the noticing skills of both teachers and PTs (Ozdemir Baki & Kilicoglu, 2021; Paksuniemi et al., 2021; Schäfer & Seidel, 2015; Sherin & van Es 2009; Shin, 2019; Woods, 2021). However, the development of PTs' noticing skills depends on integrated strategies

that guide the use of videos in teacher education context (Uygun, 2020; Warshauer et al., 2019). In those settings, research has been carried out about the PTs' professional vision regarding how they understand instructional principles and strategies based on generic pedagogical knowledge (PK) (Schäfer & Seidel, 2015; Seidel & Stürmer, 2014), and also how the PTs' subject matter knowledge (SMK) and pedagogical content knowledge (PCK) can support their noticing skills (Liston, 2015). Other studies have focused on how PTs notice students' mathematical thinking in general or related to specific domains such as algebraic or statistical reasoning (Callejo & Zapatera, 2017; Shin, 2019; Uygun, 2020).

Although there is a considerable body of studies on PTs' noticing, they have mainly focused on using resources that portray classroom episodes, with reduced attention on the teacher's planning. Moreover, research on PTs' professional vision related to promoting whole-class reflective discussion to develop students' algebraic thinking is still scarce. Examining different emerging areas of mathematics teacher noticing studies, Dindyal et al. (2021) highlight the importance of recent studies that "have continued to explore further the terrains of teacher noticing by focusing on existing ideas related to perception and the dialogic nature of classroom interactions" (p. 5). Researchers argue that such interactions demand mathematical tasks that challenge the students "in a way that builds connections to the mathematical meanings" (Stein & Smith, 1998, p. 268). Such meanings would support the emergence of diversified resolution strategies that are then discussed and legitimised among peers, in small groups, and collectively (Hähkiöniemi, 2017; Menezes et al., 2015). In this regard, the whole-class reflective discussion plays a fundamental role in mobilising and sustaining dialogic inquiry processes (Wegerif, 2010; Wells, 2004), for instance with important effects on the development of students' algebraic thinking (Oliveira & Mestre, 2014). However, the teacher's practice to promote it is so demanding that it is compared to the orchestrating activity and, therefore, needs to be prepared before that moment (Cengiz et al., 2011; Stein et al., 2008).

That practice is particularly demanding for the PTs, who only have classroom experience as students, and thus are less likely to develop a deepened professional vision of classroom events (Seidel & Stürmer, 2014). Thus, we consider that besides recognising the actions by which a collective discussion takes place, PTs need to understand how that practice is planned and the teacher's intentions for the lesson. In this study, classroom videos and several other artefacts collected from an experienced teacher's practice were selected and articulated in a MC to foster the PTs' analysis on specific aspects of the whole-class reflective discussion aimed to mobilise students' algebraic thinking in a 6th-grade class.

Conceptual Framework

Multimedia Cases and PT's Professional Vision

With a long tradition in teacher education, the use of cases from practice is founded on the notion that PTs need to develop conceptual and contextual understanding of the teaching processes, thus establishing connections between theory and practice (Gallagher, 2019). Since classrooms entail a great level of complexity, teaching and learning should be addressed in a coordinated manner in the PTs' preparation, namely, through cases that allow them to examine the teaching practice and its relation to students' activity, interactions between the different agents that unfold, and how these affect what is taught and learnt (Han et al., 2013; McGraw et al., 2007). MC

have been produced for teacher education with video and audio components, beyond written texts, providing a wide range of information and eliciting interactivity (Gallagher, 2019; Han et al., 2013). The expansion of navigation and communication possibilities of this resource can support more consistent analyses, since it allows multiple readings and interpretations of classroom practice elements, leading PTs to deepen their reflections in several domains (Han et al., 2013; McGraw, et al., 2007; Uygun, 2020). In particular, the use of video has afforded “the opportunity to study the characteristics of effective teaching and then, to apply that knowledge to professional learning” (Blazar et al., 2018, p. 131). In initial teacher education programs, this opportunity has showed even more relevant when field experiences were not possible due to the COVID-19 pandemic (Perry et al., 2022).

The decomposition and approximation of the teaching practice (Grossman et al., 2009), through video analysis, can favour the development of teachers’ noticing skills, leading them from skimming to in-depth interpretations (Estevam et al., 2017; McDuffie et al., 2014). This resource’s use in initial and in-service teacher education has been diversified, both regarding the source and nature of the videos and the focus of the analysis that is intended. Based on full lesson videos, Santagata et al. (2007) asked specific questions to direct PTs’ analysis to the lesson’s goals, to the students’ understanding and the teaching approach. In other studies, shorter video clips, sometimes extracted from teachers’ classes, are analysed, and the issues that seem pertinent to them are discussed (van Es & Sherin, 2002). In the study by Ozdemir Baki and Kilicoglu (2021), six video episodes containing different types of evidence related to students’ thinking were selected to be analyzed by experienced teachers. The results showed that oral evidence of student thinking was more considered than non-verbal evidence. To advance beyond a general analysis by the PTs limited access to teachers’ actions, several studies report the selection of short video excerpts focused especially on students’ mathematical thinking (Jacobs et al., 2010; Rodrigues et al., 2019; Sherin & van Es, 2009). In some cases, PTs were able to identify mathematical elements in use by students nevertheless they mobilized them in a limited way to interpret students’ understanding (Cabral et al., 2021; Callejo & Zapatera, 2017; Shin, 2019). In other words, the development of professional noticing skills relates not only to highlighting and describing critical events in classroom but it also requires making sense of those events based on the (prospective) teachers’ knowledge (van Es & Sherin, 2002). For PTs to make sense of such events, a process of mobilisation of knowledge of both general principles and the specific mastery of their area of activity must be triggered (Estevam et al., 2021; Schäfer & Seidel, 2015; Seidel & Stürmer, 2014).

In this sense, the development of the teacher’s professional vision may function as a productive lens to understand teacher learning (Sherin & van Es, 2009). Professional vision is considered as “socially organised ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group” (Goodwin, 1994, p. 606). In what PTs’ learning concerns, activities that guide the use of video in teacher education require the coordination of various aspects of the teaching practice strategically embedded for this purpose (Kang & van Es, 2019).

Two distinct subprocesses of the dynamic interplay of the teacher’s professional vision have been described by Sherin (2007): *selective attention* and *knowledge-based reasoning*. The *selective attention* highlights the aspects the teacher chooses to guide his/her attention to, amid the classroom complexity. For example, given the various ideas the students explain, the teacher recognises those that may be important to the purposes of the class. The

knowledge-based reasoning concerns how the teacher assigns meaning to what he/she sees, and draws conclusions based on what is known about the analysed event (Sherin & van Es, 2009). These two subprocesses, that do not develop separately, function as major components in the perceptual and interpretive processes (Sherin & van Es, 2009; van Es & Sherin, 2002). They interrelate dynamically, since “on the one hand, what stands out to a teacher will certainly influence the reasoning that takes place. But in addition, a teacher’s expectations and knowledge also drive what a teacher perceives” (Sherin, 2007, p. 385).

Sherin and van Es (2002) distinguishes three forms of reasoning related to teachers’ professional vision, but highly interrelated: *description*, *explanation*, and *prediction*. According to Seidel and Stürmer (2014) the *description* represents the ability to accurately distinguish and report the relevant aspects of the focused teaching and learning components. It proved to be an essential support for explanation or prediction. *Explanation* refers to the ability to mobilise what is known to reason about classroom events, linking the aspects described to their fundamentals. *Prediction* refers to the ability to, from what was observed in the situation, make inferences about students’ learning, predict/suggest consequences of teaching strategies or learning outcomes. This process showed the need to relate noticed events to broader or more consistent elements of teaching and learning. Due to their demands, research indicates that explanations and predictions are usually less achieved by prospective teachers than by experienced teachers (Schäfer & Seidel, 2015; Seidel & Stürmer, 2014).

In this study, the components of the teacher’s professional vision – *selective attention* and *knowledge-based reasoning* – and the aspects that conceptualize the nature of reasoning – *description*, *explanation*, and *prediction* – function as lenses to understand how the professional vision of prospective mathematics teachers on *whole-class reflective discussion* develops throughout the exploration of a MC. Next, we delve into aspects of that phase of the lesson.

Whole-class Mathematics Discussions based on Dialogic Inquiry

In clarifying the term *dialogic*, Wegerif (2010) describes that it literally means “reason across difference” (p.25). In this process, students are invited to explain their own meanings, but that also to consider those of others, that is one expects engagement, understanding, and exploration of the different points of view that are shared (Wegerif, 2010; Wolfe & Alexander, 2008). Without leaving aside their references, students may agree or disagree, but above all, weave coherent lines of inquiry, while challenging and deepening their understandings (Wegerif, 2010; Wolfe & Alexander, 2008). Wells (2004) emphasizes the importance of the “whole-class reflective discussion” in creating a collaborative environment “in which knowledge is co-constructed, as students and teacher together make meaning on the basis of each other’s experiences” (p. 160).

Similarly, promoting whole-class reflective mathematics discussions begins with the student’s activity and focuses on their discursive participation. Due to the challenges that this involves, Stein et al. (2008) propose five practices for discussion facilitation. The first, Anticipating, refers to aspects such as setting objectives for the class, selecting or preparing challenging tasks for students, predicting possible students’ resolutions and difficulties or errors, and have clear the intentions. Monitoring involves supporting students in task solving and,

at the same time, preserving students' autonomy over their resolutions. These two practices assist the teacher in selecting and sequencing students' resolutions that will be shared and discussed with the class. From time to time, the practice of selecting resolutions needs to consider the aspects that distinguish them from each other, and that may be relevant to students' learning. The sequencing also needs to be judicious, namely considering the resolutions developed by most groups, or that may contain some error, or starting from the simplest to the most complex students' strategies (Stein et al., 2008). The fifth practice – connecting student responses – has as its main goal “to have student presentations build on each other to develop powerful mathematical ideas” (p. 330). Enacting such practices imply important challenges to the teacher but that can be overcome if students “are publicly credited as the ‘authors’ of their ideas” (p. 332), i.e., by nourishing an environment that recognizes and promotes student's mathematical authority.

Some studies carried out with mathematics teachers who adopted an inquiry-based mathematics teaching report that they seek to promote the mathematical quality of students' presentations as well as creating an environment conducive to discussion and promoting a culture of genuine respect and interest in others' ideas (Hähkiöniemi, 2017; Menezes et al., 2015). In that context, the teacher's questioning stands out as “an essential component of classroom interaction” (Hähkiöniemi, 2017, p. 973). Within this teaching perspective, what is at stake is not the accuracy or validity of student's ideas by themselves, but how they are triggered and leverage the extension or improvement of student's thinking and that of the others (Wolfe & Alexander, 2008). These activities echo in support of a dynamic of “talk and action that provides a platform for the development of common knowledge” (Wolfe & Alexander, 2008, p. 10), linked to the encouragement and conditions for students' participation.

In this study, we understand that the development of the PTs' professional vision on this set of aspects underlying the collective discussion, considered from the point of view of dialogic inquiry, can be supported by exploring diverse multimedia resources that come from the practice, including elements from the teacher's planning and videos of this phase of the lesson. Thus, this study seeks to understand how PTs' professional vision about whole-class reflective discussion develops as they explore one multimedia case in a mathematics teacher education course. Two research questions unfold from this main goal: (1) What aspects of whole-class reflective discussions do PTs attend to? (2) What is the nature of their reasoning on these aspects?

Method

Setting and Participants

This study was carried out in the context of one methods course untitled Teaching Practice and Laboratory (TPL) in a 4-year undergraduate Mathematics program, in the State of Mato Grosso do Sul – Brazil. The program qualifies PTs for teaching mathematics in upper elementary school (students aged 11-14) and secondary school (students aged 15-17). The course was offered in the 4th semester of the course, with the first author as teacher educator. Previously, the PTs have attended courses that aimed, separately, at PK or SMK development. TPL was the first course that focused essentially on the development of PTs' PCK. More specifically, it encompasses mathematics education themes related to the curriculum, planning and teaching approaches (inquiry-based mathematics teaching, communication in mathematics classroom, and task selection and design), and Algebra

teaching and learning processes. The complete course had 31 sessions, lasting 1h 40m each, mainly twice a week.

The instructional unit described in this study was based on the analysis and discussion of an online multimedia case, *The necklaces*, during 11 sessions. The case is hosted on an online platform (<http://rmfp.uel.br/>) with restricted access. The case brings together different multimedia resources that seek to highlight certain aspects of the teaching practice of an experienced teacher, in a lesson following an inquiry-based mathematics teaching (Menezes et al., 2015). The study's participants are 15 PTs who attended at least 75% of the sessions.

The Use of the Multimedia Case in the Course

The 6th-grade class from which the multimedia case (MC) was designed focused on the exploration of the task *The necklaces* (see Figure 1), aiming at students' algebraic thinking. The introduction of the MC in the course intended to promote PTs' knowledge about inquiry-based teaching and algebraic thinking to broaden their views on algebra teaching and learning. Thus, the instructional unit was designed to support the development of PTs' professional vision about an ambitious mathematics pedagogy (van Es et al., 2017).

Task *The Necklaces*

Inês made three necklaces with black and white beads, as shown in figures 1, 2, and 3.



Fig. 1

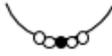


Fig. 2

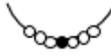


Fig. 3

Number of beads in the necklace

1. Find out the total amount of beads in each figure.
2. Without drawing, consider the pattern of this necklace sequence, how many beads would the necklace corresponding to figure 4 have?
3. How many beads would the necklace corresponding to figure 8 have?
4. How many beads would the necklace corresponding to figure 19 have?
5. Is there any necklace in this sequence that has 55 beads?
6. Describe the rule that determines the number of beads in any sequence figure.

Figure 1. Task *The Necklaces*, adapted from Pedro (2013)

The MC is organised chronologically (see Table 1). PTs first access the Before the Lesson (BL) section, which portrays the teacher's planning phase, where they are requested to analyse the teacher's lesson plan, the mathematical task, and the audio-recorded interview with the teacher on her intentions for this lesson. Then PTs analyse the Lesson (TL) section, which is organised according to the four phases of the lesson (Introduction, Students' autonomous work, Discussion and Synthesis), and where 10 video-recorded classroom episodes are provided. PTs are guided on the analysis of each artefact by open questions focused on the teacher's actions and how these may contribute to students' learning, and they are also asked to analyse students' written work on the

task (Cyrino & Oliveira, 2016). Finally, in the Reflection section of the MC, PTs are invited to analyse excerpts of the teacher’s audio-recorded interview which took place after the lesson.

Table 1. Landscape Multimedia Case *The Necklaces* Design

Sections of case	Teacher’s actions	Resources analysed by PTs	Timing of the teacher education sessions
Before the lesson	Anticipating the lesson	The task The necklaces Five audios on the teacher’s intentions for the task and each phase of the lesson; One detailed lesson plan with predictions on students’ activities and the intended teacher’s actions	Session 12 Sessions 13, 14
The lesson	Proposing the task	One video-episode	Session 15
	Monitoring students’ autonomous work	Three video-episodes	Sessions 16, 17
	Selecting and sequencing	Nine students’ written-solutions of the task	Session 18
	Discussing and connecting students’ responses	Two video-episodes	Sessions 19, 20
	Synthesising	One video-episode	Session 21
After the lesson	Reflecting	Five audios with the teacher’s reflections on each phase of the lesson after its conclusion	Session 22

Before PTs started exploring the MC, the teacher educator discussed some papers considered essential for supporting their work on the case, focusing on: the characteristics of direct teaching, inquiry-based teaching and mathematical tasks, cognitive demand levels and the mathematical activity developed in the classroom (Stein & Smith, 1998); development of algebraic thinking in upper elementary school; and mathematics teacher’s communicative actions. A synthesis that came out from those lessons served as a framework that could scaffold the PTs in their noticing skills (Warshauer et al., 2019). For the MC analysis, the participants were separated into seven groups. Subsequently, in a collective moment, the PTs held discussions with the teacher educator’s support on the aspects that most attracted their attention in the MC’s resources they analysed.

Data Collection and Analysis

The data for this study focus on the seven groups’ written responses to the questions provided in MC concerning the teacher’s actions about the collective discussion with the class, and their contribution to students’ learning,

both in BL and TL. The first dataset included the seven groups' written responses on their analysis of the lesson plan and of the teacher's interview regarding her intentions for the whole-class discussion in BL section (see Table 2).

Table 2. Description of the Resources Content, in the BL

Lesson plan	Summary of teacher's audio-interview (4m33s)
<ul style="list-style-type: none"> - The discussion will focus not only on the expression of the rule, but also on how students discovered it. - For the students' presentations, I will first choose those about which they reasoned by recurrence and/or the ones for which they used a table. Then, resolutions that used the rule in natural language will be chosen, and finally, those that used symbolic language to represent the rule. - During the students' presentations, the teacher should promote the groups participation in the discussions; to encourage students to recognise the different strategies/procedures for solving the task; to relate the ideas conveyed by the students' strategies with the formalised mathematical representations and to promote the acknowledgement of the importance of rules or generalisations. 	<ul style="list-style-type: none"> - Select different ways to solve the task. - Ask students to explain how they thought. - Gradually, have students get used to overcoming shyness to be in front of the class to explain their resolution. - Reach those students who have not yet been able to complete the task, or those who are not confident in their reasoning. - Draw students' attention to the aspects of the resolutions that can help in their understanding. - Students can perceive important aspects of the task through their classmate's speech. - Orchestrate the discussion so that students can reach a generalisation or systematise the rule. - Ask students to compare the different representations and strategies by analysing them from different perspectives.

The second dataset included the PTs' written responses to questions regarding two video-clips that they watched (Episodes 7 and 8 - about 5 minutes long each), in the TL section. Episode 7 concerns the beginning of the Collective Discussion phase, when students present their solutions to question 3 of the task (see Figure 2), and Episode 8, when students' answers to question 4 are compared and contrasted (see Figure 3).

Considering the research questions, we assumed an interpretative perspective in data analysis (Erickson, 1986), with data coding procedures based on the grounded theory (Charmaz, 2006) to identify the main ideas and meanings emerging from the data. We draw on the selective attention construct (Sherin & van Es, 2009) as an indicator of the aspects the PTs considered relevant in the video, since it highlights the ideas that became the main focus during their analysis. We also make use of the knowledge-based reasoning construct which covers the meanings that the PTs reconstructed about those ideas. In this study, those analytical strategies guided the analysis of the data that emerged from the PTs' exploration of the resources in their different formats.

In search of elements concerning whole-class reflective discussion that became PTs' main focus, the data were coded into two sections according to the grounded theory: the initial coding and the focused coding (Charmaz, 2006). We started with an identification of the aspects concerning whole-class reflective discussion that were significant to each PTs' group, in the set of written responses concerning that lesson's phase in two sections of the MC (BL and TL). An initial code was created for each aspect.

Episode 7	Some scenes
<p>The teacher invites representatives of four groups to present their solutions and asks them to justify their strategies. The first student explains that his group got to answer 17 because they saw that figure 3 had 7 beads, so they increased by twos until they got to figure 8. The second student shows that they performed the operations "$8+8=16$, $16+1=17$", explaining that this "one" is the black bead. The third one exposes that "8 on each side with the black bead gave 17", explaining that the figure number is the same as the number of white beads on each side of the necklace with the black bead in the middle. The fourth student registers the result on the blackboard and explains that "if each side has 8 white beads, the result is 16, plus the black bead, it equals 17".</p>	

Figure 2. Description of Episode 7, in the TL

From a large number of initial codes, we subsequently selected and grouped the ones that were more similar, creating units of greater analytical relevance. After that, a focused coding was initiated, conducted by comparative processes of the units with each other, based on data retrieving by the three researchers, to validate or review the recognition of their common or distinct characteristics. This analytical process resulted in six aspects highlighted by the PTs for both MC's sections (BL and TL), which were constituted as categories of analysis: Questions and prompts for promoting students' explanations (A1); Collaboration for collective mathematical learning (A2); Recognising other ways to solve the task (A3); Comparing resolution strategies (A4); Selected solutions and their presentation order (A5); and Students' mathematical authority (A6). Each category was recomposed into a two columns table, according to the source (BL and TL sections), with excerpts from each group's responses that best characterised it. Thus, it was possible to register the incidence with which each aspect was identified in each

section (BL and TL), by each group. Although the noticed elements in both sections were the same, we observed differences in the groups' incidence who were able to notice them, and with significant progress from one section to another.

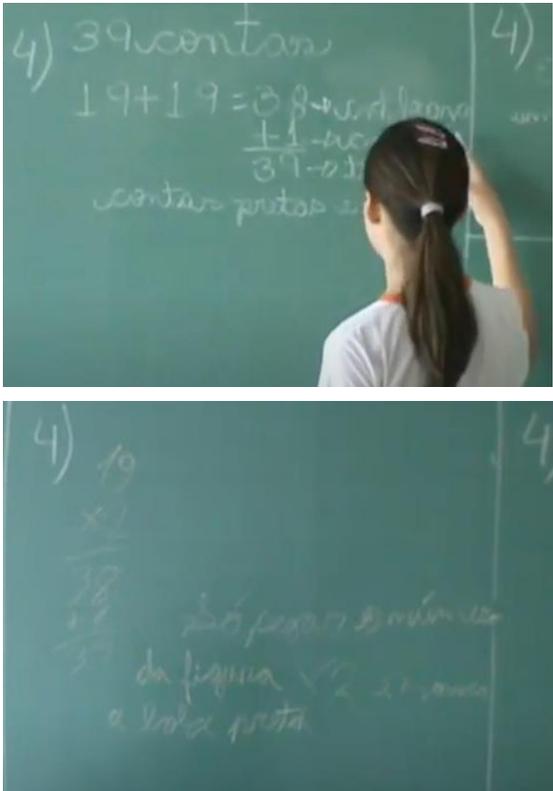
Episode 8	Some scenes
<p>The teacher invites representatives of four groups to present. The first student reports that starting from the total of beads in figure 8 (task's previous question), they added by twos until they got to figure 19, and that this was performed in a table. The second one states that they calculated "2×19, $38 + 1 = 39$". The third student shows that they calculated "$19 + 19 = 38$, then $38 + 1 = 39$", explaining the meaning of each component: "19 is the figure number (position) and also the number of white beads on each side of the necklace and that the "+1" is the addition of the black bead". Focusing on the two previous resolutions, the teacher questions the class about what is similar and what is different between them. Then, the fourth group's representative presents his resolution in natural language: "we took nineteen on each side, and we added, and the result was thirty-eight. Then, we added the black bead, placing the plus one, giving thirty-nine".</p>	

Figure 3. Description of Episode 8, in the TL

Later, based on the excerpts of PTs groups' answers provided in the tables representative of the six elements (categories), the data were codified under the framework concerning the nature of PTs' knowledge-based reasoning, according to the three ways of reasoning – *description*, *explanation* and *prediction* –, based on the literature review. The data analysis was performed by the first author, but three researchers altogether discussed the codification and the emerging categories, in several stages of the research. In the face of disagreements in the analysis, the researchers reviewed the criteria in use and its application to reach a consensus.

The seven PTs groups will be nominated as follows: João and Túlio (G1), Isac and Fred (G2), Alex and Lara (G3), Toni and Ari (G4), Davi and Joel (G5), Nina, Caio and Diana (G6), and Tainá and Íris (G7). The transcripts presented as data evidence are coded according to the nature of PTs' reasoning and its source (BL or TL; Episode 7 or 8).

Results

This section consists of six subsections that seek to represent each of the aspects of the whole-class discussion noticed by the PTs and the three ways of reasoning about them. As mentioned, certain aspects were noticed by more groups than others. Thus, to present them, we established an order that goes from the most to the least aspects the PTs noticed.

Questions and Prompts for Promoting Students' Explanations

Initially, when analysing the resources in the BL section, only three groups highlighted that, in the collective discussion phase, the teacher has the intention of asking students to explain their answers and how they got them. This idea was presented in the forms of description (G3; G4) or explanation (G5):

[the teacher] will ask the students to go to the board to present their resolutions and explain the method by which the result was obtained. (Description, G3, BL, Session 14)
... at this phase, the students, can learn to speak, to better expose their way of thinking to others. (Explanation, G5, BL, Session 14).

More expressively, in the TL section, through three descriptions (G1, G6, G7), five explanations (G2, G3, G4, G5, G7) and three predictions (G4, G5; G6), all groups recognise the importance of students' explanations and interpret the teacher's questions and prompts as a way to promote them:

The teacher keeps questioning the students' resolutions and asking them to speak loudly for the other students to hear. (Description, G1, TL-Ep. 7, Session 19)
... as usual, the teacher keeps questioning the students, so that they speak as clearly as possible, so the whole-class understands their peers' resolution. I believe that her main intention is to get clarity in their explanations, not so that only she understands it, but also for the other students to understand what their classmates have made ... (Prediction, G5, TL-Ep. 7, Session 19)

In the answers provided (in TL), G1 noticed that students' discourses should be directed to their classmates and not to the teacher. In the same line of reasoning and improving it from an explanation (in BL) to a prediction (in TL), G5 emphasises the teacher's action has the intention of leverage the student who is presenting (as interpreted in BL), but also to support the other students' understanding. Another result to be highlighted is the evolution of the reasoning nature within the TL case's section. From one video episode to another, G6 advances from a descriptive to a predictive way of reasoning:

The teacher wants to know what the student's thinking was; how they got to that result. (Description, G6, TL-Ep. 7, Session 19)
... it is necessary to questioning students so that, through their presentation, the peers perceive what they did wrong or how they manage to reach a rule in question 6. Because that rule is built and thought from

question 1 to 2, where there is an understanding of the task. (Prediction, G6, TL-Ep. 8, Session 20)

Thus, G6 makes inferences about the potential of the task's questions, which are strategically linked, to mobilise fundamental processes for promoting students' algebraic thinking – generalisation and its expression.

Collaboration for Collective Mathematical Learning

The role of collaboration for collective mathematical learning is a key element in whole-class reflective discussion that is noticed by the PTs. In the BL section, this is an aspect that is noticed by four groups (G1, G5, G6, G7), and that reveals in a description (G7) and several explanations, such as:

. . . this phase is important for students to come to a generalisation and also for those who have not yet understood the task. Perhaps the discussion is the moment where they will formalise their thinking. (Explanation, G5, BL, Session 14)

Possibly based on the teacher's intentions expressed in the BL interview, PTs begin to associate the whole-class discussion with the opportunity to involve all students in the mobilisation of their algebraic thinking, especially those who were not yet able to generalise or present symbolically the rule.

Subsequently, with the video analysis (in TL), the same groups (G1, G5, G6, G7) corroborate this idea and two other groups recognise it (G3, G4), and rely on it to provide explanations. G2 and G5 add more meaning to the interpretation of students' learning outcomes and make predictions. Next, situating the whole-class discussion as a chance to add something to the other's thinking, G5 seems to have understood it, from its dialogic dimension, as an opportunity to reason algebraically through difference:

We observed that some students had not yet realised that the figure number was the same as the number of the white beads on each side. So, the discussion of this issue helped to highlight this, especially in the resolution of Helena and Manuel. Another point that is well clarified is that the black bead never changes, we observe this in these students' resolutions. Thus, their explanations contributed to the understanding of those who had not yet visualised these patterns. (Prediction, G5, TL-Ep. 8, Session 20)

Thus, through some examples, G5 emphasises how important it was that the discussion clarified two fundamental points related to identifying patterns in the figure and consequently to get to the rule. The first explains the relationship between the figure number and the number of its elements, and the second produces meaning to the expression "+1" in the context of the task.

Also paying attention to certain aspects of algebraic thinking, emerging from the mathematical sequence, the prediction of G2 is associated with one of the objectives of the lesson – to identify the relationship between the variables –, and their collective development through collaboration:

At this stage of the lesson, even those who did not grasp what happens on each side of the necklace and its relationship with the figure number comes to realise – through their classmates’ resolutions and the teacher’s questions – that this dynamic helped them to understand that the relationship is valid for any figure number. (Prediction, G2, TL-Ep. 8, Session 20)

These data illustrate a deeper PTs’ understanding of the relationship between the teacher’s actions in whole-class discussion, the lesson’s objectives, and the promotion of students’ algebraic thinking.

Recognising Other Ways to Solve the Task

In the BL section, the PTs relate the whole-class discussion phase to students’ chance to recognise other ways to solve the task, expressed through three descriptions (G2, G3, G5) and an explanation (G6), for instance: “students can understand that there are other ways to solve the task” (Description, G3). In the TL section, this idea emerges in six groups, where we can find two descriptions (G1, G3), four explanations (G1, G2, G5, G7) and one prediction (G6). Initially, the PTs reason about the discussion as an opportunity to “open students’ horizon” (G2) as to the breadth of mathematical ideas on which they can rely.

However, with the analysis of Episode 8, the PTs recognise the importance of validating the different strategies through the attribution of meanings to support the collective discussion. In particular, G1 starts from a descriptive analysis on one aspect (Ep. 7), then pays attention to more aspects (Ep. 8), and establishes a relationship between them and elaborates an explanation:

[the students] will see other ways to solve the task (Description, G1, TL-Ep. 7, Session 19)

They will know that there is not only one way to solve . . . not by teacher or others saying that their answer is wrong. It is challenging for the teacher to make students understand that all resolutions are valid and that there are different ways to solve the same task, and that this depends on the knowledge acquired by each one. The knowledge differs, but all students can contribute. (Explanation, G1, TL-Ep. 8, Session 20)

This excerpt points out the PTs’ understanding about the importance of students acknowledging different ways to solve the task but that depends on how error, incompleteness, or difference are deemed by the teacher in whole-class discussion.

Comparing Resolution Strategies

In the BL section, this aspect draws the attention of three groups, conveyed through two descriptions (G3, G4) and one prediction (G2). Descriptively, G3 highlights the comparison of strategies and their feasibility for solving the mathematical task, although very concisely. Whereas G2, possibly based on the teacher’s previous intentions, predicts possible consequences of the teacher’s action to students’ perception of mathematically crucial strategies:

. . . have students compare resolutions to see what is feasible (Description, G3, BL, Session 14)

The discussion is important to counterstrategies of different resolutions, so that one can understand the usefulness of generalisation in some cases where the use of recurrence will not be reasonable... (Prediction, G2, BL, Session 14)

Although having not watched the whole-class discussion, G2 seems to infer that the teaching strategy of countering two different resolutions can facilitate students' understanding of the potential for algebraic generalisation and the limits of recurrence in solving both this mathematical task and others involving more complex situations.

In TL, all groups directed their attention to the action of comparing students' strategies, through three descriptions (G1, G3, G7), two explanations (G4, G5) and two predictions (G2, G6). It is important to emphasise that the PTs rely on ideas related to the specific characteristics portrayed in Episode 8 to reason about how the teacher's action unfolds. G4, for example, is attentive to the teaching event in which the teacher contrasts two ways of expressing the rule:

For students to compare the groups' resolutions on the board, the teacher asked what they had in common ... An example of that was the comparison of the resolutions of Helena's group and Manuel's group, so the students could recognise that one group used multiplication in their rule, and the other used the sum. (Explanation, G4, TL-Ep. 8, Session 20)

The PTs have noticed that the teacher's promotion of comparative processes of students' resolution strategies, which seek to put in evidence different ways of thinking, can help students in detailing each strategy. This action may shed light on points that connect those strategies with others and thus broadening the class mathematical knowledge.

Selected Resolutions and their Presentations Order

One result about the interpretive process of this aspect is the change of sense making on the observations, from one MC's section to the other. In the BL, the selection and sequencing of students' possible resolutions that is anticipated by the teacher are considered by five groups through two descriptions (G4, G6) and three explanations (G2, G3, G5). Then, in the TL section, the PTs present three explanations (G2, G5, G6) and two predictions (G3, G6) that express new views on this aspect. For example, PTs in G2 considered that only resolutions that were "clear to students" should be selected, but then they have noticed that this selection can be made more broadly:

Select resolutions that are clear to students who have not reached the final answer or are not yet too confident ... (Explanation, G2, BL, Session 14)

[the teacher] chooses several groups with different strategies to present, showing the students several ways to reach the correct answer, giving the student the possibility to know a different solution (Explanation, G2, TL-Ep. 7, Session 19)

Also G6, who had previously anticipated that the teacher would ask all groups to go to the blackboard, afterwards explain the teacher's criteria to select some of the student's resolutions and to sequence them. In other words, the group understands that the presentations start from the least to the most complex resolution in terms of generalisation and symbolisation processes developed by students. Moreover, PTs notice that the adopted sequence of students' presentations enabled the discussion of the differences and connections between the resolutions.

[the teacher] will mind that everyone goes to the blackboard to explain. (Description, G6, BL, Session 14)

The teacher requests that the group that used a recursive strategy presents its solution first, because it is the simplest. Then she selected two similar answers, but one solved by multiplication and another by sum so that students could understand that it is the same strategy, but using two different operations. Finally, to assure that all students understand everything that was made... she chose a group that used natural language, which is a 'description' of the idea used by the two previous groups. (Explanation, G6, TL-Ep. 7, Session 19)

The PTs interpret the order the teacher establishes for the presentations based both on key aspects of algebraic thinking associated with each of them, and how they may be articulated to promote a broader understanding of students in the Algebra field.

Students' Mathematical Authority

This aspect that is related to the engagement and participation of students, in BL, is noticed only by G6 (through a description and an explanation). In TL, students' mathematical authority is resumed by G6 and three other groups who present explanations (G3, G5) and predictions (G1, G6). In BL, G6 described and explained that the teacher would assure that students themselves explained their resolutions and linked her intention to the role of students' interactions and responsibility on their own and other's learning.

The teacher will take care that the students themselves explain how they got to the answers. (Description, G6, BL, Session 14)

[the discussion] can also promote students' integration and interaction by changing the class format; the way students learn is no longer that 'stuff' that only the teacher teaches and the student listens and learns. In this method, students learn by their own effort and with other students. (Explanation, G6, BL, Session 14)

Based on the analysis of Episode 8, G3 resumes this point and highlights that the teacher takes advantage of the students' discourse to support them:

The teacher attempts to help those who did not accomplished the question or did not reach the rule by re-explaining the resolution, but using the same language as the students (Explanation, G3, TL-Ep. 8,

Session 20)

This group noticed that, instead of elaborating a different explanation, the teacher maintains the students' language, retelling them in the same terms, and thus not interfering in the students' mathematical authority. Integrating several ideas, G1 seems to have understood that students positioned themselves as authors of their productions, relationships, and explanations, and that this authority is made visible and valued through the inquiry-based teaching instructions adopted by the teacher:

[The whole-class discussion] favours learning in a differentiated way, where the students themselves create the relationships between one and the other [resolution]. ... through this discussion, the student has a voice and you see that is giving great importance [to what he has produced]. Besides that, comparing his resolution with that of the other, he makes relationships and realises that he could solve [the task] in another way that he had not thought yet. (Prediction, G1, TL-Ep. 8, Session 20)

Thus, PTs seem to associate the position that students occupy in the whole-class reflective discussion with the development of their mathematical authority.

Discussion and Conclusion

The present study was carried out in the context of the exploration of a multimedia case (MC) by PTs enrolled in an initial teacher education course. The main objective was to understand how their professional vision on whole-class reflective discussion develops, from the analysis of resources with different formats and content, based on the practice of an experienced teacher. The results show the six aspects about the whole-class reflective discussion that have been noticed by the seven PTs groups, from (A1) to (A6) which have been discussed in the previous section.

Each of these perceived aspects, in association with the ways of reasoning about them and with the respective MC's section, shows that the analysis carried out by the groups resulted in descriptions, explanations and predictions in both sections of the case (BL and TL). However, the perceived aspects emerged more consistently in the form of explanations and predictions, in the TL section. Therefore, there seems to be an important relationship between the structure and sequencing presentation of the resources in the MC and the reformulation or expansion of the ideas that underlie the PTs' reasoning about the teacher's actions, students' learning and the dynamics of interactions in the whole-class discussion, which are consistent with the dialogic inquiry perspective. Those results reveal the articulated development of two components of the PTs' professional vision: noticing relevant aspects of challenging classroom situations and reasoning based on knowledge about them.

These results are consistent with previous research that shows changes and differences in relation to what PTs notice and the meanings attributed to what was highlighted throughout the interpretive process (van Es et al., 2017). This variety of results on the aspects of the whole-class discussion, associated with the three ways of noticing, when compared with the particular characteristics of the different resources analysed in this study, allows

us to conclude that there is a close relationship between the resources content and the PTs' noticed aspects, as well as between the resources format (text, audio, or video) and the nature of the PTs' reasoning. For example, explanations and predictions emerge more predominantly in video analysis, reaffirming their potential for PTs' professional learning, as evidenced in other studies (Santagata et al., 2007; Schäfer & Seidel, 2015; van Es et al., 2017). In the analysis of videos, the students discourse was considered an important evidence to interpret their thinking, in line with the study by Ozdemir Baki e Kilicoglu (2021). On the other hand, several explanations and predictions produced in TL present indications of having been constituted from the resumption of aspects previously noticed by the PTs in BL, and on which new meanings were added, which shows the relevance of that analysis.

Initially the PTs' selective attention focused only on the teacher's actions, so their reasoning was more descriptive and they presented mostly superficial explanations. However, as they reason on the consequences of the teaching strategies from their interpretation of students' mathematical thinking depicted in the videos, explanations and predictions further emerge. Unlike other studies, in which PTs did not focus on specific aspects of student's mathematical thinking (Callejo & Zapatera, 2017; Warshauer et al., 2019), in this research PTs interpreted the dialogic approach as a context that aims at students' algebraic thinking through the communication that takes place in the classroom and, particularly, with the recognition of the several strategies and their discussion focusing on mathematical ideas.

The results also show that the aspects highlighted in the BT section, although in different ways, attracted again the PTs' attention in the TL section. In this sense, it seems that the teacher's intentions, portrayed in her lesson plan and interview, that express what she expects of students in an inquiry-based mathematics teaching, have also become previous expectations of PTs, influencing them as to what to notice in the video-clips in TL. Thus, the expectations and knowledge built in the BL section may have helped the PTs to notice fundamental aspects of the collective discussion. These conclusions corroborate the existence of a dynamic interrelationship between selective attention and knowledge-based reasoning (Sherin, 2007), mainly because the PTs' expectations and knowledge do not only support their ways of reasoning, but also guide what can be noticed in the situation. Thus, PTs' learning about the teacher's role in anticipating the lesson also supports the development of their professional vision in the process of recognising and rebuilding meanings about classroom interactions depicted in the videos. This is not so encompassed in previous settings aiming at PTs' professional vision and might be considered in teacher education courses.

This study meets the identified need for conceiving "learning environments that create opportunities for preservice teachers to develop new ways of attending to and making sense of instruction" (van Es et al., 2017, p. 167). In particular, we could evidence strategic approaches to support PTs' perception of "ambitious" teaching practice (Kang & van Es, 2019) in the specific case of a quite demanding and unfamiliar practice for the PTs, and even for experienced teachers – the whole-class reflective discussion. Although the analysis in this study focuses in one lesson's phase, we consider that an added value of this intervention was, on the one hand, to focus PTs' attention on specific moments with particular aims, and to which some teachers' actions are associated, and, on the other hand, to enable PTs to examine the lesson as a whole to access this teaching practice and understand its complexity

in the specific context in which it unfolds.

We found that the diversity of content, sequencing and format of resources in the MC, are intervening factors in the mobilisation of multiple PTs' readings and interpretations of the teacher's practice, which is in line with the results of other studies (Kang & van Es, 2019; McGraw et al., 2007; Oliveira & Cyrino, 2013; Rodrigues et al., 2018). Naturally, the aspects that the PTs have noticed only emerged due to the richness of the teacher's practice portrayed in the MC, which poses the challenge to teacher educators of providing representations of practice that, besides being recognised by the PTs as authentic, are appropriate to the particular purposes of each teacher education context.

The potential that multimedia cases encompass, by allowing several approximations to practice, allow PTs to understand a complex teaching practice and develop the ability to reason about the planning it requires, as well as a language to talk about it. Thus, it can be seen as a complement to direct contact with teaching practice, namely, to the observation of cooperating teachers' lessons, especially when the access to that practice may be undergoing several restrictions (Perry, et al. 2022). In this way, the digital resources integrated into the multimedia cases, namely videos (Blazar et al., 2018), can be used as an alternative to meet the needs of an online teacher education, required by coping with the recent COVID-19 pandemic. These resources may mitigate the impacts that the abrupt and prolonged interruption of access to professional practice may cause to initial teacher education programmes (La Velle et al., 2020).

Recommendations

As future research, we suggest involving the PTs in planning and teaching classes from a dialogic inquiry perspective, which was not addressed in this teacher education course and that it is less present on the researchers' agenda in the noticing field. Also, individual differences between the PTs were not considered, due to the methodological options assumed, but they can be addressed in future uses of the multimedia case in teacher education.

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