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The Contribution That Utilizing Generative AI for Problem Posing Makes to Pre-Service High School Mathematics Teachers' TPACK

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Abstract

Fifteen pre-service mathematics teachers studying in a teachers college participated in a unique program entitled "From high-tech to teaching" as part of a specialization program for teaching advanced secondary school mathematics. During the course, they solved various mathematical problems and then were asked to pose additional problems and upgrade them to meet precise educational goals by furnishing "prompts" to a generative AI environment (ChatGPT) and refining them as necessary according to predefined steps. To understand how this process contributed to the pre-service teachers' body of technological, pedagogical, and content knowledge (TPACK), they were asked to respond to a questionnaire (ten Likert-type statements; six open-ended questions). The findings indicate that posing problems in an AI environment increased their technological knowledge vis-à-vis the advantages of the environment itself and how to generate effective prompts to obtain relevant responses. They also became aware of some disadvantages. With respect to their technological-pedagogical knowledge, integrating AI at challenging pedagogical junctures (e.g., tailoring problems to students' individual needs) provided diverse pedagogical ideas. Although AI involves some challenges (e.g., all suggestions must be critically examined), overall they expressed satisfaction with the concept and indicated they would use it in the future.

Introduction

Mathematical problem posing involves formulating original problems or re-formulating given ones (Silver & Cai, 1996). To effectively pose mathematical problems, teachers must have mathematical and pedagogical knowledge, be familiar with the different types of problems, understand the various pedagogical strategies and how to implement them (such as "what if not" strategy that invites posing new inquiry problems based on an existing one (Brown & Walter, 1983), and be comfortable working in a technological environment (Segal et al., 2018). Indeed, mathematical problem-posing is a challenge for teachers who often have to adapt existing problems (e.g., from a textbook) to meet their educational goals or their students' knowledge base (Cai & Jiang, 2017; Chan, 2023; Lim et al., 2023). Mallart et al. (2018) emphasized the difficulties teacher face when posing problems related to

educational needs, such as offering problems that allow students to recognize the relevance of mathematics in their everyday lives and or problems adapted to a specific educational level. Hence, scaffolding that will offer new perspectives and expose mathematics teachers to novel mathematical and pedagogical ideas when posing mathematical problems would be highly advantageous.

The recent emergence and accessibility of artificial intelligence (AI) technology environments are yielding fresh opportunities for education systems, teacher educators, and teachers (Baidoo-Anu & Ansah, 2023). Although its integration into the education system is still in its infancy, teacher educators and teachers are trying to find appropriate paths. The ever-developing AI algorithms can now be utilized at points where pre- and in-service teachers have difficulty, such as posing problems. This point is addressed in the current study, where pre-service teachers (PSTs) explored the concept of mathematical problem solving: different approaches, integrating various technologies, and understanding the process of posing the problem and how it can effectively enhance their students' learning and appreciation of mathematics and alleviate difficulties. Since posing problems in mathematics is an inherently challenging task, and since these teachers had not yet practiced it, we decided to have them pose problems to meet pre-defined educational goals while working with an AI "assistant" (in this case, ChatGPT) in accordance with Cai and Rott's (2024) graphical mathematics problem-posing model. Following this, we aimed to assess the contribution that AI made to the mathematical problem-posing process and to the technological, pedagogical, and mathematical content knowledge (TPACK) of the teachers.

We chose to use ChatGPT based on our experience as instructors as this app is already used in our colleges in other courses and therefore a familiar and accessible tool for the students. The choice was also driven by its many advantages, including a user-friendly interface and ease of use. Additionally, ChatGPT is built on advanced natural language processing models that have demonstrated accurate understanding of user intentions and the generation of content with high complexity. Nevertheless, it is important to note that the technological environments based on AI are developing rapidly and dynamically, so it is possible that in the future another environment may prove more optimal.

Literature Review

Below we highlight some literature on problem posing in mathematics, specifically the issue of problem posing in technological environments and the challenges faced by teachers in the process. We then discuss various aspects of integrating AI into teaching and learning in general and mathematics education in particular to provide a response to challenges in posing mathematics problems. Finally, we will present the TPACK model for characterizing technological, pedagogical, and content knowledge of teachers in general and mathematics teachers in particular.

Mathematical Problem Posing, Especially in a Digital Environment

Mathematical problem solving and problem posing are primary goals and important processes in mathematics education (Brown & Walter, 1983; Polya, 1954; Silver, 1994). In fact, problem posing may be complementary to

problem solving (Mallart et al., 2018). Problem posing often involves formulating new problems, but it can also involve re-formulating situations by varying a given problem (Silver & Cai, 1996). These processes can be aided by digital technologies, leading to a fundamental change in how teachers look at problem posing as they are able to examine diverse examples, identify and investigate special cases, present hypotheses, and search for counterexamples (Gros, 2016; Santos-Trigo & Moreno-Armella, 2016; Segal et al., 2018). Problem posing may use a combination of pedagogical techniques such as the "what if not" strategy, which invites the fabrication of new problems based on an existing one (Brown & Walter, 1983). As an example, using this strategy with dynamic geometry software leads to unique interactions between technology, teachers' actions, and understanding the processes. Changing even one aspect of a given problem's data can lead to processes of filtering, translating, understanding, and revising a new problem, and fosters the transition from problem-solving to problem-posing activities (Lavy, 2015; Segal, et al., 2018).

Cai and Jiang (2017) stressed the need to integrate and systematically teach problem-posing activities into the curriculum at teachers colleges, and pointed out that the Chinese and USA curricula strongly emphasize problem posing in school mathematics. At the same time, they pointed out the many challenges that hinder its implementation. Liljedahl and Cai (2021) supported these claims based on their survey of problem-solving and problem-posing processes and suggested that there is still much work to be done to gain sufficient insight into different situations and contexts, namely, how teachers can improve the problem-posing competencies, approach mathematical tasks properly and prepare appropriate questions, and make intelligent use of problem posing to teach mathematics. Mallart et al. (2018) presented a case study that showed that pre-service mathematics teachers had difficulty in adapting (textbook) problems to the specific school curriculum or educational level, in creating problems that can be self-corrected, or in designing problems that their students could recognize as relevant to their everyday lives, a facet that is important because it convinces students that mathematical knowledge has real-world applications (Nedaei et al., 2021).

The problem-posing process is reflected in a number of models presented in the literature. Cai and Rott performed a literature review on problem posing (Cai & Rott, 2024), and suggested that a generic problem-posing task incorporates a cycle comprising four phases: (1) *orientation* (understanding the task and the given problem, the strategies for solving it, what the poser notices in the problem, and what the poser wishes to improve); (2) *connection* (developing ideas for new problems by varying the problem's given situation or adapting it to specific educational needs); (3) *generation* (recognizing the potential connections made in the previous phase and developing new problems to connect it to educational needs); and (4) *reflection* (reassessing the problem and optimizing it by reformulating it). Cai and Rott encouraged researchers to empirically examine the usefulness of this problem-posing model to help better understand the processes in different contexts, and we used their model in the current study with ChatGPT.

Generative AI and Its Implications

The COVID-19 pandemic challenged the world, as well as education systems, where teachers had to suddenly face integrating technology for remote teaching and learning (Klemer et al., 2023; Turnbull et al., 2021). Certainly,

the emergence and accessibility of AI technologies environments in recent years exposes the education systems to a new reality (Baidoo-Anu & Ansah, 2023).

Generative AI (GenAI) refers to a group of machine learning algorithms designed to allow users to create new content by asking questions (prompts). By imitating human communication, GenAI environments are able to provide factual information, answer questions, edit existing texts, or produce new texts and images (Baidoo-Anu & Ansah, 2023; Chan & Hu, 2023; Marienko et al., 2020). ChatGPT is a GenAI with the potential to promote personalized and interactive learning. In the context of education, it can serve as an "assistant or partner" that can provide suggestions and prompts for teachers and instructors who are preparing courses and designing teaching/learning resources and materials. The use of GenAI in education systems is still not widespread enough. For example, Chan's (2023) study indicates that both teachers and students in various higher institutions in Hong Kong reported relatively low experience with AI, suggesting that there is significant room for growth in its integration into future teaching and learning practices as teachers begin to recognize the potential benefits of AI technologies.

However, a subsequent study by Chan and Hu (2023) with 399 undergraduate and postgraduate students from various disciplines in Hong Kong and an exploratory study by Baidoo-Anu and Ansah's (2023) that synthesized recent extant literature suggest that that along with GenAI's advantages come a number of disadvantages. Its benefits include personalized tutoring, immediate learning support, writing and brainstorming assistance, and interactive and adaptive learning. However, it lacks human interaction, is subjected to GenAI's limited mathematical understanding, does not exhibit creativity, and is highly dependent on data. Furthermore, students expressed concern about accuracy, privacy, ethical issues, and the impact on personal development, career prospects, and societal values. Through understanding teachers' perceptions and addressing their concerns, teacher educators can establish strategies to effectively implement of GenAI, ultimately enhancing teaching and learning experiences (Chan & Hu, 2023).

To use GenAI, teachers must have AI literacy, that is, skills to evaluate, communicate, and use AI effectively based on a basic understanding of its essence, the ability to interact with the various tools, and the ability to exercise critical thinking about the process and the product obtained as a result of that interaction (Druga et al., 2019). There is a correlation between AI literacy and frequency of use (Chan & Hu, 2023), and effective implementation requires gathering experience. The critical role of experience while utilizing AI was emphasized by the results of Kim's study (2024), which describes three phases of teacher-AI collaboration evolution: teachers as passive AI recipients; teachers as active AI users; and teachers and AI as constructive partners. Hence, teachers need opportunities to effectively integrate GenAI technologies into the pedagogical dimensions of their teaching and learning practices, including assessments, and developing student competencies, meaning that teacher educators must encourage a balanced approach to adopting AI in various disciplines of education.

Utilizing GenAI in Mathematics Education

Utilizing GenAI effectively to support mathematics teachers' learning is a meaningful challenge for educators as

GenAI offers the possibility of providing personalized feedback and specific support to PSTs while learning mathematical subjects or understanding complex content (Jia, 2015). This personalized GenAI feedback can promote pre- and in-service teachers' mathematical knowledge and increase their proficiency in solving mathematical problems (Rane, 2023; Wardat et al., 2023). Most importantly, a qualitative study conducted by Wardat et al. (2023) that focused on the perceptions of PSTs and educational professionals regarding the use of ChatGPT in mathematical instruction, found that most of the participants believed that ChatGPT, given its ability to engage in intelligent conversation, could facilitate solving various mathematical problems and provide instructional support. In fact, utilizing GenAI for solving mathematical problems in various mathematics subjects (geometry, algebra, etc.) can assist learning processes by explaining and visualizing mathematical concepts and theorems and presenting the properties of the geometrical shapes. GenAI facilitates personalized and adaptive learning experiences and enables teachers to receive immediate feedback, participate in interactive problem-solving dialogues, and access step-by-step solutions that align with their requirements. As a result, teachers have the opportunity to strengthen and expand their understanding of mathematical concepts and to improve their learning experience and mathematical competencies (Kasneci et al., 2023; Rane, 2023; Wardat et al., 2023). GenAI can also be implemented as a lever for promoting collaborative learning while participating in discussions with peers (Rane, 2023).

Teaching and learning mathematics with GenAI also presents challenges. For example, the accuracy and dependability of AI-generated solutions are not always flawless, and ChatGPT does not have a deep understanding of some mathematical topics (e.g., geometry) and thus may provide incorrect or incomplete solutions to a given mathematical problems and not allow effective handling of misconceptions (Rane, 2023; Wardat et al., 2023). As examples, in one study (Wardat et al., 2023), it generalized the Pythagorean theorem to encompass all triangles (not only right triangles), which has the potential to mislead those relying on it, and it created "mysterious" arithmetic errors that a simple calculator would not, such as when using the definition of the derivative of a given function, $f(x)$, it provided an incorrect answer when substituting $x = -1$ into $f'(x)$. This lack of deep understanding prevents providing effective tailored feedback or correcting misconceptions. Hence, students and teachers need to be aware that ChatGPT can provide inaccurate information and be ready to critically evaluate every suggestion it makes. In any event, to strengthen ChatGPT's accuracy and reliability it is essential to consider the specific context and field of knowledge, to give ChatGPT precise, well-defined input, and to cross-reference the information provided with other sources (Wardat et al., 2023). Users also need to be aware of ethical concerns, such as data privacy, intellectual property, liability issues, and the ethical implications of AI-driven decision-making processes (Rane, 2023; Wardat et al., 2023). Baidoo-Anu and Ansah (2023) called for further studies to explore how to integrate GenAI tools into teacher-education programs to prepare in- and pre-service mathematics teachers to use AI tools safely and constructively and to support student learning and teachers' TPACK.

TPACK: A Framework for Teacher's Pedagogical and Content Knowledge and Their Integration with Technology

In this study, we use the TPACK model to describe the knowledge that the participants acquired while utilizing ChatGPT for mathematical problem posing. The literature describes various models in the context of teacher's

knowledge, especially those concerning integrating technology into teaching processes. TPACK, an amalgamation of technological knowledge (TK), pedagogical knowledge (PK), and content knowledge (CK), is an important model for describing teachers' knowledge (Koehler & Mishra, 2009, based on Shulman, 1986). The TPACK model implies that all three bodies of knowledge intersect at various levels of complexity (Jahangiril et al., 2021; Segal et al., 2021), thus forming technological content knowledge (TCK) and technological pedagogical knowledge (TPK). This model has become a useful framework for understanding the goals involved in integrating technology into teacher education as well as mapping teacher knowledge and developing research tools because it defines the knowledge teachers require to effectively integrate technology into their teaching (Caniglia & Meadows, 2018; Nantschev et al., 2020; Polly & Orrill, 2012). The model's complexity is reflected in the components and combinations of basic knowledge: TK is knowledge about the various technologies available for planning and teaching in the classroom and online environments (Tondeur et al., 2020) whereas TCK is knowledge about how technology can create different representations for a specific concept (which demands that teachers be aware and understand how using specific technologies can affect learners' skills and understanding of the relevant concepts and content), and TPK implies familiarity with the range of technologies that can be integrated into teaching and understanding how they can affect teaching methods and promote teachers' educational needs. Teachers who have deep, expansive TPK can use technologies to expose their students to a variety of physical and virtual means of thought, thereby improving their thinking and understanding skills (Özerem, 2012; Stoilescu, 2015). They can use technology for student assessment in a variety of ways, including self-evaluation, peer evaluation, and more (Torres-Madroño et al., 2020). It also enables the promotion of students' inquiry skills and active learning (Klemer & Rapoport, 2020; Segal et al., 2018).

Being aware of one's TPACK may assist teachers' professional development and vice versa. Teachers who are aware of the TPACK components will deepen their content, pedagogical, and technological knowledge and by continually doing so, promote their students' learning and performance as well as their ability to solve and pose problems in dynamic environments within various contexts (Jahangiril et al., 2021; Koh et al., 2013; Lavy, 2015; Leikin, 2015; Segal et al., 2018). Teachers with strong TPACK use technology to promote inquiry-based activities for their students and use a variety of representations of mathematical and scientific concepts to assist their student's development of content knowledge (Guzey & Roehrig, 2012; Segal et al., 2018; Yang et al., 2023). One strategy for expanding mathematics teachers' TPACK is by involving them in designing tasks in a digital environment where they can concretize abstract concepts, establish technological knowledge, and become aware of the technological possibilities for learning and teaching mathematics (Kirişçılar & Yildiz, 2018).

Research Question

Posing mathematical problems presents a challenge for mathematics teachers. While this may be aided by GenAI, its use in education is still underdeveloped. Nevertheless, initial studies indicate the potential of GenAI to serve as a "partner" for teachers not only in their instructional tasks, but also for posing mathematical problems and improving their TPACK. This study aimed to examine how ChatGPT, a specific GenAI tool, can aid pre-service high school mathematics teachers in posing mathematical problems and contribute to their TPACK. The research question is:

What is the contribution of ChatGPT to the TPACK of pre-service high school mathematics teachers when used to assist in posing mathematical problems according to defined educational needs?

Method

Participants

The participants were 15 individuals from the fields of high-tech and engineering studying in a special academic program for transitioning to education, specifically, teaching high school mathematics. The program included several mathematical and pedagogical courses aimed at teaching advanced, high-level mathematics in high school. During the program, the participants became familiar with the curriculum's content, pedagogical and didactical methods for teaching the subjects, and various ways of problem solving and posing. The program included a year-long course entitled "Teaching Advanced Mathematics" which involved integrating technology into teaching and learning mathematics. Two sessions of this course focused on problem posing utilizing GenAI/ChatGPT, during which the participants were asked to follow specific steps (see below, "Problem posing process using ChatGPT") to compose and refine prompts for the ChatGPT so that it would assist them in posing problems that would meet their educational goals. This research was conducted during these sessions.

Research Tools

The study used a mixed method (combined quantitative and qualitative data collection and analysis) protocol. Data was gathered from documentation and a questionnaire.

Documentation

During the sessions, the researchers documented all stages of the problems and prompts that the teachers gave ChatGPT by integrating the links to the AI environment into a Google Form file that guided the teachers, step by step, through the process.

Questionnaire

After the two sessions regarding problem posing utilizing ChatGPT, the participants were asked to fill out a two-part questionnaire (constructed and validated by three mathematics education experts) to assess their feelings about working with GenAI and to determine if and how it affected their TPACK. The first part of the questionnaire comprised 10 Likert-type statements which the participants were asked to rate from 1 (strongly disagree) to 5 (strongly agree). (The questions are listed in the section on Data analysis, below). The second part comprised the following six open-ended questions, the purpose of which was to expand our understanding of the participants' responses in the first part and gain broader insight into their perceptions of teaching mathematics in high school, specifically regarding how using GenAI for problem posing contributed to their TPACK:

1. How has using ChatGPT improved your ability to pose a problem efficiently and creatively?
2. How did ChatGPT deepen your understanding of the mathematical content related to the problem you

- posed? Please provide an example.
3. How did ChatGPT help you develop critical thinking about the problem-posing process? Please provide an example.
 4. What are the advantages of using ChatGPT in the problem-posing process? Please elaborate.
 5. What are the disadvantage of using ChatGPT in the problem-posing process? Please elaborate.
 6. How do you think you will implement ChatGPT for posing problems in mathematics in the future?

Problem-Posing Process Using ChatGPT

During the two sessions, the teachers were required to employ ChatGPT during the process of writing a problem-posing task. The process included the four main stages (based on Cai & Rott, 2024) illustrated in Figure 1 and detailed below.

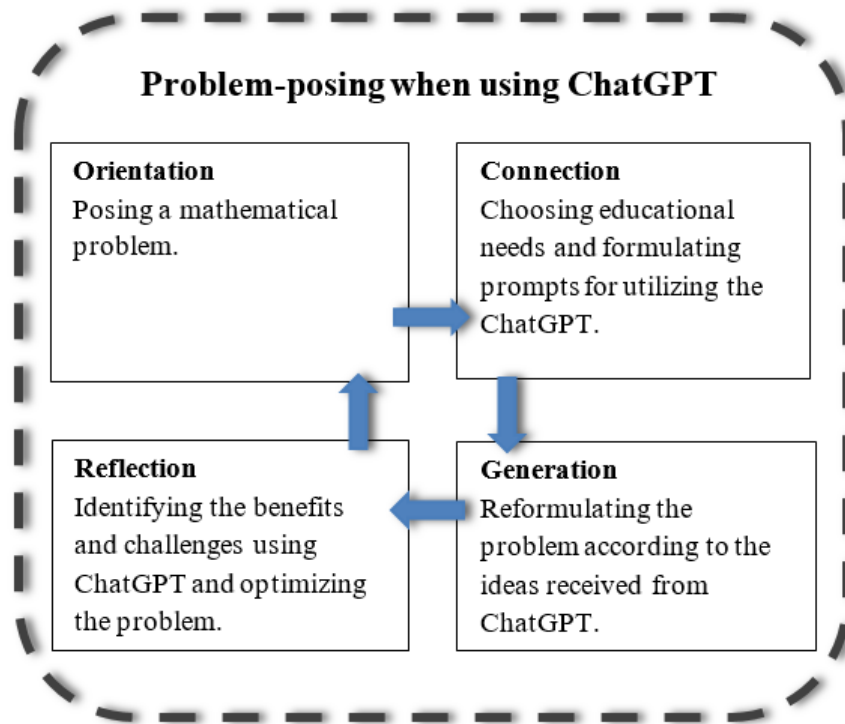


Figure 1. Mathematical Problem Posing Stages Utilizing ChatGPT (based on Cai & Rott, 2024)

Stage 1: Orientation

The teacher poses a mathematics problem that would be applicable to an advanced-level subject (sequences, Euclidean geometry, calculus, etc.). They are then asked to list the knowledge required to tackle the problem and potential difficulties a student might confront while solving the problem.

Stage 2: Connection

The teacher defines their educational needs and formulates prompts for the ChatGPT to receive recommendations

for improving the problem. They were provided with preliminary guidelines and examples on how to conduct a dialogue with ChatGPT and refine the prompt process according to their specific needs. Here is one example:

I am a pre-service teacher of mathematics education. As part of a course teaching advanced mathematics, I am tasked with creating a problem in calculus. Here is the problem I posed: "...". I would appreciate it if you could provide me with three recommendations on how to improve my problem.

Following the suggestions supplied by ChatGPT, the teachers could continue the dialogue with additional questions to focus the prompts on specific aspects and educational goals. For example:

1. Is the problem clear and understandable? How can it be made clearer?
2. Is the problem sufficiently challenging, or does it seem too simple? How can its difficulty be increased?
3. Does the problem contain all the necessary information for its solution? What additional details could enhance its value?
4. Can the problem include real-world applications or connections to other scientific fields?

Stage 3: Generation

The teachers consider the ideas received from the ChatGPT and decide if and how they relate to their defined educational needs and whether they (the teachers) have the knowledge and skills to implement them. They decide which suggestions are suitable and reformulate the problem accordingly.

Stage 4: Reflection

The teacher identifies the benefits and challenges of using ChatGPT for the process of mathematics problem posing and decide whether to continue working with ChatGPT for another task. If so, they repeat Stages 1-3, this time posing a mathematical problem that connects the mathematical concept to everyday life.

Data Analysis

The study used a mixed method, sequential explanatory procedure to acquire a complete understanding of the contribution that utilizing ChatGPT for mathematical problem posing made to the participants' TPACK. Quantitative and qualitative data were collected via questionnaires (Creswell, 2014) and additional qualitative data was obtained from the documentation. Quantitative data analysis (frequencies, means, and standard deviations) of the Likert-type statements allowed analyzing the contribution that ChatGPT made to the mathematics teachers' technological-pedagogical and mathematical knowledge, as perceived by them. To qualitatively analyze the data obtained from the teachers' responses to the open-ended questions, their responses were divided into "units of meaning" (from one word to a sentence or paragraph that focused on one idea), themes were identified and categorized using "constant comparative analysis" (Charmaz, 2014; Glaser & Strauss, 2017), and then the themes were divided into primary and secondary content areas (Fram, 2013).

Results

Before presenting the analytical findings, we present two examples dialogues between participants and ChatGPT, demonstrating how they fine-tuned the problems according to their defined educational needs. This will help readers better understand the process by which ChatGPT was integrated into the problem-posing process.

Example A: Nir's (pseudonym) problem-posing process using ChatGPT: Exploring a function.

Stage 1: Orientation: Posing the problem

Explore the function $f(x) = \frac{2x^2}{x^2+6x+8}$ as follows:

- a. Find the domain.
- b. Find the asymptotic equation.
- c. Find the intersection points with the axes.
- d. Find the extrema.
- e. Find the areas of rise and fall.
- f. Sketch the graph.

Next, Nir defined the required knowledge and the potential difficulties that a student might confront.

Knowledge required to solve the problem: Investigating a quotient function, finding the derivatives of a quotient function, finding the domain, asymptotes, and intersection points with the axes, extrema, areas of rise and fall, and sketching.

Possible difficulties: Students may err when finding the derivative function; students may have difficulty drawing a discontinuous sketch.

Stage 2: Connection

Nir then formulated prompts to stimulate suggestions from ChatGPT:

I am a student teaching mathematics. As part of a course... I was asked to pose a problem for investigating a quotient function.

Here's the problem I posed. ...

Please write down three recommendations on how to improve my problem,

specifically:

1. Is the problem challenging enough or does it seem too simple? How can we increase the challenge?
2. Does the problem contain all the information needed to solve it? What missing details can add value?
3. Can real-world applications or connections to other areas of science be included in the problem?

Stage 3: Generation

ChatGPT responded with suggestions, and after considering them, Nir reformulated the problem according to his educational needs.

ChatGPT's response

Increasing challenge: Although the problem is good, you can increase the challenge by adding questions that require more in-depth analysis or additional techniques. For example, asking to describe the behavior of the function next to its vertical asymptotes (i.e., whether the function approaches positive or negative infinity).

Possible difficulties. Consider that the students might have difficulty sketching the area of the graph near the vertical asymptote. You might wish to have them use a graphic tool to provide a graph that they can compare with their own graph.

Missing details: It may be helpful to include a reminder about the definition of asymptotes and how to find them algebraically, especially for students who may be encountering these concepts for the first time. Make sure you provide clear guidance on how to approach each part of the problem. For example, to find the domain of the function, you might want to include guidelines for identifying values of the function at points where the function is not defined (such as values that make the denominator zero).

Linking the problem to everyday life: Although the problem itself is purely mathematical, you might consider adding real-world context or application to make it more appealing and relevant to students. For example, a scenario in which the function represents a physical state (such as a growth model or financial investment) and then ask students to interpret the results of their calculations in this context. Thus,

you can improve the depth, clarity, and applicability of your problem, making it more engaging and valuable for students.

Nir considered the suggestions and decided to implement them as follows

- a. He added a question asking for the behavior of the function graph near the asymptotes. This would help the student deal with the difficulty of drawing the graph of the function around the points of non-definition where it is not continuous.
- b. In this section, he introduced a recap of the process of finding asymptotes to assist those who have difficulty or those who are doing so for the first time.

Thus, the additional question was worded as follows:

Find the asymptotes (pay attention to the behavior of the function in close proximity to the horizontal asymptotes). Reminder on how to find asymptotes: vertical asymptotes in a quotient function occur when the function is not defined (that is, the denominator equals 0 and the numerator is NOT 0); horizontal asymptotes in a quotient function occur when the highest power in the expression's numerator is smaller than or equal to the highest power in the expression's denominator.

Stage 4: Reflection

Nir considered his dialogue with ChatGPT and decided which suggestions he should accept.

Nir added a section directing the students to check their sketch using a graphic tool, and then asking them to compare and investigate if there is a difference between their graph and the graph presented and what such a difference might stem from.

Nir chose NOT to implement ChatGPT's recommendation to integrate the problem into daily life by linking the graph to a model of growth or financial investment.

Example B: Gal's (pseudonym) problem-posing process using ChatGPT: A problem relevant to daily life

Stage 1: Orientation.

Gal wanted her students to deal with the concept of similar triangles. The following problem was posed:

Estimate the height of the lamppost outside with a laser tape measure and a ruler.

She then defined the required knowledge and the potential difficulties that a student might confront.

Knowledge required to solve the problem: similar triangles, thinking outside the box.

Difficulties: understanding how to arrive at a solution, using the means given, calculating distances.

Stage 2: Connection

She then prompted ChatGPT with the educational needs for improving the problem.

I am a student teaching mathematics and as part of a five-unit mathematics teaching course. I was asked to compose a problem in which the solution is based on the similarity of triangles.

Here's the problem I wrote: "Estimate the height of the lamppost outside using a laser tape measure and ruler."

Please write down three recommendations on how to improve my problem.

Stage 3: Generation.

Gal considered the new ideas received from ChatGPT and reformulated the problem based on her educational needs.

ChatGPT's response

Here are some suggestions.

- 1) Add context to a problem to make it more engaging. For example, specify a scenario where the light pole is located. This context can help students understand why this might be important.
- 2) Include additional information about the tools used (e.g., the laser tape measure) and any limitations to consider when making the estimate. It helps students understand the constraints of the problem and encourages critical thinking.
- 3) Clarify the purpose. Make sure that it is clear what students are expected to do.

In this case, the task is NOT to directly measure the height of the light pole but to use the laser and ruler to estimate its height indirectly using similarity of triangles. Giving explicit instructions can help students focus on the relevant mathematical concepts.

- 4) Since the goal is teach about similarity of triangles, state this explicitly in the problem statement. For example, "Using the laser tape measure and ruler, estimate the height of the lamppost by applying the principles of similarity of triangles."
- 5) Consider including guiding questions to scaffold the students' problem-solving process. For example, ask students to identify the similar triangles formed by the laser tape measure, ruler, and light pole, and then define and resolve proportion to find the lamppost's height.
- 6) Break down the problem into smaller stages and guide students through each step with inspiring questions. This helps students build confidence, especially if they are new to applying the concept of similarity of triangles in real-world contexts.

Gal considered the suggestions and decided to implement them as follows

- a. She improved the formulation of the problem:

"Today we are going to measure the height of the lamppost in the sports field. Yes, the pole is high, and we cannot measure it with a tape measure. But we do have a laser tape measure and a ruler. Before we go out on the field, try to think how we can almost accurately measure the height of the pole using the tools we have and what we learned in school.

Stage 4: Reflection

Gal considered her dialogue with ChatGPT and optimized the problem by reformulating it. She commented on the benefits and challenges as follows:

1. It helped that it recommended breathing a bit of "life" into the question; it also gives a little more framework for the students' thinking and a little more interest.
2. When I asked for more suggestions, it suggested giving the students some initial information that would serve as a starting point. It also suggested asking the students

at the end to use critical thinking as to whether there might be errors in measuring the height of the pole and what they could result from and/or asking them to explain why they thought the way they measured was "sufficiently" accurate. Another interesting suggestion was to divide the students into pairs/groups.

The examples illustrate the degree of missing pedagogical knowledge. For example, Nir was unsure whether the level of complexity was appropriate for his students, and Gal was uncertain as to how to make the problem more interesting. With respect to mathematical-pedagogical knowledge, Gal was also unsure how to improve the problem's mathematical complexity to make it sufficiently challenging. The responses from the chat demonstrate how AI technology can help formulate a problem and bridge the "holes" in pedagogical or pedagogical-mathematical knowledge.

Quantitative Analysis

The distribution of the PSTs responses to the 10 closed statements in the questionnaire are presented in Table 1 (Likert scaled from 1 – strongly disagree to 5 – strongly agree). Mean, standard deviations, and frequency of responses for each scale value are presented. Overall, it appears that the teachers seem satisfied with using ChatGPT as the mean scores for all statements range from 3.20 to 3.93, indicating moderate to high agreement with the presented statements.

Table 1. Distribution of PSTs' Answers to the 10 Closed Statements on the Questionnaire

Statement	Mean	S.D.	Frequency				
			1	2	3	4	5
1. ChatGPT helped me better understand how to improve the problems to meet my educational goals.	3.20	1.21	2	2	3	7	1
2. ChatGPT offered new ideas for posing problems.	3.20	1.26	2	2	4	5	2
3. The feedback from ChatGPT was useful.	3.33	1.11	1	2	5	5	2
4. I feel more confident in posing mathematical problems after using ChatGPT.	3.33	1.29	2	2	2	7	2
5. The process with ChatGPT made me think more creatively about posing a new problem.	3.80	1.47	2	1	2	3	7
6. I believe that using ChatGPT in the future will enhance my ability to independently pose problems.	3.73	1.16	1	1	3	6	4
7. Using ChatGPT strengthened my understanding of how to relate a problem I posed to its application in everyday life.	3.40	1.55	3	1	3	3	5
8. The task helped me learn how to focus ChatGPT to meet my mathematics educational needs.	3.93	0.96	0	1	4	5	5

Statement	Mean	S.D.	Frequency				
			1	2	3	4	5
9. I would recommend to other teachers to use ChatGPT to help them in posing problems.	3.73	1.22	1	1	4	4	5
10. Using ChatGPT encouraged me to continue asking questions and seek help.	3.67	1.18	1	1	4	5	4

Qualitative Analysis

Qualitative analysis of the open-ended questions revealed five themes, which are presented in Table 2, along with their frequency and sample responses.

Table 2. Themes (frequency) and Sample Quotes of Pre-Service Teachers' Responses to the Open-Ended Questions

Theme (Frequency)	Sample Quotes
Exposure to new pedagogical ideas (ChatGPT introduces new ideas and approaches to problem posing.) (n=10)	<ol style="list-style-type: none"> 1. ChatGPT suggested ideas that I had to implement myself. 2. ChatGPT gives ideas, and you can focus it on what you want to achieve. 3. ChatGPT helped me easily add cases and examples and expand the question beyond focusing on a specific element. For example, beyond focusing on the amount of an arithmetic progression. 4. ChatGPT gives a variety of ideas for constructing an effective complex problem. 5. ChatGPT offers ideas that stimulate thinking and connects to understanding rather than procedures.
Develop critical thinking (ChatGPT promotes a deeper understanding and critical reflection on the problem-posing process.) (n=8)	<ol style="list-style-type: none"> 1. ChatGPT brings up points of reference from a broader perspective that we don't always think about. 2. ChatGPT helped me understand that I should add more sections to encourage understanding. 3. ChatGPT can help fine-tune a question, but you have to keep your finger on the pulse and not accept every suggestion. 4. ChatGPT identified that the wording in a problem I posed was not clear enough and proposed corrections. 5. In one case, ChatGPT gave me an unreasonable answer that showed a negative distance. Clearly, this solution was rejected.
Continued (future) implementation of ChatGPT (n=7)	<ol style="list-style-type: none"> 1. I will definitely use it, mainly to test understanding. 2. I intend to use ChatGPT to make sure the questions I pose are clear enough and don't lack data; or for correcting wording. 3. I intend to use ChatGPT to tailor existing textbook questions

Theme (Frequency)	Sample Quotes
	for the class I teach. 4. I'm going to use ChatGPT for the upcoming exam I'll be writing.
Challenges and limitations (while using ChatGPT for problem posing) (n=6)	1. Difficulty in the implementation and accuracy of recommendations received from ChatGPT. 2. Translation of mathematical terms into English and translation back into Hebrew. 3. Correct formulation of the prompt to get the desired result. 4. I have to explain to ChatGPT what I want and "fine-tune" it several times. 5. I need to make sure that the ChatGPT suggestions are not wrong, because I've found that it can't be relied on blindly; it often has errors in both the questions suggested and the solutions.
Accessibility and efficiency (for generating ideas for problem posing). (n=5)	1. ChatGPT confirms the data and understanding of the question, including the solutions. 2. Fast, accessible, comprehensive, but not always accurate. 3. An accessible connection to a rich database that can offer explanations I hadn't thought of before. 4. ChatGPT can suggest topics to ask about and what to focus on. It's similar to the feedback we would get if we consulted with colleagues, but without judgment. 5. We receive a quick response with a broader view of areas that may interest the student, such as connecting the subject to daily life.

Discussion

Technology enables access to a rich and diverse set of tools that can serve as a platform for research and provide a deeper understanding of learning and teaching (Schleicher, 2019). In the current study, the innovative GenAI-based technology – ChatGPT – enabled pre-service advanced-mathematics teachers to access an endless variety of pedagogical ideas and obtain recommendations in the context of posing mathematical problems, and this served as a basis for expanding knowledge, improving critical thinking, and applying the problems in practice.

The process of posing mathematics problems is challenging and fraught with difficulty (Cai & Jiang, 2017). For the advanced-mathematics PSTs who participated in the current study, this was their first experience posing mathematics problems, and the integration of ChatGPT as a "partner" that was non-judgmental and fostered personalized and interactive learning (Baidoo-Anu & Ansah, 2023; Lo, 2023) allowed them a safe environment, alleviated the challenge, and provided them the experience of success, which can strengthen their confidence. Working with ChatGPT helped advance their personalized teaching methods, each according to their pedagogical

needs, without any limitations on time, how much instruction they might require, and the accuracy of the prompts they fed ChatGPT. It also allowed them to customize their problems to meet the unique needs of the learners. Indeed, the findings of the current study, similar to those of Wardat et al. (2023) and Rane (2023), suggest that the personalized feedback the participants received from GenAI promoted their proficiency and capabilities in posing mathematical problems. The findings indicate that the participants recognized the added value of working with ChatGPT and expressed interest in using it in the future. The process of utilizing AI according to one's pedagogical needs is an ongoing one of accumulated experience (see Kim, 2024). As the teacher gains more experience using this technology, they become better able to exploit GenAI as a collaborating peer.

Contribution to TPACK

The current study comes as a response to Chan's (2023) call for teacher educators to integrate AI into teacher education. One of the goals of the current study was to determine how utilizing ChatGPT when posing mathematical problems contributed to high school mathematics PSTs' TPACK. Below the contributions to each individual facet of TPACK – namely technological knowledge (TK), pedagogical knowledge (PK), content knowledge (CK), pedagogical content knowledge (PCK), technological pedagogical knowledge (TPK), and technological content knowledge (TCK) – are discussed. Note that we define TPK as “understanding how teaching and learning can change when particular technologies are used in particular ways” (Koehler & Mishra, 2009, p. 65).

Contribution to TK

During the course, the PSTs learned about an innovative technological environment that could aid in posing mathematics problems: the GenAI environment allows access to a technological knowledge base that efficiently offers an endless plethora of innovative pedagogical examples and ideas that can serve as a basis for posing meaningful and interesting problems. While learning how to work in this environment, they realized that the quality of prompts they provided influenced whether they would obtain the most relevant (and sometimes novel) information to meet their educational goals, similar to the findings from studies by Baidoo-Anu and Ansah (2023) and Lo (2023). Thus, they gained insight into the central importance of the repetitive process of formulating clear and precise prompts to reap the best results.

Enhanced TK is also expressed in their increased awareness of the limitations of the environment and the need to sometimes reject the proposals made. The teachers became aware of the advantages and challenges of working with this environment and how to implement critical thinking to properly formulate prompts and to conduct an ongoing dialogue to hone their teaching needs.

Contribution to PK, CK, and PCK

There were many stages in which use of ChatGPT enhanced the teachers' PK, CK, and PCK. In the *orientation* stage, the teachers were required to obtain in-depth familiarity with the problem they wished to pose for the chosen

curriculum topic, the knowledge required to solve the problem, and the difficulties that may arise in the process. This suggests that the process helped improve their mathematical CK and PCK. In the *connection* stage, they had to further consider their students' educational needs (PK) and refine them properly to efficiently communicate this to the GenAI. During the *generation* stage, the dialogue with ChatGPT contributed to three aspects: improving the clarity of the wording of the problem (PK), ascertaining whether all the data in the question was required (CK), and determining whether any information was missing (PCK). This exposed them to new, sometimes surprising, pedagogical ideas to improve the formulation of the problem or to connect it to real-life experience (PK). In the *reflection* stage, they had to think critically whether the ideas and proposals offered were appropriate to use in their circumstances or even – as some noted in their responses to the open-ended questions – were actually incorrect or inappropriate (PCK). They had to carefully examine the proposals and decide whether the problem was suitably adapted to their defined educational goals and their students' skill level (PK), exploring how to improve the problem to increase the challenge to the students (PK), improve their students' skills, or add interest by showing how the mathematics involved could be applied to daily life (CK). They also had to be certain that the proposals aligned with their ability as teachers. In addition, the recursive prompting process, which introduces an ever-more specific dialogue between teacher and AI, helped teachers learn how to conduct a fruitful dialogue with their students regarding the problem and its solution (PK).

Contribution to TPK

With respect to TPK, the PSTs learned that, given the proper prompts, ChatGPT could help pose problems to meet pedagogical needs such as making their teaching more relevant (e.g., linking the problem to daily life), ensuring the wording was clear, meeting specific goals (e.g., making a problem more challenging), adding explanations when required (e.g., explaining what a laser tape measure is or explaining the meaning of a point where the function is not defined or function behavior in the asymptotic environment). Thus, they had to develop their skills for formulating relevant prompts and ever more finely tune these prompts depending on the responses received.

ChatGPT also contributed to TPK by enabling the PSTs to successfully cope with challenging pedagogical junctions during the process of posing problems, such as posing problems relevant to daily life or considering whether students have the appropriate knowledge to be able to deal with the problem, both difficulties reported by Mallart et al. (2018) and Nedaei et al. (2021). The contribution of ChatGPT to TPK is also reflected in their understanding that intelligent use of this environment is required.

Another contribution to TPK noted by the PSTs was the necessity of refining their critical thinking and creative thinking, similar to the findings of Chan & Hu (2023), Rane (2023), and Wardat et al. (2023). This was manifested a number of ways in the current study: (1) the PSTs reported that they identified inaccuracies and inapplicability in the suggestions received from the GenAI, realizing that every suggestion must be considered carefully; (2) they realized that they were receiving pedagogical ideas from a source not "officially" approved by any entity (that provides approved tests, worksheets, websites, textbooks, etc.), thus conferring upon them the responsibility of ensuring that suggested ideas and content were suitable; (3) they noted that some ideas pointed to "holes" in their knowledge or crossed the boundaries of their knowledge, which may serve as a lever for their future professional

development; and (4) they noted that sometimes the suggestion required skills to teach mathematics according to the curriculum that they did not have or which required excessive time or effort to implement them.

Contribution to TCK

TCK differs from TPK in that the former involves the relationship between mathematical concepts prescribed by the curriculum that are reflected in real-life events, whereas the latter involves pedagogical ideas with respect to the wording of the problem that expose the students to processes or real-life events. Working with AI exposed the teachers to new mathematical ideas, which made it possible to link mathematical concepts and processes to real situations from everyday life, such as linking graphs of functions to trends in financial investment or the link between the concept of similarity of triangles and the real-life need to measure the heights of tall objects in the environment, and more. In this way, the use of AI technology expands the repertoire of examples that can be presented to students linking mathematics to everyday life. The use of GenAI also expanded the PSTs' TCK knowledge in the context of the limitations of AI in terms of mathematical content and skills, such as inaccuracy in calculating the value of the function derived at a point, thus requiring them to develop critical thinking about the proposals offered by AI.

Conclusion

The accelerated technological development of AI-based environments require researchers in mathematics education to rise to the challenge of continuously studying the subject: rapid progress leads to ever-new research opportunities, and today's limitations may be resolved in a relatively short timeframe, leading to countless opportunities for teachers and their educators. The findings of the current study reinforce Chan's claim (2023) regarding the critical role that teacher educators in general and mathematics-teacher educators in particular have in providing PSTs with practical learning opportunities in the ChatGPT environment, thereby improving their TPACK. Although the current study focuses on integrating GenAI into the process of posing mathematics problems, the scope for using this environment is broad, and its use can help respond to the challenges, difficulties, and concerns of both pre-service and in-service teachers. Furthermore, given the optimistic findings presented herein, it seems credible to suggest that beyond GenAI's usefulness for mathematics teachers, it can also provide an excellent resource for developers of mathematics curricula and the authors of mathematical textbooks.

Study Limitations and Suggestions for Future Research

The current study was conducted with a group of 15 PSTs enrolled in a unique retraining program for teaching secondary school mathematics. The small sample set means the findings cannot be generalized; a similar study with a larger group of PSTs and/or in-service teachers would allow better generalization and more accurate findings. It is also important to note that the participants in this study were academics in high-tech professions seeking to enter the realm of teaching. It is reasonable to assume that most of them would have a positive attitude and high motivation to integrating technology. It would be interesting to compare the current results PSTs from other backgrounds. Finally, it would be interesting to explore how GenAI can be used for other pedagogical needs,

such as developing teaching processes, developing tasks combined with technologies, and more.

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