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Discovering Concepts of Geometry through Robotics Coding Activities

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Abstract

In recent years, mathematics classrooms in the U.S. and around the world have seen an increasing integration of educational robotics with interest from both students and teachers. Through their robotics coding activities, students in the present study discovered the concepts of special angle pairs in geometry—namely, complementary and supplementary angles—as they learned to navigate the immediate feedback from the robot Sphero SPRK+ into a trial-and-error mathematics problem-solving process. Students’ experiences in these three coding activities revealed, to a certain extent, that engaging in reflective play could be shaped into meaningful teachable moments where students could participate in a “doing with learning” pedagogical method using educational robotics. These activities had transferability implications that might afford STEM learning access and opportunities for students to develop not only mathematical reasoning skills, but also problem solving and critical thinking skills operable to a coding environment. This paper presents students’ use of educational robotics in a school geometry curriculum setting to demonstrate the possibility that mathematics concepts could be gathered and mastered in a playful and informal manner, and that robotics games and computer coding could be performed and framed in a thoughtful and challenging manner.

Introduction

During recent decades, the use of robotics in school mathematics as an instructional means of engaging and motivating students at the elementary grade levels has found growing favor with classroom teachers (Alimisis, 2013; Bers et al., 2002; Kim et al., 2015). Students, in turn, view this learning method positively as they become empowered to apply various abstract concepts in mathematics to concrete situations in the real world (Atmatzidou & Demetriadis, 2016; Jara et al., 2011; Nugent et al., 2010). Notwithstanding student engagement as intrinsic personal motivation to learn and a key to effective teaching in educational settings (Bandura, 1997; Kahu, 2013), some researchers are not without reservations about the pedagogy of educational robotics (Barak & Assal, 2018).

In this paper, we examine the experience of students in exploring, identifying, and understanding geometry concepts through robotics-coding activities at the elementary grade levels. In doing so, we address the concern

of “doing without learning” (Barak, 2012) in a seemingly overly-focused-on-robotics-play environment by presenting the opportunity of “doing with learning” as an alternative approach of teaching and learning mathematics.

We designed robotics-coding activities using Sphero SPRK+ (Sphero, 2019a) and the Sphero Edu application (Sphero, 2019b). Sphero SPRK+ is a ball-shaped robot capable of being encoded with specific operating instructions. Essential commands of Sphero SPRK+ in the Sphero Edu app (such as speed, travel time, initial heading, and angular direction) were the focus of our current study as students discovered the concepts of special angle pairs in geometry—namely, complementary and supplementary angles. Through their robotics-coding activities, students learned to translate the immediate feedback from Sphero SPRK+ into a trial-and-error mathematics.

Technology in Mathematics Education

Technology has played an increasingly important role in the teaching and learning of mathematics (Brown, 2015; Geiger et al., 2012; Hardy, 2008; Powers & Blubaugh, 2005; Shaffer & Kaput, 1998). Mathematics teacher educators recognized both advantages and disadvantages in its implementations in classrooms when considering the attitudes, beliefs, and practices of not only pre-service and in-service teachers (Chuang, 2013; Kersaint, 2003; Pierce & Ball, 2009; Sahin & Thompson, 2007; Zbiek, 1998), but of their students as well (Drijvers, 2015; Goos et al., 2000; Pierce et al., 2007; Schmidt et al., 2009; Young et al., 2017).

Some researchers highlighted significant improvements in student performance in mathematics assessments over time, while others noted a comparable increase in the quality of students’ mathematical understanding as a whole (Brown et al., 2004; Estapa et al., 2017; Huang et al., 2010; Leong & Lim-Teo, 2003; Mayes, 1992). This positive effect has also been shown to be exhaustive and extensive, albeit with gradual empirical evidence, when it comes to different branches of mathematics topics in K-12 curriculum (Hollebrands, 2007; Kertil & Gurel, 2016; Kumar, 2014; Lagrange et al., 2003; Oates, 2011; Özgün-Koca et al., 2010).

Beyond handheld graphing calculators (e.g., Bostic & Pape, 2010; Ellington, 2006; Waits & Demana, 2000), recent studies also analyzed the use of technology ranging from educational mathematical software—such as Computer Algebra System (e.g., Mallet, 2007; Özgün-Koca, 2010; Palmiter, 1991), GeoGebra (e.g., Bhagat & Chang, 2015; Botana et al., 2015; Hohenwarter et al., 2009), and Geometer’s Sketchpad (e.g., Kesan & Caliskan, 2013; Meng & Sam, 2011; Weaver & Quinn, 1999)—to computer-coding software and robotics, such as Python (e.g., Frassia, 2018; Grandell et al., 2006; Orfanakis & Papadakis, 2016) and Scratch (e.g., Amador & Soule, 2015; Calao et al., 2015; Rodríguez-Martínez et al., 2020). It is also worth noting that a greater proportion of classroom applications of educational mathematics software has been oriented toward geometry than toward other branches in mathematics (Aydoğdu, 2014; Ferrara et al., 2006; Hohenwarter et al., 2009; Laborde et al., 2006; Sinclair & Bruce, 2015; Sinclair et al., 2016).

Further developments in robotics established that a classroom community’s exposure to educational robotics can

enrich and integrate with the school mathematics curriculum (Anwar et al., 2019; Benitti, 2012; Eguchi, 2014; Ioannou & Makridou, 2018; Zhong & Xia, 2020). At the same time, educational robots have also been used outside classroom settings where games, competitions, or tournaments might be involved (Barker & Ansorge, 2007; Menekse et al., 2017; Sklar et al., 2003; Yudin et al., 2017). This paper presents students' use of educational robotics (specifically Sphero SPRK+) in a school geometry curriculum setting to demonstrate the possibility that mathematics concepts may be gathered and mastered in a playful and informal manner, and that robotics games and computer coding may be performed and framed in a thoughtful and challenging manner.

Method

The current study aims, through a set of three robotics-coding activities and by building on students' prior geometric knowledge of measures of single angles (specifically, acute, right, obtuse, and straight angles), to introduce elementary school students to the concepts of special angle pairs in geometry (namely, complementary and supplementary angles). The current study was specifically guided by the following research question: to what extent would robotic coding activities interact with mathematical problem solving and critical thinking skills in the process of the development of new mathematical concepts in measures of complementary and supplementary angles at the elementary school level?

The current study involved 24 elementary school students (four 4th graders and 20 5th graders, nine males and 15 females, ages 9 to 10). An announcement to solicit the participation of students in grades 4 to 6 in a two-week science, technology, engineering, and mathematics (STEM) summer school program was distributed to one large school district in a southern state of the U.S. The 24 students who enrolled in this two-week STEM summer school program participated in the current study voluntarily. Students' participation was part of the two-week, three-hours-per-day STEM summer school program that was led by two mathematics education faculty (the first and second authors) and four preservice elementary teachers, and was geared towards the Hispanic community in a southern state of the U.S. Students in the current study had little to no prior exposure to any computer programming activities. Specifically, they were not familiar with Sphero SPRK+ or the Sphero Edu app prior to the current study.

A pre-test assessment was administered to students. Included in the pre-test assessment were elementary items in geometry involving measures of single angles (e.g., acute, right, obtuse, and straight angles) and those of special angle pairs (e.g., complementary and supplementary angles). It was expected that the 4th and 5th graders in the current study would not have been aware of the terms and concepts of complementary and supplementary angles, as those topics were part of the 7th grade common core state standards of mathematics. An assessment similar to the pre-test assessment was administered to students as the post-test assessment. Following the pre-test assessment and preceding the post-test assessment, students participated in three robotics-coding activities: driving, boomerang, and bowling.

The three robotics-coding activities incorporated geometric concepts of measures of single angles (e.g., acute, right, obtuse, and straight angles) in the 4th grade and measures of special angle pairs (e.g., complementary and

supplementary angles) in the 7th grade (Common Core State Standards Initiative, 2010; International Technology Education Association, 2007; National Science Teaching Association, 2013; Texas State Mathematics Standards, 2012; Texas State Science Standards, 2017). Table 1 shows examples of science, technology, engineering, and mathematics practices and standards specifically addressed by the three robotics-coding activities.

Table 1. Examples of STEM Practices or Standards addressed by the Three Robotics-coding Activities

Activities	STEM Practices or Standards	
Activity 1: Driving	State Math Standards	State Math Standards
	<p>Grade 4, (6) Geometry and measurement. The student is expected to: (A) identify points, lines, line segments, rays, angles, and perpendicular and parallel lines; and (C) apply knowledge of right angles to identify acute, right, and obtuse triangles.</p> <p>Common Core State Standards (CCSS) for Math</p> <p>CCSS.MATH.CONTENT.4.G.A.1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</p>	<p>Grade 6. (5) Proportionality. The student is expected to: (A) represent mathematical and real-world problems involving ratios and rates using scale factors, tables, graphs, and proportions.</p> <p>Grade 6. (6) Expressions, equations, and relationships. The student is expected to: (A) identify independent and dependent quantities from tables and graphs; and (C) represent a given situation using verbal descriptions, tables, graphs, and equations in the form $y = kx$ or $y = x + b$.</p> <p>Common Core State Standards (CCSS) for Math</p>
Activity 3: Bowling	State Math Standards	State Science Standards
Activity 2: Boomerang	<p>Grade 4, (6) Geometry and measurement. The student is expected to: (E) determine the measure of an unknown angle formed by two non-overlapping adjacent angles given one or both angle measures.</p> <p>Grade 7. (11) Expressions, equations, and relationships. The student is expected to: (C) write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships.</p>	<p>CCSS.MATH.CONTENT.6.EE.C.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation</p>

Grade 8. (11) Expressions, equations, and relationships. The student is expected to: **(D)** use informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Common Core State Standards (CCSS) for Math

CCSS.MATH.CONTENT.7.G.B.5.

Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Grade 6. (8) Force, motion, and energy. The student knows force and motion are related to potential and kinetic energy. The student is expected to: **(B)** identify and describe the changes in position, direction, and speed of an object when acted upon by unbalanced forces; **(C)** calculate average speed using distance and time measurements; and **(E)** investigate how inclined planes can be used to change the amount of force to move an object.

Science & Engineering Practices in the Next Generation Science Standards (NGSS) (e.g., Grades 3-5)

Asking Questions and Defining Problems. Ask questions about what would happen if a variable is changed.

Planning and Carrying Out Investigations. Make observations and/or measurements to produce data to serve as the basis for evidence for an explanation of a phenomenon or test a design solution; Make predictions about what would happen if a variable changes.

Analyzing and Interpreting Data. Analyze and interpret data to make sense of phenomena, using logical reasoning, mathematics, and/or computation.

Constructing Explanations and Designing Solutions. Use evidence (e.g., measurements, observations, patterns) to construct or support an explanation or design a solution to a problem.

International Technology Education Association (ITEA/ITEEA) Standards for Technological Literacy

Standard 2. Students will develop an understanding of the core concepts of technology. (M) Technological systems include input, processes, output, and, at times, feedback (6-8).

Standard 9. Students will develop an understanding of engineering design. (C) The engineering design process involves defining a problem, generating ideas, selecting a solution, testing the solution(s), making the item, evaluating it, and presenting the results (3-5).

Activity 1: Driving

Activity 2: Boomerang

Activity 3: Bowling

Standard 11. Students will develop abilities to apply the design process. (F) Test and evaluate the solutions for the design problem; (G) Improve the design solutions (3-5).

Standard 16. Students will develop an understanding of and be able to select and use energy and power technologies. (D) Tools, machines, products, and systems use energy in order to do work (3-5).

Activity 1: Driving

This activity was an introduction to coding Sphero SPRK+ using the Sphero Edu app. Students were introduced to the elementary commands of Sphero SPRK+ in the Sphero Edu app using iPads, Apple-based tablet computer devices. Figure 1 illustrates the Sphero Edu app's basic coding blocks, including start, delay, speed, travel time, initial heading, angular direction, light change, speak, and sound play commands.

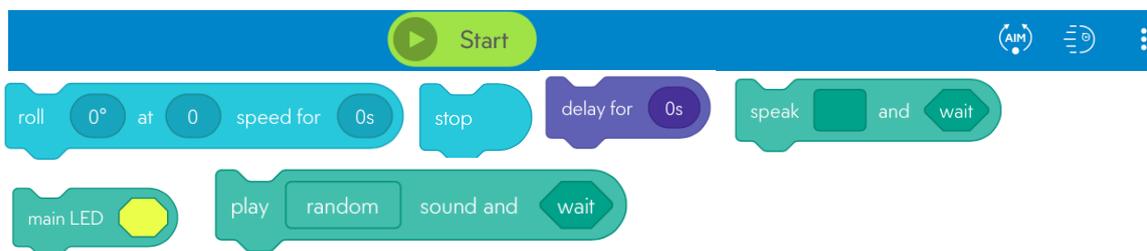


Figure 1. Example of Coding using the Sphero Edu App

Students were also shown how to modify the initial heading of Sphero SPRK+. For example, to rotate Sphero SPRK+ to an initial heading of 0° , students learned to orient the aim button until the blue tail-light faced them—that is, calibrating to the opposite direction of 180° from the initial heading of 0° (see Figure 2).

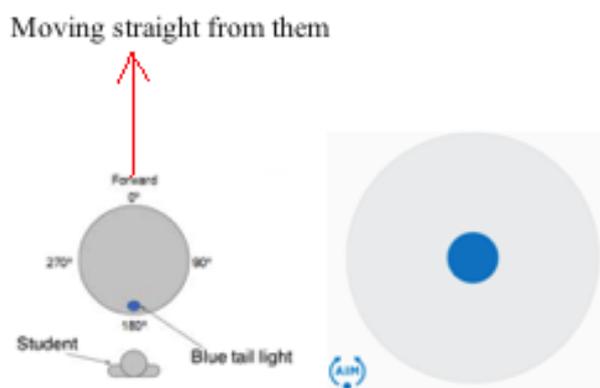


Figure 2. Orienting Sphero SPRK+ to an Initial Heading of 0°

As students organized these code blocks in a sequence, they learned to program the movement of Sphero SPRK+. In the beginning, speed, duration, and “Start” were the common sequential movement commands that students used as a practice while maintaining the initial heading of 0° . For instance, students in “Activity 1:

Driving” were instructed to drive Sphero SPRK+ to reach a certain distance (see Figure 3). After several practices, students in “Activity 1: Driving” had the opportunity to drive Sphero SPRK+ in a specific direction in a “Mission Clear!” drive (see Figure 4).

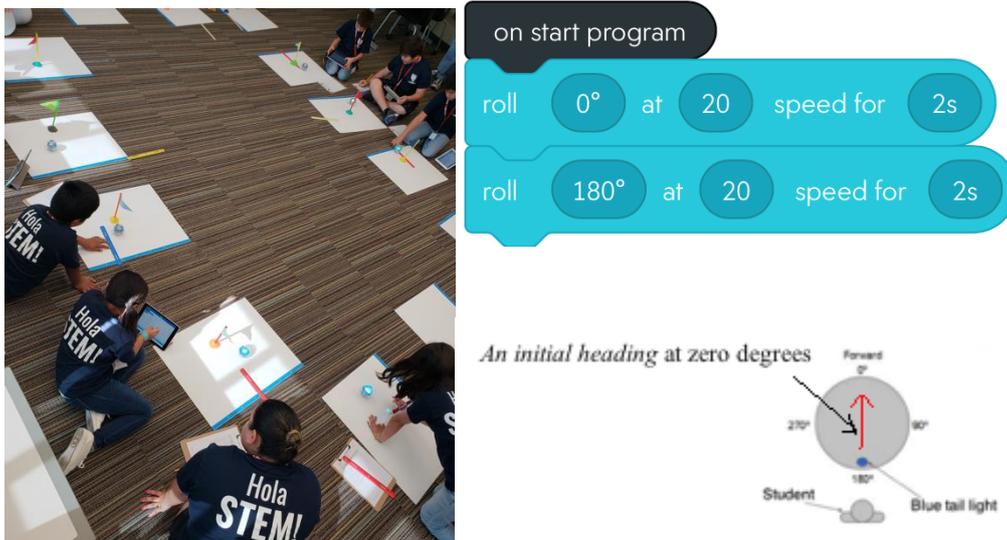


Figure 3. Driving Sphero SPRK+ for a 40-cm Distance

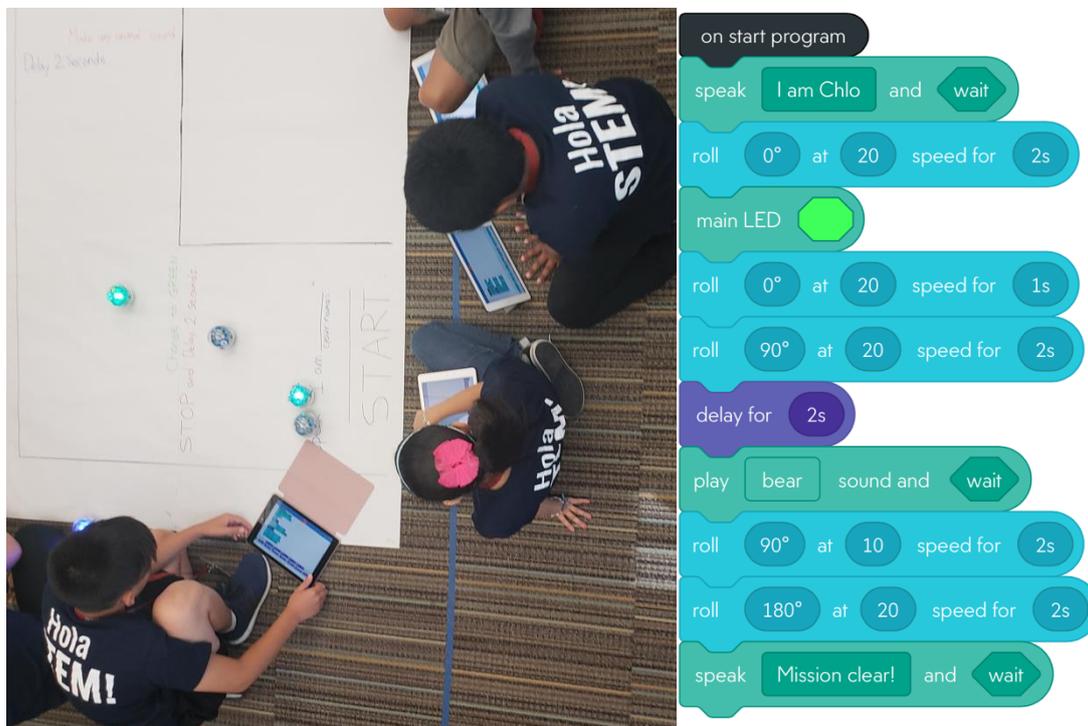


Figure 4. Mission Clear! Example

Activity 2: Boomerang

This activity required students to code Sphero SPRK+ so that it traced common types of polygons, such as squares, isosceles right triangles, and equilateral triangles. Students applied their reasoning and coding skills to

drive Sphero SPRK+ in a boomerang-style pathway. The goal of this activity was to develop and later fine tune students' understanding of complementary and supplementary angles.

Activity 3: Bowling

This activity offered students an opportunity to apply the knowledge of complementary and supplementary angles they learned from the previous two activities into a playful game of bowling. The goal of this activity was to make the connection between the mathematical knowledge they learned and the real-world problems that they needed to tackle in this game. Students were to code Sphero SPRK+ in a bowling motion to bring down as many bowling pins as possible in order to earn the highest scores.

Results and Discussion

The finding of the pre-test assessments revealed that, by and large, students were familiar with measures of single angles (e.g., acute, right, obtuse, and straight angles). On average, students scored approximately 94% on problems involving measures of single angles in the pre-test assessments. However, they showed little to no understanding of measures of special angle pairs (e.g., complementary and supplementary angles). Students scored approximately 35% on average on problems pertaining to measures of special angle pairs in the pre-test assessments.

Activity 1: Driving

This activity provided students with a warm-up example to drive Sphero forward to reach a distance of 40 cm with an initial heading of 0° at a speed of 20 cm/sec (a 2-second travel time), as shown in the first sequence of commands (see Figure 3). Students were given time to practice different distances with the same initial heading of 0° . After the initial practice drive, students were presented with particular sequences of commands to drive Sphero SPRK+. Figure 4 demonstrates an example in a "Mission Clear!" drive. Following the on-start program in this example, students were instructed to code Sphero SPRK+ to: (i) speak "I am (your first name)"; (ii) drive for a 40-cm distance using a rolling angular direction of 0° at a speed of 20 cm/sec with a 2-second travel time; (iii) change the color of the main LED light to green; (iv) drive for a 20-cm distance using a rolling angular direction of 0° at a speed of 20 cm/sec with a 1-second travel time; (v) drive for a 40-cm distance using a rolling angular direction of 90° at a speed of 20 cm/sec with a 2-second travel time; (vi) delay for 2 seconds; (vii) make an animal sound; (viii) follow a path on a drive for a 20-cm distance using a rolling angular direction of 90° at a speed of 10 cm/sec with a 2-second travel time, followed by a drive for a 40-cm distance using a rolling angular direction of 180° at a speed of 20 cm/sec with a 2-second travel time; and (ix) speak "Mission clear!" From the "Mission Clear!" drive, students were able to review their understanding of special angles such as the zero angle (0°), right angle (90°), and straight angle (180°).

Returning to the first example in Figure 3, students were provided with the opportunity to problem solve. In a mathematical modeling problem, they were asked to drive Sphero SPRK+ back to its original position after the

initial 40-cm drive. To achieve this, students employed the same speed of 20 cm/sec with a 2-second travel time to cover the same pathway. Students realized that keeping the rolling angular direction of 0° drove Sphero SPRK+ further away from the original position. Provided that the initial heading was still at 0° , they then reasoned through trial and error that the rolling angular direction should be positioned to 180° , as opposed to the starting 0° . This is shown in the second sequence of commands (see Figure 3).

To some extent, the experience in “Activity 1: Driving” was their first informal exposure to the idea of special angle pairs. The fact that they made a connection between the initial heading of 0° and the appropriate angular direction of 180° demonstrated their initial understanding of the supplementary nature of these two angles after only a few prompts by their instructors.

Activity 2: Boomerang

Like the connection between a singular angle and a straight angle discussed earlier in “Activity 1: Driving,” a similar connection between a singular angle and a right angle was generalized by students in “Activity 2: Boomerang.” Instead of a straight-line pathway found in “Activity 1: Driving,” students were asked to code Sphero SPRK+ to follow three types of polygonal-shaped pathways—namely, squares, right triangles, and equilateral triangles.

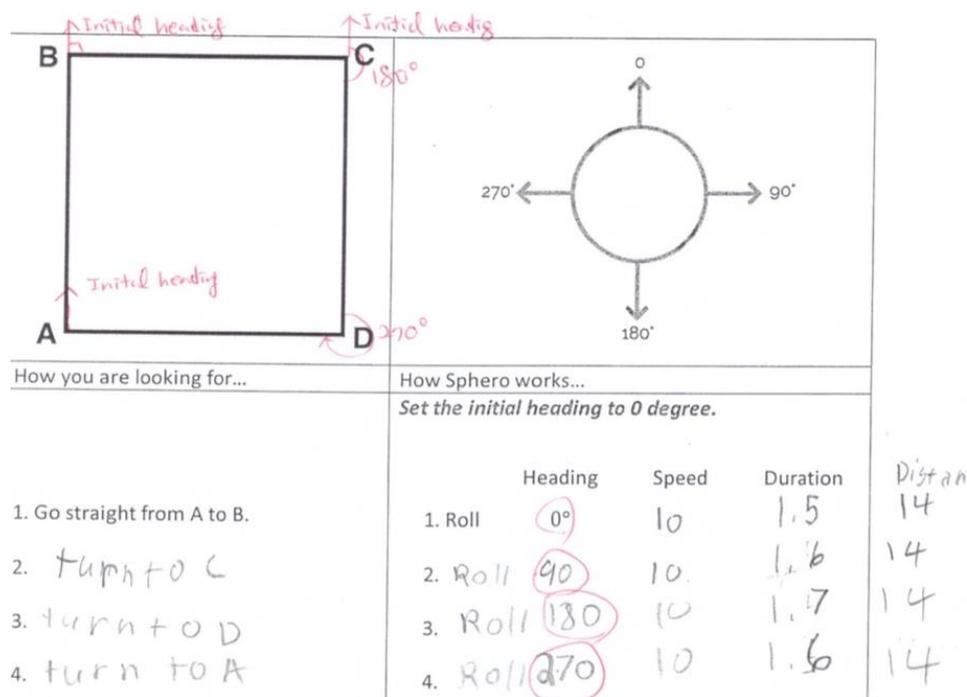


Figure 5. Tracing Sphero SPRK+ in a Square-shaped Pathway

Figure 5 shows a written response by Student 1, who worked out a square-shaped ABCD pathway. Through trial and error (and with some rounding errors in the measurements due to possible friction), Student 1 was able to devise a plan to: (i) trace from A to B in a 14-cm drive using a rolling angular direction of 0° at a speed of 10 cm/sec with a 1.5-second travel time; (ii) trace from B to C in a 14-cm drive using a rolling angular direction of

90° at a speed of 10 cm/sec with a 1.6-second travel time; (iii) trace from C to D in a 14-cm drive using a rolling angular direction of 180° at a speed of 10 cm/sec with a 1.7-second travel time; and (iv) trace from D to A in a 14-cm drive using a rolling angular direction of 270° at a speed of 10 cm/sec with a 1.6-second travel time. Student 1 was successfully able to execute her plan and drive Sphero SPRK+ on the square-shaped ABCD pathway. Student 1 brought the experience of adjusting the rolling angular direction while keeping the initial heading of 0° from “Activity 1: Driving” with her into “Activity 2: Boomerang.” This was evident in her coding from C to D and from D to A, as she properly modified the rolling angular directions to 180° and 270°, respectively, and as she noted that the direction of the blue tail-light was still facing her.

Although students were familiar with all the interior angles of a square being 90°, the connection between interior angles of a square and the angular direction of Sphero SPRK+ did not become obvious to them until they were met with an isosceles-right-triangular-shaped pathway. For example, when tracing the square-shaped ABCD pathway, Student 1 noticed only the rolling angular directions of 90° and 180° that turned Sphero SPRK+ from B to C and from C to D corresponded to the complementary and supplementary angles of the initial heading of 0°, respectively. Student 1 saw that only the former angle (that is, the rolling angular direction of 90°) was consistent with the 90° interior angle of the square, making both angles supplementary to each other. This was clear to Student 1 that the rolling angular direction of 90° functioned in two roles: (i) as a complementary angle in the first case with regard to the initial heading of 0°; and (ii) as a supplementary angle in the second case with regard to the 90° interior angle of the square. However, this experience was later contrasted with that of tracing the isosceles-right-triangular-shaped pathway.

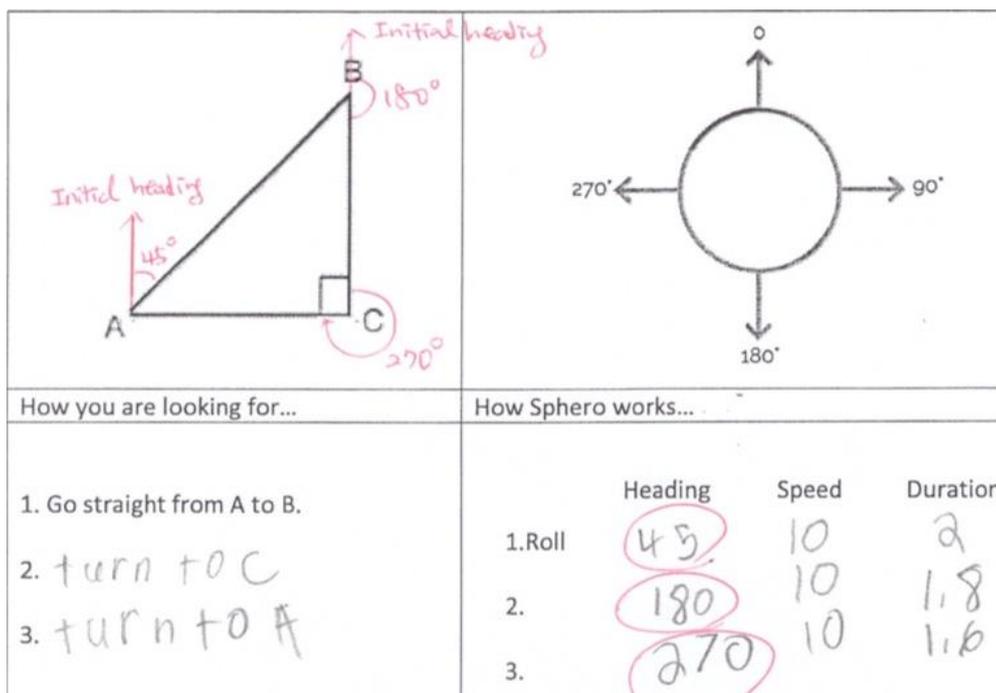


Figure 6. Tracing Sphero SPRK+ in an Isosceles-right-triangular-shaped Pathway with an Initial Heading of 0°

Figure 6 shows the written response of Student 2, who worked out an isosceles-right-triangular-shaped ABC pathway. To accomplish this goal, the student coded Sphero SPRK+ to: (i) trace from A to B in a 22-cm drive

using a rolling angular direction of 45° at a speed of 10 cm/sec with a 2-second travel time; (ii) trace from B to C in a 16-cm drive using a rolling angular direction of 180° at a speed of 10 cm/sec with a 1.8-second travel time; and (iii) trace from C to A in a 16-cm drive using a rolling angular direction of 270° at a speed of 10 cm/sec with a 1.6-second travel time.

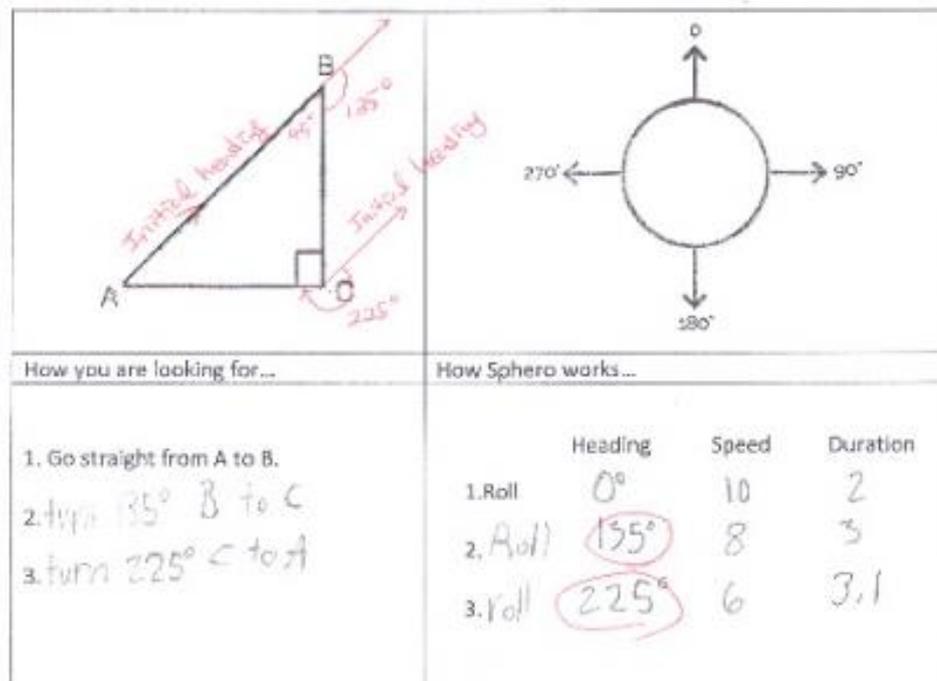


Figure 7. Tracing Sphero SPRK+ in an Isosceles-right-triangular-shaped Pathway with an Initial Heading of 45°

Student 2 then became intrigued to explore the same isosceles-right-triangular-shaped ABC pathway using a different initial heading. Recognizing the interior angles of an isosceles right triangle as being 45° – 45° – 90° , Student 2 used a 45° angle as the new initial heading angle (see Figure 7). To this end, Student 2 modified her coding to: (i) trace from A to B in a 22-cm drive using a rolling angular direction of 0° at a speed of 10 cm/sec with a 2-second travel time; (ii) trace from B to C in a 16-cm drive using a rolling angular direction of 135° at a speed of 8 cm/sec with a 3-second travel time; and (iii) trace from C to A in a 16-cm drive using a rolling angular direction of 225° at a speed of 6 cm/sec with a 3.1-second travel time.

Student 2 was able to generalize her encounters with “Activity 1: Driving” and “Activity 2: Boomerang” and effortlessly make two conjectures. First, she observed that keeping the initial heading of Sphero SPRK+ to 0° set up a complementary-angle correlation between the interior angle of any polygon and the immediate rolling angular direction at its original position. Second, she noted that changing the initial heading of Sphero SPRK+ to mirror the interior angle of any polygon along its identical pathway established a supplementary-angle link between the particular interior angle via the initial heading and the corresponding rolling angular direction. In the first conjecture, Student 2 recognized the complementary-angle connection between the 45° interior angle CAB via the 0° initial heading and the immediate rolling angular direction of 45° . In the second conjecture, she saw the supplementary-angle connection between the 45° interior angle CAB via the initial heading and the corresponding rolling angular direction of 135° .

Other students, following the group discussion with Student 2, became encouraged to test the two conjectures on a different geometric shape—namely, the 60° – 60° – 60° equilateral-triangular-shaped ABC pathway (see Figures 8 and 9). It gradually became evident to all students that one needed to anticipate in the equilateral-triangular-shaped pathway: (i) the complementary-angle relationship between the 60° interior angle CAB via the 0° initial heading and the immediate rolling angular direction of 30° (see Figure 8); and (ii) the supplementary-angle relationship between the 60° interior angle CAB via the initial heading and the corresponding rolling angular direction of 120° (see Figure 9).

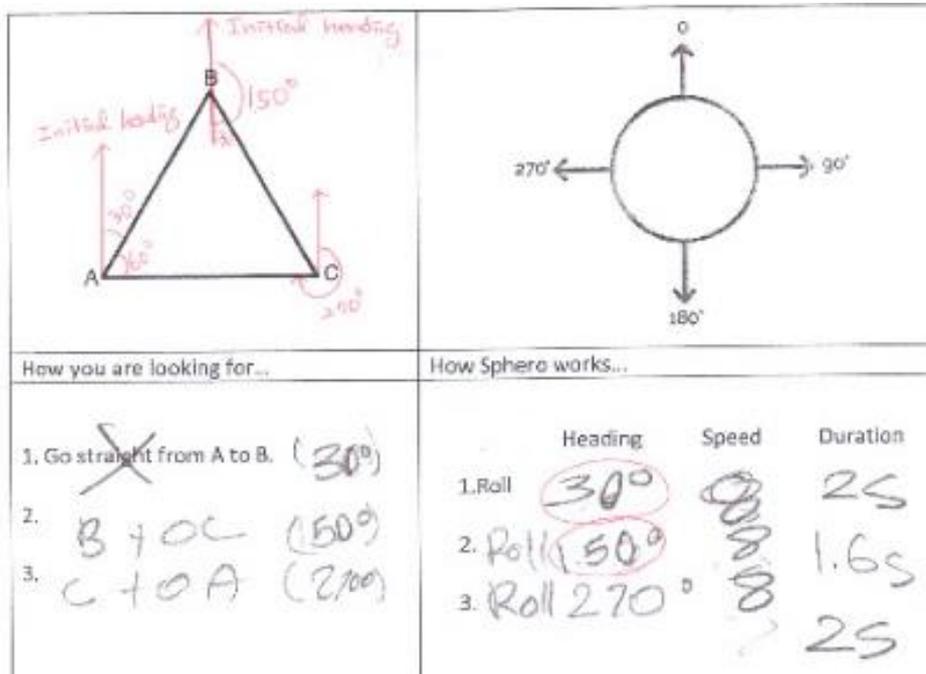


Figure 8. Tracing Sphero SPRK+ in an Equilateral-triangular-shaped Pathway with an Initial Heading of 0°

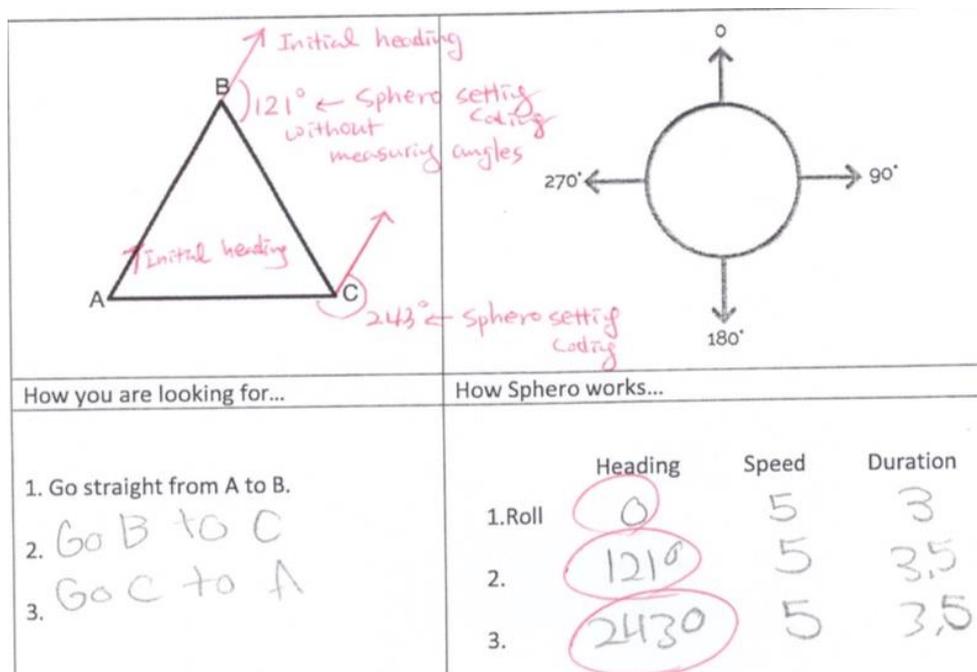


Figure 9. Tracing Sphero SPRK+ in an Equilateral-triangular-shaped Pathway with an Initial Heading of 60°

A further generalization was proposed by Student 3, who adopted the possibility of a different starting position. He maintained that, setting aside the explicit instructions of the provided worksheets, one could find complementary- and supplementary-angle relationships for all interior angles of any polygonal pathway by altering the starting position of Sphero SPRK+ as it traced the pathway. To some extent, students collectively recognized the coding of Sphero SPRK+'s pathway in "Activity 2: Boomerang" as an opportunity to refine their growing awareness of the geometric concepts of complementary and supplementary angles from "Activity 1: Driving."

Activity 3: Bowling

Towards the end of "Activity 2: Boomerang," students became excited and anxious to see how they could apply the geometric concept of complementary and supplementary angles. "Activity 3: Bowling" was an example of "doing with learning," where students' doing and playing brought their learning to fruition. Students worked on larger sets of polygonal pathways where the length of each side became enlarged to at least double its original measure. They were informed of the goal of the game—that is, to "bowl" Sphero SPRK+ in order to bring down as many bowling pins as they could. They noted that a set of bowling pins consisted of 10 pins (see Figure 10).

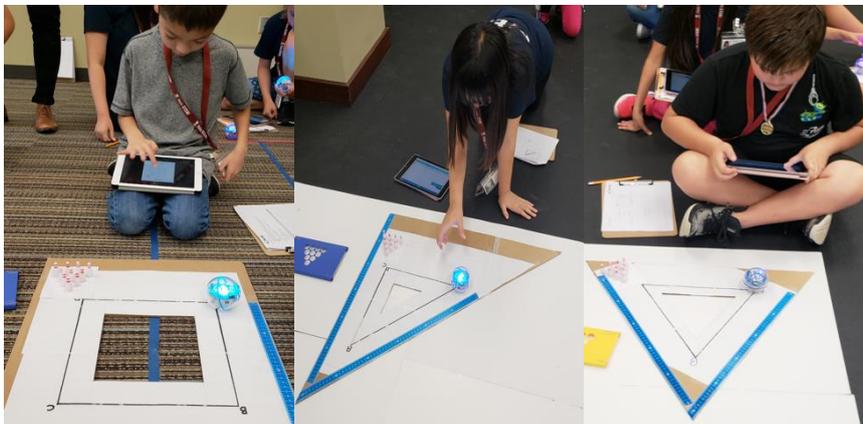


Figure 10. "Bowling" Sphero SPRK+ to Bring down Bowling Pins

After a few attempts, students realized that a "strike" was rather challenging. (A "strike" happens when, in one try, students are able to successfully code Sphero SPRK+ to bowl down all 10 bowling pins on any pathway.) This difficulty was due to the fact that the area covered by all 10 bowling pins was larger than the size of Sphero SPRK+. Consequently, students had to adapt their coding to the number of bowling pins remaining to be brought down. When a strike did not occur, students learned to modify their codes to include a different initial heading, rolling angular direction, speed, and travel time, among others. If a second version of code was able to bring down the remaining bowling pins left from the first try, this was counted as a "spare." Students played this bowling game many times, as they became persistent and determined to achieve a "strike" or a "spare." It was this immediate feedback from observed movements of Sphero SPRK+ that allowed the constant correction and adjustment of code, enabling the students to respond instantly. In this sense, students learned to reinforce the geometric concept they just learned by integrating it in solving mathematical modeling problems presented as playful game activities.

Following the end of the three robotics-coding activities, post-test assessments were administered to all students. Similar to their performance in the pre-test assessments, students scored, on average, approximately 94% on problems related to measures of single angles in the post-test assessments. In contrast, students scored approximately 66% on average on problems connected to measures of special angle pairs in the post-test assessments—a statistically significant increase, according to a paired samples *t*-test using their 35% score on similar problems in the pre-test assessments as a comparison ($p < 0.001$).

While their understanding of measures of single angles (e.g., acute, right, obtuse, and straight angles) remained the same because of their already developed understanding of these geometric concepts, students' understanding of measures of special angle pairs (e.g., complementary and supplementary angles) increased to a considerable degree. The increase indicated that the three robotics-coding activities with Sphero SPRK+ played an important role in establishing and advancing students' geometric understanding of measures of complementary and supplementary angles. In connection to the current study's research question, these results offered additional evidence for effective pedagogical practice through an integrated learning experience in mathematics, science, and technology—in particular, the interplay of robotic coding activities and the consequential ability to leverage problem solving and critical thinking in acquiring new geometric concepts of complementary and supplementary angles at the elementary school level.

Conclusion

In this paper, we presented students' experiences in learning the concepts of special angle pairs in geometry (namely, complementary and supplementary angles) through coding activities with Sphero SPRK+. "Activity 1: Driving" acquainted students with the fundamental knowledge of basic coding of Sphero SPRK+, as well as a general review of special angles. "Activity 2: Boomerang" expanded students' discussion of special angles to the extrapolation of special angle pairs through different conjectures. "Activity 3: Bowling" allowed students to apply the concepts of geometry they just learned into concrete challenges in the form of an engaging, playful game of bowling.

Despite lacking in traditional classroom instruction on formal mathematical terminologies of complementary and supplementary angles, students progressed their mathematical learning promptly and rapidly through their own informal conceptualization of those geometric concepts. The learning experience simulated in the current study confirmed the hypothesis of favorable pedagogical outcomes in mathematics associated with educational robotics indicated in earlier studies (Barker & Ansorge, 2007; Yudin et al., 2017). Furthermore, it highlighted the need for students to explore creative multiple problem-solving approaches as recommended by previous research (Star & Rittle-Johnson, 2008; Tjoe, 2019). Students' experiences in these three coding activities revealed, to a certain extent, that engaging in reflective play could be shaped into a meaningful teachable moment where students participated in a "doing with learning" pedagogical method using educational robotics. These activities had transferability implications that might afford STEM learning access and opportunity for students to develop not only mathematical reasoning skills but also problem solving and critical thinking skills operable to a coding environment.

Recommendations

The current study suggests that learning mathematics concepts might be accomplished through fun and playful activities involving the use of technology, as in the coding of Sphero SPRK+ via the Sphero Edu app. The current study also suggests that coding activities (with Sphero SPRK+ as an example) embracing not only visual, but also kinesthetic learning modalities, might be profitably employed in a school mathematics curriculum setting. The current study is limited to the extent that research subjects identified through the voluntary, convenient sampling technique might be considered to be more highly motivated than average students, as well as the fact that the increased achievement scores on the concepts of special angle pairs demonstrated in our findings might be particularly dependent on an already above average mastery and prior content knowledge of the concepts of single angles. Future research might therefore consider different sampling methods, including a cluster-sampling technique, to further understand any difference in student performance by grade level and mathematical background. By raising awareness of technology use in mathematics classrooms, mathematics teacher preparation programs might be better able to respond adaptively to different needs. The findings of the current study might be an argument for equipping their pre-service teachers with more training in the integration of technology in mathematics education. The findings of the current study might also prompt school districts to explore professional development workshops that empower their in-service teachers with pedagogies that leverage hands-on computational thinking and quantitative reasoning skills to foster success in an ever-changing digital world.

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