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Abstract

This study investigated the features of elementary students' problem solving skills, when teachers provide scaffolding in the process of solving an ill-structured problem in an elementary school mathematics classroom in Seoul, South Korea. In this study, participants solved the ill-structured problem following the phases of Analyze, Browse, Create, Decision-making, and Evaluate. When problem solving was completed without the phase of the Evaluate, to provide metacognitive scaffolding helped to analyze the information of the problem in more depth by returning to identifying related information, which was the sub-phase of Analyze and Browse. When there were difficulties in deepening their understanding of the information from the problem situation, to provide strategic scaffolding helped to access information in an organized way and facilitated solving an ill-structured problem. Based on these results, this study draws implications about scaffolding that can help in the process of solving ill-structured problems, and ultimately suggests the direction to advance to improve problem solving ability in mathematics education.

Introduction

Mathematics education around the world has emphasized the mathematical processes that can help solve real-world problems encountered outside of school. From this point of view, the use of ill-structured problems can be suggested. The elements contained in the ill-structured problems are not specified, and their nature itself can be ambiguous (Chi & Glaser, 1985; Jonassen, 1997). Therefore, when solving the ill-structured problem, students need to be concerned about their understanding of the problem situation and be provided with appropriate help as needed (Bransford, Brown, & Cocking, 2000). Jonassen (1997) compared the solution of structured problems to that of ill-structured problems and the help needed at each stage of solving ill-structured problems. Since then, research into providing scaffolding has been ongoing for research on how to provide help in solving ill-structured problems (Chen & Bradshaw, 2007; Davis & Linn, 2000; Ge, Chen, & Davis, 2005; Ge & Land, 2003, 2004; Greene & Land, 2000; Jang, 2014; Kim, Park, & Lim, 2015; Lee, Chen, & Chang, 2014; Song & Shin, 2010). These studies are empirical studies that analyze the effectiveness of scaffolding as a treatment for solving the ill-structured problem and can be highly appreciated in terms of their contribution to establishing a theoretical basis for scaffolding in solving the ill-structured problem. However, since these studies are in a technology-based learning environment, the contents of scaffolding are pre-determined and presented unilaterally. There are limitations that do not show consistent results regarding the effectiveness or method of utilizing scaffolding.

One of the objectives of school mathematics in Korea is to develop the ability of problem solving and this is also stated and emphasized in other countries' curriculum (CCSSI, 2010; Ministry of Education, 2015). The goal is to help transfer the problem solving skills learned in textbooks into everyday life, and ~~to do this,~~ the use of ill-structured problems in mathematics classes will have an educational implication for this. However, the studies on solving ill-structured problems and providing scaffolding, as the previous studies did, have not been done in the classroom. Therefore, in this study, in order to help students develop their problem solving skills by utilizing ill-structured problems in mathematics classes, teachers provide scaffolding as needed.

This study aimed to investigate the characteristics of solving the ill-structured problem, when teachers provide scaffolding in the process of collaborative problem solving in an elementary school mathematics classroom in South Korea. This study was guided by the following research questions:

- How does teacher scaffolding help elementary school students solve an ill-structured problem?
- What difficulties do elementary school students experience in solving an ill-structured problem?

Solving an Ill-structured Problem in Mathematics Education

One of the "good problems" that are seen in the mathematics classrooms is a problem that can transfer knowledge and experience gained in problem solving to other problem situations (Lenchner, 1983). Non-routine ill-structured problems are the types of good problems to use for enhancing student learning, because the solving of these problems lead to various conclusions which were achieved through active inquiry by learners and enabled a continued higher-order thinking by the learners (Charles & Lester, 1982; Holmes, 1995; Jonassen, 1997). Because of the characteristic of ill-structured problems that the solution could be varied, Abdillah, Mastuti, & Rahman (2018) reported that students solved ill-structured mathematical problems in various ways and those experiences had a positive effect on the development of creativity.

It is noted, that the mathematical thinking required to solve ill-structured problems is different from that required in convergent problem solving skills of well-structured problems (Dunkle, Schraw, & Bendixen, 1995; Ge & Land, 2003; Jonassen, 1997; Lave, 1988). The process of solving well-structured and ill-structured problems is divided into the phases of representing, solving, and monitoring in common, but there are the phases of justifying and evaluating in solving ill-structured problems (Ge & Land, 2003; Jonassen, 1997). Kim, Heo, & Park (2014) analyzed the characteristics of the process of solving ill-structured problems (Ge & Land, 2004; Jonassen, 1997) and developed the ABCDE model for the process of cooperatively solving ill-structured mathematical problems. Ge (2002) proposed a framework consisting of problem representation (Define problems, Set the goals, Identify related information, Collect information), solution creation (Develop solutions), and justification and selection (Construct arguments), evaluation of solutions (Evaluate solutions and justification).

The characteristics of successfully solving ill-structured problems are similar to those commonly seen by people with high levels of problem solving skills (Qualification and Curriculum Authority, 2004; Rowland, 1992). In addition, in solving ill-structured problems, the results of problem solving can be different depending on how to understand the problem situation (Artzt & Yaloz-Femia, 1999; Kim, Heo, Cho, & Park., 2012; Kintsch & Greeno, 1985; Voss & Post, 1988). Prayitno, Subang, Susiswo, & Abdur (2020) analyzed the use of visual and symbolic representations in understanding problems to investigate how mathematical knowledge and experience affect the solution of ill-structured problems. As can be seen from the studies presented above, it is important to understand problems in the process of solving ill-structured problems, unlike well-structured problems.

Also, studies on solving an ill-structured problem in elementary mathematics classrooms have explored various aspects of mathematical thinking: the type of mathematical reasoning (Kim, Heo et al., 2012), the developmental stage of proportional reasoning (Kim & Park, 2013), and the process of mathematical abstraction (Hong & Kim, 2016). There are also studies that analyze the characteristics of decision-making (Kim, Lee et al., 2012), problem solving strategies and justification types (Joo & Kim, 2014). The motivational characteristics, mathematical attitudes, and solving ill-structured problems of elementary pre-service teachers were also analyzed. Also, Araiku, Parta, & Rahardjo (2019) said that having an experience of continually solving ill-structured problems can help formulate questions, construct solutions, or make reasonable decisions based on relevant information. Taken together, the aforementioned studies have identified features of ill-structured problem solving and emphasized the importance of such experiences in the mathematics classroom, which can be seen to ultimately lead to the development of mathematical thinking.

Scaffolding and Solving an Ill-structured Problem

Wood, Bruner, and Ross (1976) described scaffolding as a framework for helping children achieve higher levels beyond their unassisted efforts. In this way, the teacher providing the scaffolding that is appropriate to the level of the learner allows the leaner to do what they can't do on their own, and thus works towards gradually reducing the scope and amount of scaffolding. It is contingent support that the teacher provides scaffolding according to the learner's situation. This means that the teacher correctly grasps the level of understanding of the learner and presents a degree of challenge suitable for the level. At this time, authentic learning takes place for learners and teachers help learners to successfully complete tasks (Van de Pol & Elbers, 2013). From this point of view, scaffolding is an interactive process that occurs between a teacher and students when both of them actively participate in the learning process, and it is characterized by the ideas related to 'contingency,' 'fading' and 'transfer of responsibility' in relation to the student's learning of these problem solving skills (Van de Pol, Volman, & Beishuizen, 2010). In addition, according to Model of Contingent Teaching (Van de Pol, Volman, Oort, & Beishuizen, 2014), teachers first need to grasp the level of how students currently understand and diagnose their condition by asking questions or looking at learner activities. After that, the scaffolding suitable for the condition should be provided.

Scaffolding needs to be distinguished from the general aids provided in the teaching and learning process (Belland, 2014). First, scaffolding helps to simplify the process in the complexity of the task, but general aids only involves simplifying the process. Also, scaffolding involves complex procedures and knowledge due to the complexity of the task, but general aids are primarily concerned with the simplification of processes. Second, scaffolding is used temporarily and contextually to solve the problem, but general aids can be used continuously and universally in any situation. Depending on the purpose of scaffolding, it can be divided into conceptual, strategic, metacognitive, and procedural types (Cagiltay, 2006; Ge & Land, 2004; Jackson, Krajcik, & Soloway, 1998). The use of conceptual scaffolding helps students understand what knowledge is necessary for problem solving, and the strategic scaffolding helps them analyze and approach problems. In this way, the use of metacognitive scaffolding helps learners monitor the current status of problem solving and reflect on its process of problem solving, and the use of procedural scaffolding helps them understand how to use the data and tools that are needed to solve a problem.

In general, questioning is considered as one of the most effective strategies to provide scaffolding (Van de Pol et al., 2010). It helps learners focus on their learning tasks and plays a leading role in their learning process. It can also help them monitor their learning paths as the students develop problem solving skills (Rosenshine, Meister, & Chapman, 1996). Questioning also elicits learners' responses, such as explaining or reasoning, to facilitate and develop higher-order thinking as they learn problem solving skills (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; King, 1992; Lin, Hmelo, Kinzer, & Secules, 1999; Rosenshine et al., 1996). The earlier studies that provided scaffolding with questioning were mainly concerned with the process of problem solving (Davis & Linn, 2000; Ge & Land, 2003; Greene & Land, 2000), while subsequent studies were often concerned with the effectiveness of scaffolding or questioning about the content of the problem (Chen & Bradshaw, 2007; Jang, 2014; Kim et al., 2015; Lee et al., 2014). Since learners may face various difficulties in the course of solving ill-structured problems, studies have been continuously conducted on providing scaffolding to help solve those difficulties (Cho & Jonassen, 2002; Ge & Land, 2004; Jonassen, 2003; Song & Shin, 2010; Voss, Wolfe, Lawrence, & Engle, 1991). Araiku et al. (2019) also revealed that when the learner faced difficulties in solving ill-structured problems, the teacher's proper guidance also helped to improve understanding of the problem and develop a critical perspective on problem solving, and the importance of providing active scaffolding was emphasized. Based on the previous studies, the difficulties that learners may encounter in the process of solving ill-structured problems and the scaffolding that can be provided to help them overcome the difficulties were summarized in Table 1.

Table 1. Difficulties and Scaffolding in Solving an Ill-Structured Problem

Phase	Difficulties encountered in solving an ill-structured problem	Scaffolding
Understand a problem	Difficulties in identifying and defining problems due to the large space of ill-structured problems	Help in identifying the purpose of problem solving Help in understanding concepts related to problem solving
Create plans for problem solving	Difficulties in gathering and verifying information that can help problem solving	Help in selecting information needed or not needed to problem solving
Make a decision and evaluate	Difficulties in justifying by convincingly presenting information as evidence	Help in making decisions by justifying Help in reviewing and monitoring the process of problem solving

The previous studies have shown that scaffolding with questioning has a positive effect on solving an ill-structured problem and have emphasized that the type or level of scaffolding should be adjusted and applied according to the learner's level and the learner's familiarity with scaffolding. Summarizing the above, continuing research on teacher-provided scaffolding can develop students' level of problem-solving skills and help them to experience higher-level thinking in problem solving.

Methods

Participants

Participants were selected by purposeful sampling (Creswell, 2012) to obtain the in-depth information on how providing scaffolding worked in solving an ill-structured problem in the elementary school mathematics classroom. First, 6th grade students were selected as participants for the following reason: in general, domain specific knowledge is needed to solve ill-structured problems (Jonassen, 1997; Voss et al., 1991), and the problem of ratio and proportion developed in this study belongs to the sixth grade curriculum in Korea. Second, there were four participants (two boys, Tom and Jack, and two girls, Emily and Jane) who were selected from an elementary school in Seoul. They voluntarily expressed their willingness to participate in this research and were recommended by their homeroom teachers based on their experiences actively participating in collaborative problem solving activities in school life. Also, this school operates a science curriculum based on Problem-Based Learning and conducts a large amount of educational activities based on collaborative activities among all students. For this reason, students in this school were selected because it was judged that verbal interaction would be actively conducted among students participating.

Research Setting

Solving an Ill-structured Problem

This study was conducted in the following order (see Figure 1). First, the individual activity (Phase 1), which lasted about 30 minutes, consisted of the allowance of time for analyzing and understanding problems, so that the individual would be prepared to work together in a small group. Then an individual interview (Phase 2) was conducted for about 10 minutes independently to identify~~ing~~ the individual's thoughts about how each participant understood and solved the problem. Afterwards, the group of four participants were asked to collaboratively solve the problem in the group activity. In the group activity (Phase 3), the participants shared what they had understood in the individual activity, and then worked together to solve the ill-structured problem. In a group interview (Phase 4), the researcher conducted a semi-structured interview on the group's collaborative problem solving process, and on the feelings of participation of the participants in solving the ill-structured problem.

Phase 1 Individual activity ➡ Phase 2 Individual interview ➡ Phase 3 Group activity ➡ Phase 4 Group interview

Figure 1. The Procedures for Collaboratively Solving an Ill-structured Problem in This Study

In this study, the process of solving an ill-structured problem was defined as follows (Ge, 2002; Kim et al., 2014) (see Table 2).

Table 2. The Phases of Solving an Ill-Structured Problem in This Study

Phases	Sub-factors of phases	Contents
Analyze/ Browse	Define problems (PD)	Do participants state the problem clearly and completely?
	Set the goals (GS)	Do participants state at least one goal for problem solving?
	Identify related information (IR)	Do participants identify necessary information and limitations from the information presented in the problem situation?
Create	Develop solutions (DS)	Do participants create a problem solution using the relationship between the information needed to solve the problem?
Decision-making	Construct arguments (CA)	Do participants use information from problem situations to justify solutions consistently and convincingly?
Evaluate	Evaluate solutions (ES)	Do participants discuss the adequacy of the solution using the information and evidence needed to solve the problem, and explain why the solution was selected?

Development of an Ill-structured Problem Focusing on 'Ratio and Proportion'

An ill-structured problem was developed based on the mathematical concepts of ratio and proportion, which are topics that can often be experienced in real life, and are related to proportional thinking, as well as the basis for higher mathematical thinking (Lesh, Post, & Behr, 1988). However, although ratio and proportion are the integrated concepts that can be broadly addressed in connection with other subjects, these are not integrated with the themes of other subjects (similarity, speed, concentration, density, scaling, etc.) in Korean mathematics textbooks (Jeong, 2003). Based on these studies, this study has developed a problem situation in which mathematical communication and mathematical reasoning could occur, and the problem could be solved according to reasonable judgment and decision making (Park & Jeong, 2010).

The problem was developed to reflect the characteristics of ill-structured problems, such as reality, openness and complexity. For this reason, there is a lot of information presented in a problem situation, so it is necessary to select the information necessary for problem solving and organize them well. The overview of the problem is shown in Table 3. One mathematics educator and three elementary school teachers with expertise in elementary mathematics education had consulted and developed the ill-structured problem together. They had also corrected and reviewed the terms suitable for the problem situation.

Table 3. Overview of the Problem Developed in This Study

Problem situation	Select sub-costs to be included in the cumulative costs to compare annual cumulative costs (the criteria of car-selection), and solve the ill-structured problem by comparing the costs according to the time when the cumulative costs are compared
Overview	Construct and compare cumulative costs using information about fuel costs, car supply prices, car taxes, and when to replace a car Solve the problem by negotiating the cost of one year as a reference to find the cumulative cost and comparing the cumulative cost.
Application of the ratio and proportion	To compare various reference quantities such as fuel efficiency, annual fuel demand and annual fuel consumption To generate information about annual fuel demand and annual fuel consumption using the information on mileage and fuel efficiency presented in the problem

The expected process of problem solving is shown in a flowchart (see Figure 2).

Design of Scaffolding

Based on the previous research, the details of how to provide scaffolding according to the type, purpose, and content of scaffolding are as follows (see Table 4). First, the participants solved the ill-structured problem on their own without any help. After they said they were done solving the problem, in Phase 3 of Figure 1, the researcher began to ask to diagnose the status of their problem solving and provided scaffolding. Also, the principles of providing scaffolding were established. First, this study planned to provide scaffolding with questioning. Second, this study planned to use conceptual, strategic, and metacognitive scaffolding. Third, the plans for providing scaffolding could be changed according to the needs of participants.

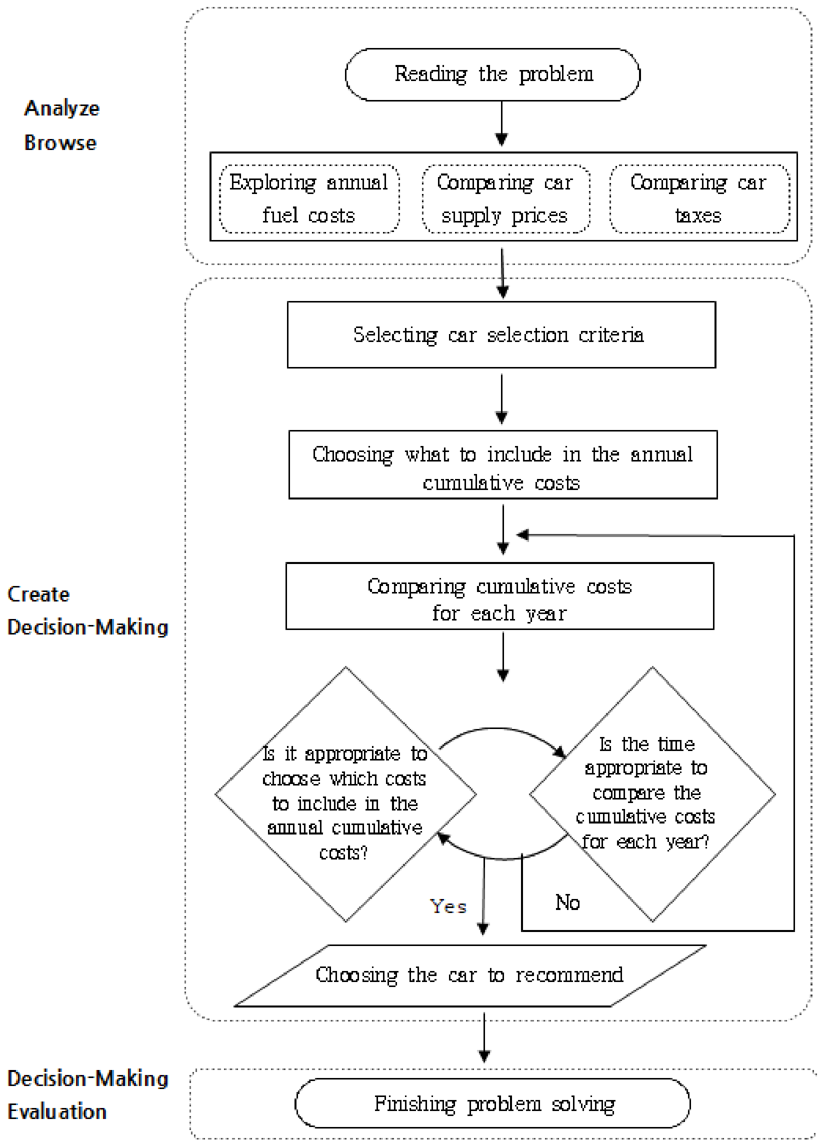


Figure 2. The Expected Process of Ill-Structured Problem Solving

Table 4. Plans of Scaffolding in This Study

Types of scaffolding	Purposes of providing scaffolding	Examples
Strategic scaffolding	Help in identifying the purpose of problem solving	What is the problem to solve?
	Help in selecting information needed for problem solving	Which of the information suggested in the problem is necessary to solve the problem? What information do you find most helpful in solving the problem? How is the information presented in the problem related to problem solving? Is there anything else I need to know other than the information presented in the problem?
	Help in making decisions by justifying	Let us explain in detail the evidence that supports your opinion.
Conceptual scaffolding	Help to understand concepts related to problem solving	What mathematical knowledge can you use to solve the problem? Have you experienced solving a similar problem or a situation similar to a problem situation?
Metacognitive scaffolding	Help to review and check the process of problem solving	Let's look back at the process of problem solving and find out what needs to be done.

Data Collection and Analysis

In this study, to collect discourse data, audio recording and video recording were performed every time with the consent of the research participants and the researchers participated in the field and recorded observations. The reason why audio recording and video recording was performed at the same time is to improve understanding of what the discourse explains, and gestures can help you understand verbal interactions properly. However, only the discourse data was taken for analysis, because the content of verbal interaction, not facial expressions or actions, represented the content of problem solving. The discourse data on video recording were collected, transcribed, and analyzed to investigate the characteristics of solving an ill-structured problem in a small group activity. In addition, interview data, individual activity sheets, and group activity sheets, which participants created during problem solving, were collected and analyzed.

In this study, in order to analyze the discourse of participants, the analysis unit was set to an episode according to the topic of the discourse. Episode analysis provides information on which topics are discussed in what ways (Choi, 2013; Hogan, Nastasi, & Pressley, 1999). Each episode begins when the topic of discourse begins to change and is composed of two or more consecutive turn-takings. The length of the episode varied depending on how long the participants' discourse continued on the topic of discourse.

Results

For this study, the results are divided into before and after providing scaffolding. In this study, the discourses data consisted of a total of 25 episodes, and the distribution of episodes was organized according to sub-factors of the phases of solving the ill-structured problem in Table 2 (see Table 5). It was shown that the episodes about IR were the highest (48%), followed by CA and ES.

Table 5. Distribution of Episodes according to Subfactors of Phase in Solving an Ill-Structured Problem

Sub-factors of phases in solving an ill-structured problem	PD	GS	IR	DS	CA	ES	Total
Frequency of episode (%)	1(4)	1(4)	12(48)	2(8)	5(20)	4(16)	25(100)

Figure 3 shows the distribution of episodes before and after providing scaffolding and compares the proportion of episodes by sub-factors of phases in the ill-structured problem solving. The ratio of IR-related discourses was highest before and after scaffolding was provided, and CA-related discourses were also consistently high. On the other hand, after scaffolding was provided, GS-related and ES-related discourses have appeared.

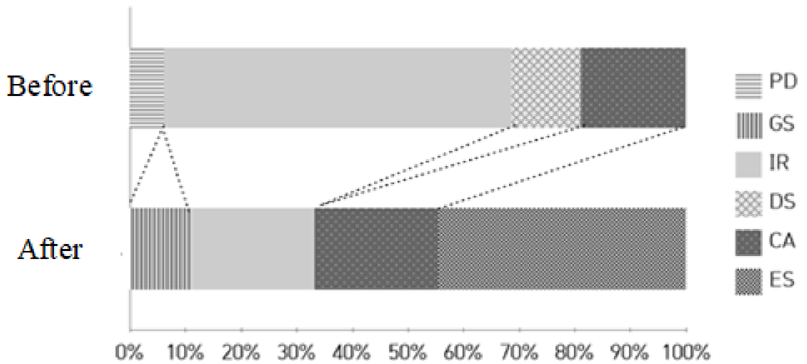


Figure 3. Distribution of Episodes by Sub-factors of the Phases in Solving an Ill-Structured Problem according to Scaffoldings

Solving an Ill-structured Problem before Scaffolding is provided

The process of solving an ill-structured problem before scaffolding is provided is as follows (see Figure 4). The phases of solving an ill-structured problem presented in Table 2 appeared sequentially in this study, but the final phase of evaluation did not.

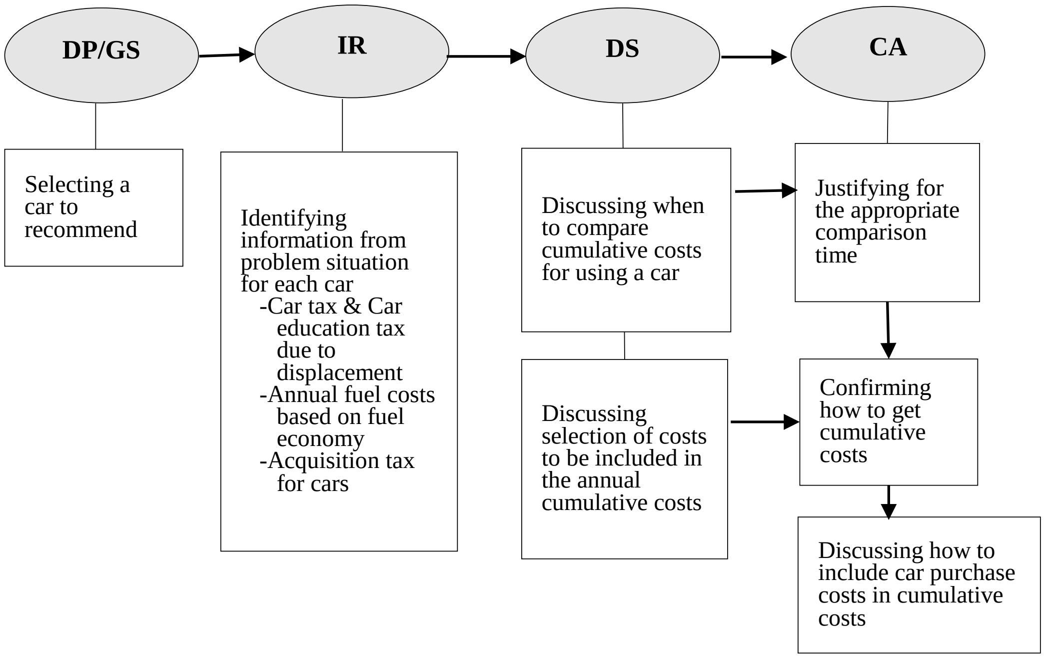


Figure 4. Solving an Ill-Structured Problem before Scaffolding is provided

Define Problems (PD)

Participants set the problem to be solved as comparing the cost of using a car to what to recommend to their fathers. They suggested how to find the total cost of buying and using a car for one year, or how to compare the cost of each part of the total cost.

Identify Related Information (IR)

The participants identified and collected information necessary for problem solving, and often used information necessary for the calculations. They gathered all the information presented in the problem situation and prepared for creating solutions. To this end, they compiled information for each car on taxes related to displacement, fuel costs, and taxes paid when buying a car.

Develop Solutions (DS)

Based on the information gathered above, the participants discussed and developed solutions in two aspects: the time to compare the cumulative costs of using a car and the cost to include in the cumulative cost. Instead, an analysis of the participant’s discussions on these two aspects suggested that the phase of IR had proceeded prematurely to the phase of CA, without a clear distinction between the information necessary or unnecessary for problem solving.

Construct Arguments (CA)

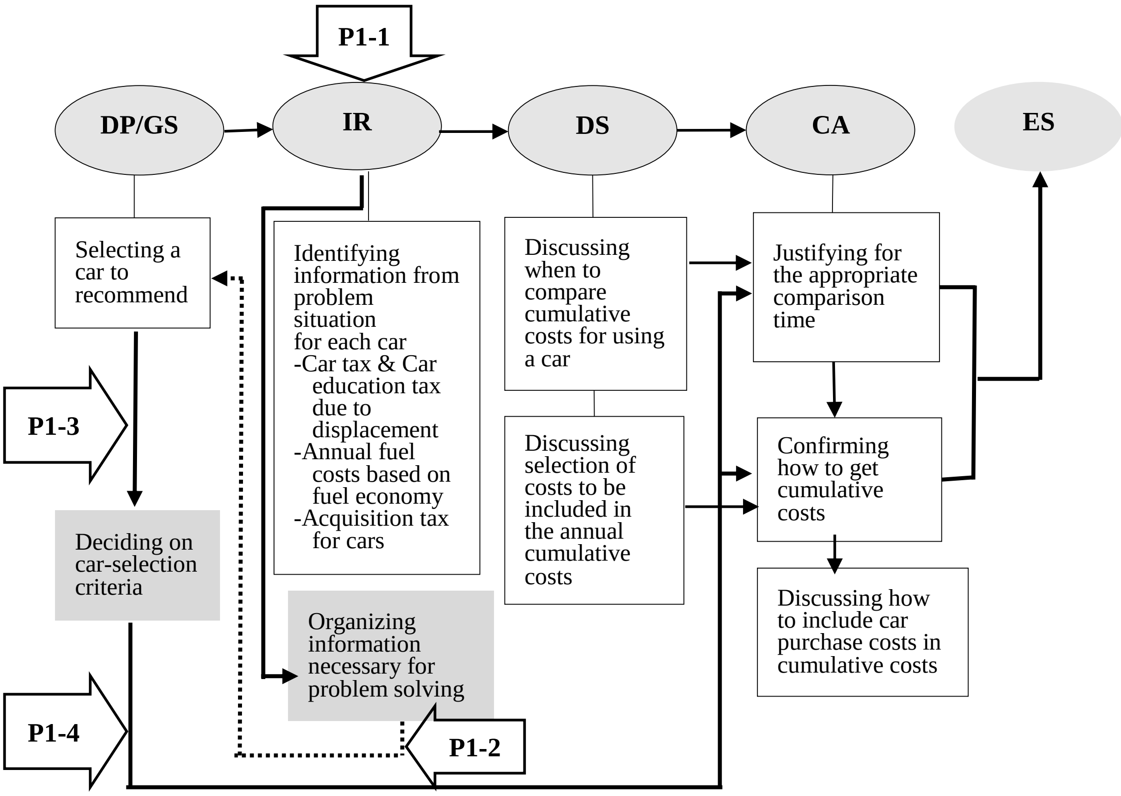
The participants justified the two aspects discussed in the phase of DS. Additionally, they justified how to compare the cumulative costs of using a car, by the cost of one year and the cost of seven years. Some participants who argued that it was appropriate to compare the cumulative costs of using a car for seven years did not solve the problem systematically, and therefore did not give convincing arguments because they presented only the principle of how to compare those costs. Similarly, others who wanted to compare costs for one year didn't provide any concrete evidence of these calculations, because it was too complicated to calculate. Subsequently, they found that it was an error to include the car supply price several times when calculating the cumulative cost, they then recalculated the cumulative cost and finally solved the problem by selecting Car B.

Evaluate Solutions (ES)

There was no process for discussing the adequacy of the solution, nor to explain in detail why the solution was chosen.

Solving an Ill-structured Problem after Scaffolding is provided

Figure 5 shows the changes in solving an ill-structured problem after scaffolding is provided.



*thin solid line: Solving an ill-structured problem before scaffolding is provided
*thick solid line: Solving an ill-structured problem after scaffolding is provided
*dotted line: Solving an ill-structured problem not facilitated after scaffolding is provided
*colored box: Features of ill-structured problem solving after scaffolding is provided

Figure 5. Solving an Ill-Structured Problem after Scaffolding is provided

Returning to Identifying Related Information

As shown in Extract 1, the researchers asked questions to diagnose the current level of the learner's understanding in solving an ill-structured problem (underlined in Extract 1). This diagnosis serves as a basis for what scaffolding to provide. In the process of diagnosis, the participants were confused about the meaning of the terms 'fuel efficiency' and 'fuel cost,' which are presented in the problem situation and played an important role in solving the ill-structured problem. Researchers continued to question this confusion, and participants were able to properly understand the confused meaning by responding. Also, through this process, participants gained a better understanding of the information needed to solve the problem.

Extract 1:

Researcher:	**<u>Do you know what you need to solve in this problem?</u>**
Tom:	Yes.
Researcher:	What should I solve?
Tom:	Which car would you recommend
Researcher:	**<u>We have to choose which car to recommend, if we recommend, what can we recommend?</u>**

Tom: Reason
Emily: Evidence
Researcher: **Yes, evidence. There must be criteria. Can you make recommendations based on criteria? What criteria do you have?**
Jack: (Pointing to the problem situation activity sheet) It is written here, to consider fuel, cost, etc.
Jane: But we did it at our price.
(omitted)
Researcher: **But what you're getting confused about is the concept of fuel efficiency**.
Jack: Fuel economy is the distance you can go per liter
Researcher: Yes. There's a problem sheet in there. What is fuel efficiency?
(omitted)
Researcher: **I think that's what you call the fuel efficiency continuously.**
Jack: Fuel cost
Researcher Yes, the fuel cost is correct.

As shown in Extract 1, participants well defined the problem to be solved, so they needed to help them progress into a phase where they needed a deeper understanding of the information. They did not understand the limitations of the information in the process of developing and justifying problem solving solutions. This can be regarded as not convincingly constructing the argument for the solution. Therefore, it was necessary to provide strategic scaffolding (P1-1) to help in-depth understanding of information for problem solving (see Extract 2).

Before scaffolding was provided, participants tried to use all the information presented in the problem. However, when strategic scaffolding (P1-1) was provided, the participants organized and classified information necessary or unnecessary for problem solving in the phase of IR (see Extract 2). By providing P1-1, information that is not necessary for problem solving is excluded, and information necessary for problem solving is organized. To organize with only the necessary information helped to access the solution more systematically. It was analyzed that this change presented by providing P1-1 facilitated ill-structured problem solving in this study.

Extract 2:
Researcher: So, **aren't there any pieces of information that you need to solve your problem, and some that you don't need? (P1-1)**
Researcher: Let's discuss it with our friends, and look back at the process we've solved so far and see if it seems to have solved it correctly or whether our approach seems right. I think we need to talk again.
(omitted)
Jack: Car education tax
Researcher: Why do you think so?
Jack: It's all the same
Researcher: I also think it's the same thing.
By the way, did you calculate car education tax or not?
Jack: No, we didn't.
Jack: (At the same time with Jane) we did.
Researcher: You didn't?
Jack: (Laughing) we did.
Researcher: You did. But should you have done it?
Jack: No.
Tom: No, we didn't.
Researcher: I think you can calculate it except that.
And do you have any other information you don't need?
Tom: Displacement
Jack: Displacement is the same.
Researcher: Yes, it's the same.

Returning to Setting the Goals

Metacognitive scaffolding (P1-2) was provided to help participants organize the information necessary for problem solving, to identify and reflect the current state of problem solving. As soon as the P1-2 was provided, one of the participants (Jack) then asked about the concerns he had had on his own before providing scaffolding, and participants began discussing the car-selection criteria.

In this context, the participants presented a variety of opinions on what could be considered as car-selection criteria (see Extract 3). One participant suggested an opinion based on the information presented in the problem situation, whereas the others were based on common knowledge used in everyday life. At that time, they were confused about car-selection criteria. Likewise, this discussion led back to the phase of specifying the sub-goals of defining the problem. This was analyzed that P1-2 made the participants return to the phase of setting the goals necessary for problem solving, and provided an opportunity to consider various viewpoints when setting the goals. After providing P1-2, participants were confused setting sub-goals.

Extract 3:

Researcher:	<u>Then let's take a look at the process you just took and talk about whether there's something you need to fix or if you missed it. **(P1-2)**</u>
Jack:	I missed… No, not missed, but the odd thing is that Tom kept saying "performance, performance", but it's not listed here.
	(omitted)
Jack:	But it's all written here (activity sheet).
Emily:	You have to look at the design.
Jack:	The design is the same.
Jane:	By the way, there are three things to consider in the activity sheet, because there is no design or performance in it.
Jack:	Performance is there. Hybrid. Electric energy utilization.
Jane:	It's all here (activity sheet).
Jack:	It is written here (activity sheet). Convenience, comfort and dynamics…

To help clarify the sub-goals of problem solving, strategic scaffolding (P1-3) is provided. Providing P1-3 helped evaluate the adequacy of various opinions on the car-selection criteria presented so far. And since the information presented in the problem situation was all about cost, participants agreed that choosing the car based on the total cost was the most reasonable way (see Extract 4). It can be analyzed that providing P1-3 helped us to realize that to justify using the information presented in a problem situation is a convincing way for providing evidence, when making diverse opinions and making decisions,

Extract 4:

Researcher:	<u>What can be the criteria with the information given here? **(P1-3)**</u>
Tom:	There's only price?
Emily:	Price (in a small voice)
Jack:	The price, it's written here (activity sheet), but the price and fuel efficiency…
Emily:	It is only price. Let's set the price.

Advancing to Creating Arguments and Evaluating Solutions

Meta-cognitive scaffolding (P1-4) was provided to help participants review and finalize the problem solving process when consensus was reached on creating sub-goals (see Extract 5).

Extract 5:

Researcher:	<u>If you decide on a price, will you re-identify what information is available and then finish problem solving? **(P1-4)**</u>

After the metacognitive scaffolding (P1-4) was provided, the participants examined the activity sheets they had recorded as they solved the problem. They tried to build a shared understanding among them by checking the calculation process and the costs to be included in comparing the overall cost, and when to compare the cumulative cost. Finally, the problem was solved by mathematically explaining the reason for choosing Car B. It is analyzed that what the participants discussed after providing P1-1, P1-2, and P1-3 helped to discuss the appropriateness of the solution and to have a shared understanding of the problem solving.

The changes in the process of ill-structured problem solving were examined by dividing them before and after providing scaffolding. Those are summarized in Table 6.

Table 4. Comparing Performance of Solving an Ill-Structured Problem according to Scaffolding

Solving an ill-structured problem before scaffolding is provided	Provided scaffolding	Solving an ill-structured problem after scaffolding is provided
Defining a problem by selecting a car to recommend	_	_
Starting to look for relevant information without having a clear discussion of the criteria to consider in selecting a car	metacognitive scaffolding (P1-2)	Suggesting various opinions about the criteria for selecting cars, but having unclear evidence, which leads to confusion in selecting criteria for problem solving
	strategic scaffolding (P1-3)	Discussing car-selection criteria using information in problem situation Clarifying sub-goals
Using information contained in problem situation and calculating to make information needed	strategic scaffolding (P1-1)	Organizing information presented in problem situation More systematic approach to creating solutions
Discussing when to compare the cumulative costs of using a car Discussing the types of costs to include in the annual cumulative costs	_	_
Justifying the appropriate time of comparison, but failing to construct persuasive arguments Discussing how to get cumulative costs using information, but stop discussing without integrating it into the process of developing a solution Comparing the cost of using Car B with the cost of using another car, and explaining why choosing Car B Not discussing whether it is appropriate to find out how much it costs to use a car	metacognitive scaffolding (P1-4)	Constructing arguments about why costs should be compared at seven years Having a shared understanding when to compare costs Having a shared understanding of the process and results of problem solving, and evaluating the suitability of the solution Finally making decisions about the solutions

All four scaffolding were provided in this study. Three scaffolding (P1-1, P1-3, P1-4) facilitated solving the ill-structured problem, whereas the other scaffolding (P1-2) caused confusion. This is analyzed because P1-2 did not fit the problem solving situation of the participants, but did not provide a convincing reason when presenting various opinions.

Difficulties Encountered in Solving an Ill-structured Problem

In this study, based on the diagnosis of the status of problem solving and the analysis of the process that provided scaffolding, participants had difficulties in two aspects in solving the ill-structured problem. First, the participants had experienced difficulties in identifying and organizing the information when developing solutions for the ill-structured problem. To help overcome this difficulty, the strategic scaffolding has been provided that leads to identifying and gathering the information necessary for solving the ill-structured problem, and understanding the relationships between the information. Generally speaking, it helped to organize the information and discuss in detail how to use it for problem solving. It also helped to find and fix errors that were made while solving the ill-structured problem. In summary, it worked to help deepen an understanding of the information elaborated and utilized in problem solving. This difficulty also appeared in responses in group interviews such as "I had a lot of things to consider," and "I had a lot to choose." In addition, as shown in the following excerpt, it was useful to help the participants understand the relationship and structure of the complex information, which were presented in the problem situation.

Researcher: If you have to solve this problem with your classmates in class, what do you think the teacher needs to do to help you?

Jack: Arrange it like this (pointing to the structure of solving the ill-structured problem that the researcher had drawn on the board).

Jane: Yes.
Jack: To structure the hints.

Second, although a process of monitoring or evaluating is necessary for the ill-structured problem solving, the participants did not voluntarily perform those process and couldn't elaborate the solution in this study. For this reason, in order to help overcome this difficulty, the researcher provided a metacognitive scaffolding to deliberately elicit the phases of monitoring and evaluating the solution. Ultimately, this led to a shared understanding among the participants.

Conclusion

Many of the studies on scaffolding in solving ill-structured problems were mostly about the effectiveness of scaffolding in a technology-based learning environment, and those studies played a major role in establishing the theoretical basis for scaffolding. However, there is little work on scaffolding for the ill-structured problem solving in elementary school mathematics classroom. This study aimed to investigate how teacher scaffolding helps elementary school students solve an ill-structured problem in mathematics classroom and what difficulties they experience in the process of solving an ill-structured problem.

First, in this study, there have been shown two major difficulties in solving an ill-structured problem: one about the difficulties in the phases of identifying and organizing the necessary information from problem situation, and the other about not monitoring or not evaluating the appropriateness of the final selected solution. The reason for these difficulties is due to the characteristics of the ill-structured problem itself: one or more of the problem elements are unknown or vaguely defined, and the problem space is large due to unclear objectives and conditions that do not appear directly in the problem situation (Jonassen, 1997; Kitchner, 1983; Spiro, Coulson, Feltovich, & Anderson, 1988; Wood, 1983). Considering the difficulties presented in this study in-depth understanding of the problem situation is important in solving the ill-structured problem, in connection with previous studies (Artzt & Yaloz-Femia, 1999; Kim et al., 2012; Kintsch & Greeno, 1985; Voss & Post, 1988) that revealed that solutions differ depending on how understanding of the problem situation is achieved,

Second, before providing scaffolding, participants solved the ill-structured problem without any help. Although the purpose of providing each scaffolding was different, it was analyzed that the changes made by scaffolding generally facilitated the ill-structured problem solving after providing scaffolding. In this study, metacognitive scaffolding helped to reset the goals and develop solution for ill-structured problem solving, and strategic scaffolding helped organize information and make good use of it to discuss the suitability of the solution. This is consistent with the findings of Ge & Land (2003) and Jonassen (1997), who said that phases of monitoring and justifying are needed to solve ill-structured problems. As reported in prior studies (Araiku et al., 2019; Chen & Bradshaw, 2007; Davis & Linn, 2000; Kim et al., 2015; Ge & Land, 2003, 2004; Ge et al., 2005; Greene & Land, 2000; Jonassen, 1997; Lee et al., 2014), providing the scaffolding can be effective in the ill-structured problem solving and qualitatively improve that. Furthermore, the fact that facilitating the ill-structured problem solving with scaffolding means that the scaffolding provided in this study was contingent upon the state of the ill-structured problem solving. Here, the fact that scaffolding was provided according to the learner's state means that with the help of scaffolding the learner can do what he or she could not do alone. Subsequently, scaffolding will gradually decrease, and the scope of transfer of responsibility for learners themselves will expand.

Based on the results of this study, providing scaffolding helped to facilitate the ill-structured problem solving. Particularly, when scaffolding was provided while solving an ill-structured problem, participants began to explore the problem situation in more depth, which led to efforts to find the best solution. This is similar to the characteristics of expert problem solving (Rowland, 1992) and those of high level problem solving (QCA, 2004). This means that providing scaffolding in the process of solving an ill-structured problem can foster the quality of problem solving solution and help to achieve a higher level of problem solving, thereby improving your problem-solving ability like an expert.

With these findings, there is a continuing need for research to identify what it means to provide "a contingent scaffolding" when a teacher provides scaffolding. As shown in Cho & Jonassen (2002), Jonassen (2003), Voss et al. (1991), learners do not come up with ideas at once or may not even try to solve it themselves, when they are struggling on their own to solve an ill-structured problem. If the teacher provides "a contingent scaffolding" for the learners' state, the learners will be able to solve the ill-structured problem only by overcoming the difficulties. As stated in Puntambekar & Hubscher (2005) and Tharp & Gallimore (1988), this will lead to an increase in mathematical problem solving ability, and will enhance the learner's self-regulatory ability to solve the ill-structured problem on their own without teacher's scaffolding.

Notes

This study was based on parts of a dissertation titled, 'A study on ill-structured mathematical problem-solving and peer interactions according to teacher's scaffolding,' by the first author.

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