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Julie Nurnberger-Haag
Kent State University, United States

Amy Scheuermann
Minnesota State University Mankato, United States

Janis S. McTeer
Ph.D.-Kent State University, United States

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Julie Nurnberger-Haag, Amy Scheuermann, Janis S. McTeer

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Abstract

Trade books are a common resource used to teach children mathematical ideas. Yet, detailed analyses of the mathematics content of such books to determine potential impacts on learning are needed. This study investigated how trade books represent whole numbers. A two-pronged approach was used a) one team documented every way 197 books represented numerical ideas and b) another team used standards to identify ideal representations. A third team validated the traits on 67 books. Greater variation than expected was documented (103 traits identified) and organized into a field guide for researchers to consult to design studies about how particular traits influence number learning. Studies could investigate how a particular trait supports learning or experimentally compare a selected combination of the 45 pictorial, 45 written symbol, 10 tactile, 2 kinesthetic, and 1 auditory trait. Implications for practice include recognizing what representations are present or missing from books used in classrooms. The study also serves as an example of how the field of mathematics education would benefit from adopting structures from disciplinary science, such as field guides, to inform how we organize phenomena of mathematics learning.

Introduction

There is much that can be used or adapted from one discipline to enhance progress in another. There are at least three ways that the field of mathematics education would benefit from drawing on science education, such as using elements of the 5E learning cycle approach (Turan & Matteson, 2021); using phenomenon organizing structures such as typologies, taxonomies, and field guides (Journal for Research in Science Teaching, 1978; Nurnberger-Haag, et al., 2020); and recognizing the importance of analyzing learning with children's books by comparing exactly what content representations and ideas were offered in these books in relation to what students learn or could learn, which has been done extensively in science education (e.g., Ford, 2006; Ganea, et al., 2011; Mayer, 1995; Rice, 2002; Trundle & Troland, 2005;), whereas analyses of the mathematics in children's books has only recently begun.

Children's trade books are ubiquitously available to families, teachers, and researchers who are increasingly being advised by authoritative sources to use them with children to support their mathematics learning (e.g., Hojnoski, et al., 2014; Hojnoski, et al., 2016; Stites, et al., 2021). Recently, a study of 250 preschool teachers confirmed the prevalence of such resources in that 92% had mathematics related books in their classroom (81% in the class library and another 11% alongside their math manipulatives; Stites, et al., 2021). Number-related books are the most common trade book involving mathematics, followed by shape-related books (Yilmaz Genc, et al., 2017). Yet, "assessments of the content of trade books that focus on numbers…are glaringly absent" from the field and are necessary as a foundation of "critical new knowledge that can directly impact instructional practice" (Hachey, 2015, p.316). The field needs to understand how numerical ideas are represented in these books before we can understand how or why students learn with such books. We determined that a field guide similar to what is used in the sciences would serve a similar purpose to document characteristics of resources available for mathematics learning.

In the United States future teachers participate in practical experience in schools or daycares, often referred to as *field experiences*. This term is used to reflect the real practical experiences which occur outside of a laboratory, similar to a science field experience. In science, a field guide helps a novice user in the field observe not only an object or phenomenon they might not have otherwise noticed, but also learn the characteristics that distinguish one from another. For example, novice observers may initially see the trees in a forest but be unable to identify the similarities or differences between them until they consult a field guide. Moreover, even those with more expertise benefit from field guides. In biology, the detailed and nuanced identifications documented and described within a field guide form a critical foundation for future research and practice (Farnsworth et al., 2013). In a similar way, mathematics teachers as well as researchers would benefit from field guides to help them notice similarities and differences between the representations of mathematical concepts found in the field. A field guide assists with the identification of objects but does not make claims about their relative value, nor does it quantify how many such objects exist in the world. In parallel, before we can determine how effective certain representations are for facilitating number learning, we must first identify the similarities and differences of these varied representations. Thus, this article contains a Field Guide to Whole Number Representations in Children's Books to denote the ways that numbers are or could be portrayed in children's books. While the version presented here is only an initial field guide for researchers familiar with the learning of early mathematics, we anticipate after additional studies are conducted about the relative impact of these differing representations on children's learning, a fully illustrated field guide could be created for teachers.

Review of Literature

Representations in Mathematics

Due to the abstract nature of mathematics, external representations are needed to express and explain mathematical concepts (National Council of Teachers of Mathematics (NCTM), 2000; National Research Council, 2001). Mathematical thinking is represented in a variety of ways using images, words, objects, and symbols (Lesh et al., 2003; Goldin, 2003; Kamii et al., 2001) and found in a variety of forms such as manipulatives, pictures, real-life situations, written and verbal expressions (Lesh et al., 1987). The ability to use representations to communicate mathematical ideas and solve problems is key to successful learning in mathematics (Huinker, 2015; Nathan et al., 2010; Pape & Tchoshanov, 2001). Within the field of mathematics education, representations research has focused on translations between modes of representations (Goldin, 2003; Goldin & Shteingold, 2001; Johnson, 2018; Lesh et al, 1987). The specific modes have been categorized as the rule of four: graphic, numeric, algebraic, and verbal (spoken and written; Mainali, 2021). This focus, however, has primarily been about secondary or post-secondary mathematics content (e.g., Bieda & Nathan, 2009; Lesh et al., 1987). In contrast, when early childhood has considered representations of number, the focus has primarily been on the three representation types some deemed critical to building early number competencies: numeral, number word, and quantity image (Baroody & Benson, 2001; Jung, 2011; Skwarchuk, 2009). To clarify, the term *number* is used to refer to the idea or concept, whereas *numeral* means the written Arabic numeral (e.g., 4) and *number word* means the oral or written word ("four"). Consequently, the analysis of children's books with whole numbers reported here, considered all of these representational types relevant across the lifespan and in relation to the sensory experiences recognized as important in child development (e.g., Elliot, 1999).

Importance of Early Number Learning

Studies of resources for learning early mathematics are important to conduct, because young children's mathematics learning matters. As early as kindergarten entry, early mathematics knowledge predicts long-term mathematics achievement (Claessens & Engel, 2013; Jordan et al., 2009; Jung et al., 2019; Watts et al., 2014). In particular, mathematics achievement prior to first grade "predicts mathematics achievement through age 15, even after accounting for early reading, cognitive skills, and family and child characteristics" (Watts, et al., 2014, p.352). How much children learn due to instruction between the ages of 4 years 6 months "and first grade is an even stronger predictor of adolescent mathematics achievement" (Watts, et al., 2014, p.352). Moreover, the importance of early math extends beyond mathematics to other content domains and overall academic achievement. In spite of the limited focus early childhood continues to place on the mathematics learning of young children, a meta-analysis of six longitudinal data sets found "early math skills have the greatest predictive power of overall school achievement, followed by reading and then attention skills" (Duncan et al., 2007, p. 1428). Specifically, early math predicted eighth grade science achievement as might be expected due to the relationships between such fields, yet Claessens and Engel (2013) also found that early math predicted eight grade achievement in reading.

Representations of Number in Children's Books

Children learn through all their varied contexts and situations; "They learn from parents and from other children. They learn by interacting with things" (Fuson et al, 2015, p. 63). One of the things from which children learn are popular press books also known as trade books. Family, child-care providers, preschool teachers, and elementary teachers regularly engage children with trade books as one way to develop mathematical ideas (Anderson et al., 2004; Bailey, 2009; Bryan & Mason, 2012; Gaylord et al., 2020; Hojnoski et. al., 2014, 2016). Yet as many have stated, the quality and quantity of research on the mathematical content contained within children's books is surprisingly limited (Björklund & Palmér , 2020; Flevares & Schiff, 2014; Hachey, 2015; Nurnberger-Haag, 2017; Yilmaz Genc, et al., 2017). Content analyses of the mathematics in children's books provide "critical new knowledge" in order to understand children's learning with books (Hachey, 2015, p.316;

Yilmaz Genc, et al. 2017). Although many of the non-mathematical aspects typically coded for in children's books are worthy of study and important to the integrated and naturalistic context of a child reader's experience (e.g., story, genre, wow factor, context, distractors, types of images as photos or illustrations, how discernable images are from one another to allow for counting; Hellwig et al., 2000; Powell & Nurnberger-Haag, 2015; van den Huvel-Panhuizen, & Elia, 2012; Ward et al., 2017), future research on learning mathematics with books also needs to attend to what students learn from the numerical representations (Björklund & Palmér, 2020). Results from studies of learning with researcher-designed books suggest the need for such research on authentic children's trade books (Björklund & Palmér, 2020). For instance, when a researcher-designed book told children the cardinality of a set of pictures before counting (e.g., *5 ducks, 1-2-3-4-5*) rather than after counting (*1-2-3-4-5, 5 ducks*), more children became cardinality-knowers (Mix et al., 2012). Such carefully designed studies inform potential learning impacts to be considered outside of laboratories suggesting that the specific representations of number within trade books might play an even greater role in learning than previously understood.

Children first need to master the rote number sequence in the language in which they are counting (e.g., "one, two, three, …"; Reys et al., 2014). Over time they also need to acquire the understanding that each successive number is one more than the previous and one less than the next number in sequence (CCSM, 2010). Children also need to learn that zero is a valid number, which precedes one. Given the expectations for the number ranges children should master during preschool and by the end of kindergarten, what opportunities do they have to learn these ideas from their books? In a sample of 120 number-related books for preschoolers, 81% included only numbers within 10 (Ward et al., 2017), and even within a sample that included books for older children, the majority of the 160 books (67%) still only provided this limited range of experiences with numbers (Powell & Nurnberger-Haag, 2015). By the end of kindergarten children are expected to orally say the number sequence from one to one-hundred, yet out of 160 books, Powell and Nurnberger-Haag (2015) found only three that provided an opportunity for children to hear this sequence as adults read a book with them. The written symbols of 100 or "one-hundred" were shown to children in 15 additional books; however, all of these skip-counted by some number, usually ten (Powell & Nurnberger-Haag, 2015). About 20 to 25% of books were shown to provide opportunities to learn numbers within 20 by providing some combination of numbers from 13 to 19 and more frequent opportunities to learn the numbers 11, 12, and 20 (Powell & Nurnberger-Haag, 2015). The concept of zero as a valid number by representing the concept with either a numeral or number word was found in 12% and 7.5% of the books from each study, respectively (Powell & Nurnberger-Haag, 2015; Ward et al., 2017, personal correspondence Mazzocco data clarification received March 17, 2021). This summarized which numbers have been found in children's books. The following is what is known about the representations of number in children's books in terms of written symbols, pictures, and tactile elements.

Written Symbols

Exposure to symbols including numerals and operational signs may support the development of number in young children (Banse et al, 2020). Children are expected by the end of kindergarten to recognize numerals 1 through 20 (CCSSM, 2010). Ward and colleagues (2017) and Powell & Nurnberger-Haag (2015) coded how frequently the written symbols of numerals and number words were found in their samples of 120 and 160 books, respectively. The content analysis of preschool books found numerals occurred twice as frequently as number words (Ward et al., 2017). Whereas the study that included, but was not restricted to preschool books, found about 75% of the books displayed numerals and 75% number words (Powell & Nurnberger-Haag, 2015). Although Ward et al (2017) also coded for written symbols of operations, neither former content analyses of written symbols of numerical ideas documented symbols of equals or inequality signs.

Pictures

Children are expected by the end of kindergarten to count up to 20 objects and solve addition and subtraction problems with the aid of objects, if needed (CCSSM, 2010). When parents and preschool teachers read with their young children, they tend to encourage children to count images even when the books were not intended to teach counting and more so when this was the focus of the book (Hojnoski, et al., 2014; Hendrix, et al., 2019; Stites et al., 2021). When teachers have read a researcher-designed book with preschool children the "design of the pictures influence which numerical aspects" children notice and begin to use (Björklund & Palmér, 2020, p.209). Thus, the ways pictures are arranged to represent quantities in trade books should be evaluated. In Powell and Nurnberger-Haag's (2015) study of 160 books, 93% used at least one picture to represent each number that was portrayed symbolically with a numeral or number word. Both Ward and colleagues (2017) and Powell and Nurnberger-Haag (2015) provided some initial insights as to how those pictures represented the ideas of number. To develop number sense and cardinality, children need opportunities to subitize, that is to see pictorial representations in common patterns repeatedly (e.g., dice or domino patterns; Clements, et al., 2019). Children's number-related books have provided this opportunity for readers to see a subitizable pattern in 14% of books for preschoolers (Ward et al., 2017) and 6% of books in a sample that included books for older children (Powell & Nurnberger-Haag, 205).

Another study that focused only on those books that presented numbers in sequence, which Nurnberger-Haag et al., (2020) defined as *counting books,* categorized these books by how explicitly the pictures of objects represented sequences. They defined three categories on the rationality dimension of the Counting Book Typology: sequences of numbers without images (*Rote Counting Books*), sequences of numbers with corresponding pictures to represent each term of the sequence without grouping (*Rational Term Counting Books*), and those books that consistently grouped images of each term (*Rational Scalar Counting Book*s; Nurnberger-Haag, et al., 2021). In other words, *Rational Term Counting Books* simply provided children the opportunity to learn the cardinality of each number that happened to be in a sequence, whereas *Rational Scalar Counting Books* provided visuals that represented the change in quantity from term to term (e.g., if a book provided written symbols of numerals of even numbers in sequence, the number four was pictorially represented as two groups of two, six as three groups of two, eight as four groups of two and so forth so that the change or addition of 2 from term to term was visually accentuated; Nurnberger-Haag, et al., 2020). After children develop facility with all counting principles, especially cardinality, children need to learn to skip count with meaning (i.e., counting by a scalar other than one). Skip counting can help students develop a conceptual understanding of multiplication as repeated addition (Sarama & Clements, 2009).

This idea of grouping or composing and decomposing quantities is crucial to number sense and foundational concepts for addition and subtraction operations. Ward et al (2017) found only 5% of their sample of preschool number-related books incorporated pictorial representations that explicitly communicated this composing or decomposing idea. When a teacher read a researcher-designed book in which particular groupings were designed into the images, this fostered a preschool child to see the individual unit berries as well as think of the "threes" as a unit (Björklund &Palmér, 2020, p.205). Because composing quantities into superordinate categories (e.g., dolls and trucks can be thought of using a superordinate category of toy) is a child appropriate precursor of learning to combine like units that is crucial for later elementary mathematics and algebra, Powell and Nurnberger-Haag (2015) coded for whether the pictorial representations did so and found that just 4% of the books provided this opportunity.

Skip counting or grouping quantities in different ways is a precursor to developing an understanding of our number system based in units of ten (Sarama & Clements, 2009). It is through thinking about the individual objects within skip-counted groups and composing/decomposing quantities that children then develop the ideas consistent with place value to think of a group as a unit itself (Fuson, 1990). Adults might see the number 17, for example, as 10 ones and 7 more or 1 ten and 7 ones. Young children, however, need time to see this base-ten structure, the way our society has developed and sees numbers. This is why it is not until the end of kindergarten in the U.S. Common Core State Standards Mathematics (CCSSM) that children are expected to master the decomposition of a teen number into 10 ones and more ones, but children need at least another year to reconceptualize groupings of ten into tens units (CCSSM, 2010; Nurnberger-Haag, 2018a). Thus, analyses of how trade books arrange images to promote composing and decomposing, particularly groupings of ten as a foundation for concepts of base-ten numbers and operations on these numbers are needed.

Tactile/Kinesthetic

Ward et al (2017) found that 10% of their sample had a tactile feature. These books must be more carefully studied for potential learning outcomes. Although this may seem like a small proportion of children's books to warrant priority as a study, when parents are choosing a book to help their children learn to count, they are more likely to choose a book with a tactile trait (Gaylord et al., 2020).

Purpose of Study

The potential learning affordances or potential problems with the opportunities to learn in any children's book should be carefully considered and reported in the design and methods of studies that intend to make claims about how children learn with books (Björklund & Palmér, 2020; Nurnberger-Haag, et al., 2020; Nurnberger-Haag, 2017; Yilmaz Genc, et al., 2017). Before we can understand affordances and problems, however, the field must identify and characterize representations of number in such books. To do this we considered the various perspectives used to categorize mathematical representations within the literature. In secondary mathematics education research, the emphasis has been on translations between particular representation types relevant at those levels of mathematics (Bieda & Nathan, 2009; Mainali, 2021). Whereas the early childhood literature focuses on three representations that differ from that used in secondary (numerals, number words, and pictorial representations; Fuson, 2019; Young-Loveridge, 1999), emphasizes multi-sensory learning experiences (Elliot, 1999), and compares pictorial and tactile number book features (Gaylord et al, 2020; Petersen et al., 2014). Thus, at the broadest level we framed this study in terms of the potential sensory experiences the representations of number in children's books intend, then looked for subcategories and traits. To this end we conducted a content analysis of the representations using a two-pronged approach: analysis of representations existing in a sample of children's number-related books with whole numbers and teachers' analyses of the US standards (CCSSM, 2010) to envision representations that educators would want in a book selected to teach that content. As will be shown in the Findings section, the representation types found were so plentiful that we created a field guide to provide accurate descriptors of representation traits as a basis for identification and future research (Farnsworth et al, 2013).

Method

A content analysis was conducted of children's books to determine in what ways ideas of whole numbers were portrayed. Rather than analyzing the representations using A Priori theoretical codes as Ward et al (2017) or Powell, et al (2015) coded, in the current study we developed inductive codes as traits of numerical representations (Berg, 2004). These traits and descriptors that compose the Field Guide to Whole Number Representations in Children's Books are the findings of the content analysis. Consequently, this methods section describes the book selection process, how traits were developed or discovered, and the validation through replication. Thus, the traits and descriptors themselves are reported as the findings.

Book Selection Criteria

To increase the chances that opportunities for learning number through children's books in the United States could be more accurately captured, books were selected from four local public libraries that reflected communities with a range of socioeconomic status or resources, the first author's collection of books acquired during other studies and in practice, as well as books recommended as high quality children's literature. Given that mathematics educators who research learning with picture books prioritize the importance of story (e.g., van den Heuvel-Panhuizen & Elia, 2012), we ensured that the selection methods included books that were on two of three lists of 100 Best Picture Books: Thomas Fordham Institute (Petrilli, 2012), Time (Time Magazine, 2020) and the New York Public Library (New York Public Library, 2013). Those who study literacy value high quality literature, but also advocate for the importance of informational texts (Strachan, 2014), so we ensured that a range of genres were captured during selection: picture books, fictional books, and informational books. Reference books such as math dictionaries, encyclopedias or workbooks were excluded. Following the definition of number-related books as books with at least two instances of numerals or number words (Nurnberger-Haag, et al., 2021), the mathematical criteria for selection in this study were two instances of whole numbers (i.e., not fractions, decimals, percents, nor measurement) on at least two separate pages represented as numerals or number words. Books were retained for analysis if the intended meaning of the pictures could be determined in relation to one or more written symbols. So, for example, operation books that only showed calculations with written symbols without corresponding images to show the meaning were excluded. These selection procedures yielded a sample of 197 books used to develop the trait identifiers and descriptors.

Development of Field Guide

Two teams conducted analyses to develop the field guide: Book Team and Standards Team. The Book Team consisted of two recently certified early childhood teachers in the US, a doctoral student who had previously taught elementary mathematics in Iran, and the first author. The Book Team analyzed the representations in the selected children's books to determine *trait identifiers* and *descriptors*. The Standards Team consisted of the first author, a second-grade teacher, a recently certified early childhood teacher, and a former high school teacher who was a doctoral candidate with a master's degree in mathematics all with experience in the U.S. Each member of the Standards Team independently read relevant standards and drew and explained ideal representations they would expect to see in a trade book that could be used to teach each of the U.S. number and operations standards related to whole numbers (CCSM, 2010). After creating a detailed list of the trait identifiers found in the book sample, any missing ideal representations identified by the Standards Team were added to the field guide, details of which described next.

Given the importance of sensory experiences for early childhood learning, we considered all five modalities but excluded the sense of smell and taste, because we could not conceive of a way that there could be countable scents or use of taste in a children's book. Thus, we began with senses: visual, auditory, and touch. Yet, we recognized that touch (tactile) and whole body (kinesthetic) experiences are different experiences, so we looked to document both tactile and kinesthetic elements in the books. The visual modality was also separated into subcategories of pictures - visual input a child could experience on their own even if an emergent reader, and written-symbols, number word and numeral symbol each of which requires learning. These five categories captured the typical representational forms number could be experienced by young children.

Within these categories of modalities, each representation was documented and named with a descriptive phrase. A constant comparative approach (Berg, 2004) to this content analysis was used each time a new representation was found and compared to the existing *trait identifiers* and *descriptors* Through constantly comparing the already documented descriptive phrases against new representations found, the descriptive phrases were either revised to better describe both representations or a new descriptive phrase was determined as a new category or representations trait. Once this initial field guide of descriptions was created, it was validated as described in the next section.

Validation of Field Guide

The purpose of the validation phase was to verify that the field guide was complete, such that no additional traits were needed. Each representation, rather than the book as a whole, was the unit of analysis in this study. Thus, the approach to validation was to have a different team analyze 67 of the books to ensure that there were not representation types present in the books that the field guide developers overlooked. Two undergraduate future early childhood teachers and one recently certified early childhood teacher were trained to use the Field Guide to Whole Number Representations in Children's Books. The only change that any validation coder required was simply a clarification that the definition of pattern included growing patterns. This was the original intent of the trait identified, but the validation coders found it was necessary to have this explicitly stated in the description. In the 67 books coded, the three coders found no new representations were needed, which validated the typology.

Findings

The Field Guide to Whole Number Representations in Children's Books begins with the modality through which a reader could notice a representation of number: Visual, Tactile, Kinesthetic, or Auditory. Once one chooses the appropriate section of the field guide based on the modality experienced, then subcategories can be identified as shown in Figure 1. This yielded 103 total trait descriptors about the mathematical ideas of the quantities: 90 Visual, 10 Tactile, two Kinesthetic and a single Auditory.

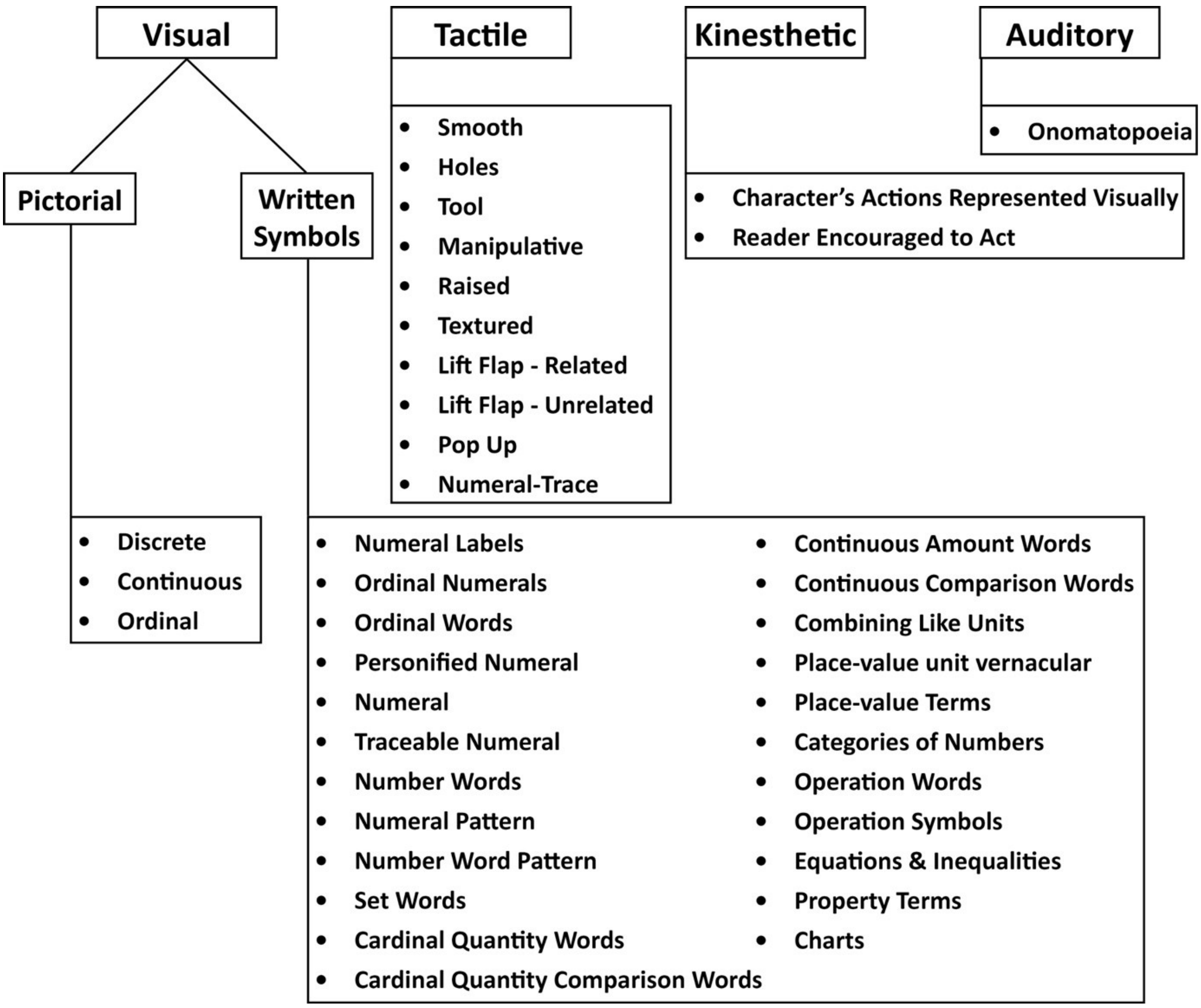

Figure 1. Overview of Top Levels of Field Guide

Visual Modality

As one might expect in a content analysis of books, the visual modality had the most traits. We separated these into Pictorial and Written Symbol categories with subcategories of each. Figure 2 shows each of the specific traits of the 45 Pictorial identifiers and Figure 3 the 45 Written Symbol identifiers. Appendices A and B, respectively, provide the field guide descriptors for each of these identified traits.

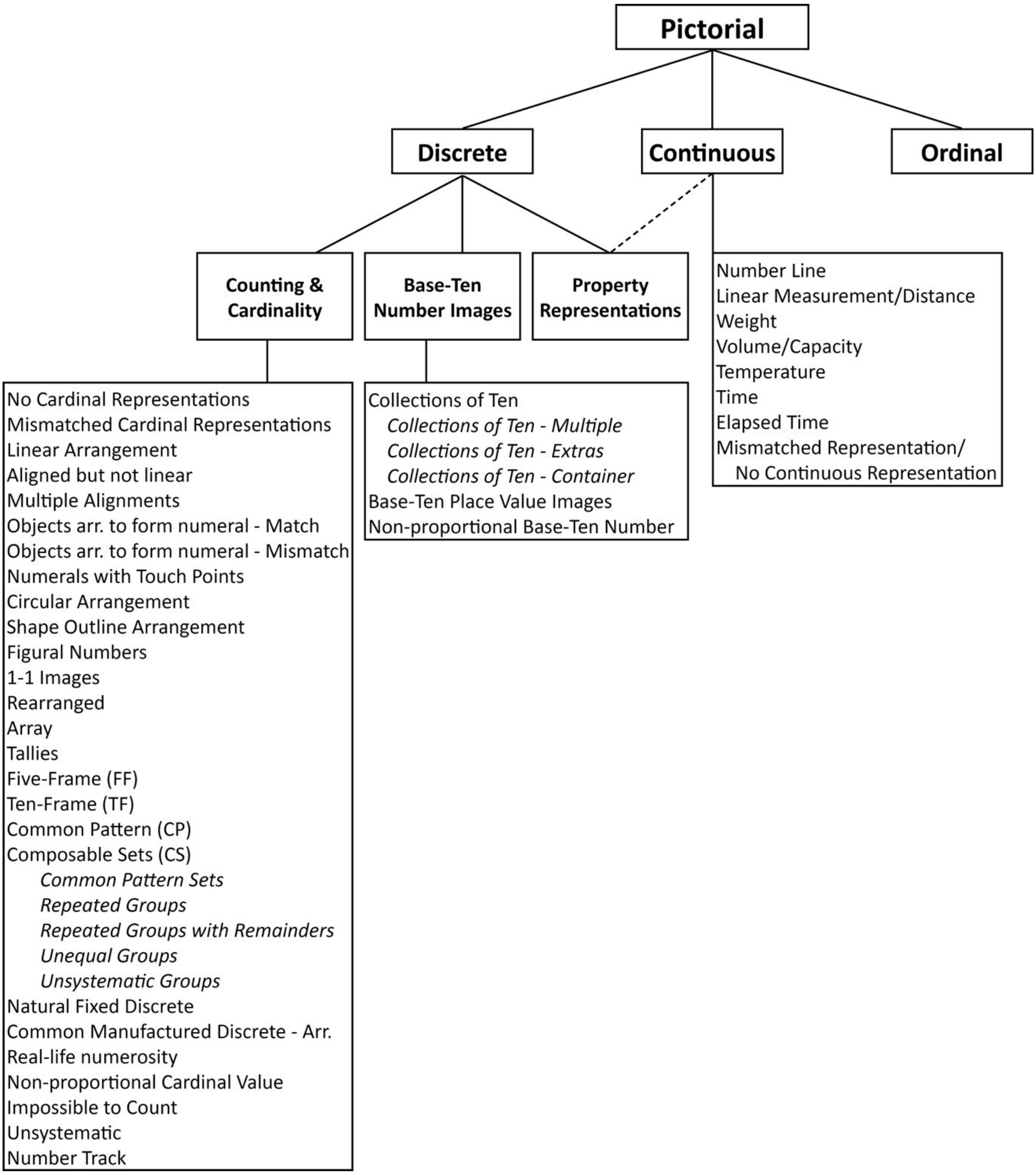

Figure 2. Pictorial Trait Identifiers

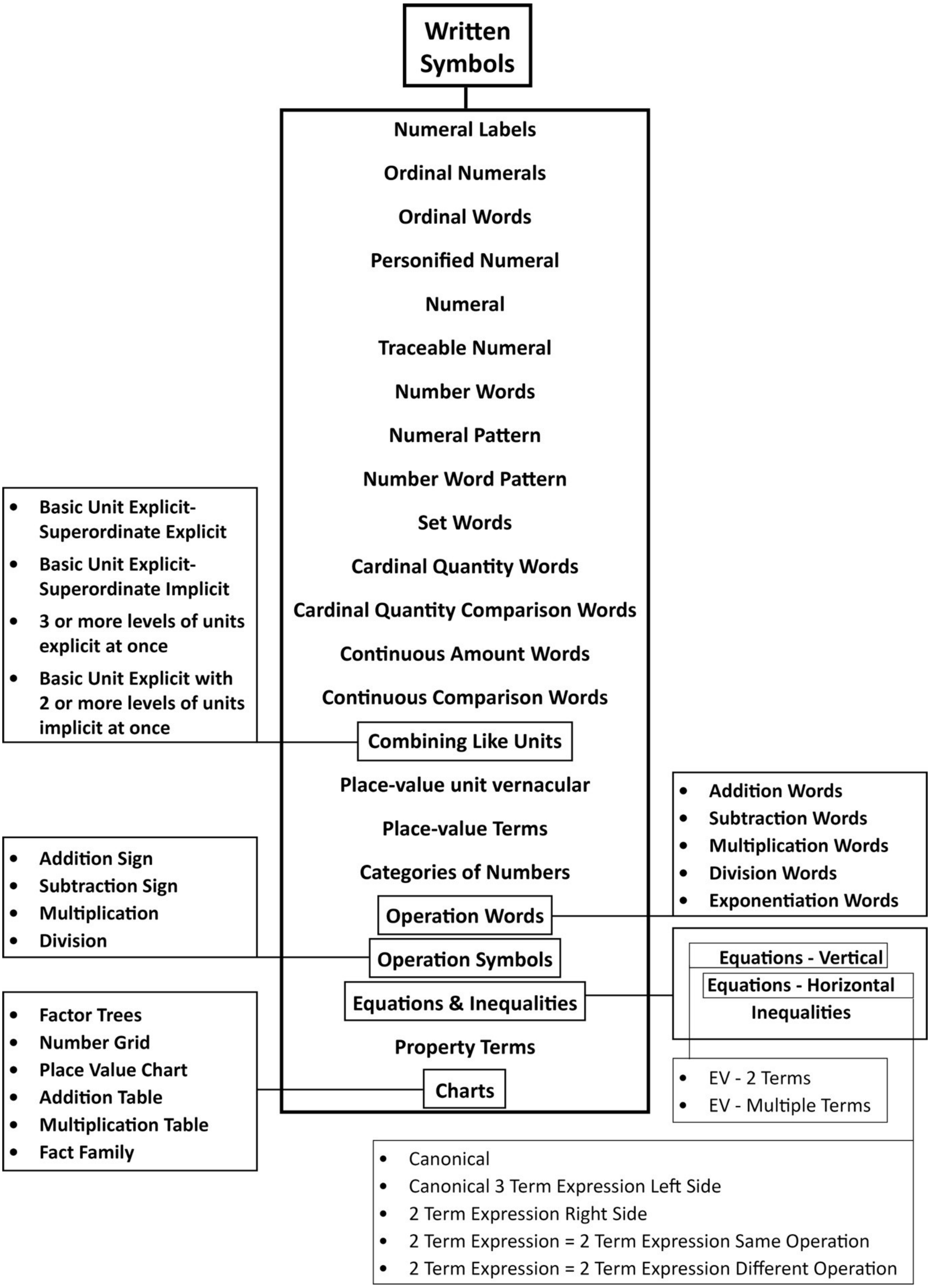

Figure 3. Written Symbol Trait Identifiers

Tactile Modality

Ways that a children's number book could provide a tactile way of experiencing quantities while reading were informed by Ward et al (2017) and Gaylord et al. (2020) and then additional ways books in the sample were found to be tactile were incorporated. The field guide consists of 10 tactile trait identifiers as listed in Figure 1 and described in Appendix C.

Kinesthetic Modality

Given the importance in child development of big body play and gross motor development (Elliot, 1999), we included kinesthetic as a modality to look for in the books. We did not find any encouragement in the books for children to experience a quantity through physical motions such as encouraging a child to stand up and hop on one foot five times to experience the number 5. However, this is a common early childhood classroom practice, so we included this as a trait identifier that some book in the future might include. The other kinesthetic trait identifier we found were stories that involved characters engaging in a kinesthetic experience a certain number of times, which the book represented using written symbols. For example, the number of laps swam in a pool was recorded with tallies and in another book the number of hops were recorded with arrows. Thus, the field guide consists of 2 kinesthetic trait identifiers (*Character's Actions Represented Visually* and *Reader Encouraged to Act*) as listed in Figure 1 and described in Appendix C.

Auditory Modality

Although children who are read these books would hear the written symbols translated into the auditory modality, how children might interact with adults with these books was not the focus of the analysis. The auditory trait identifier was used to document those sounds that are not words, rather *Onomatopoeia* type portrayals of quantity. For example, in the book *Blueberries for Sal*, "ker plunk, ker plunk, ker plunk" provides an auditory experience of the quantity of three blueberries going into a metal pail (McCloskey, 1976). No other approaches to auditory expressions of quantity were identified, so the field guide consists of one auditory trait identifier as listed in Figure 1 and described in Appendix C.

Discussion and Recommendations

With 103 trait identifiers found, the potential affordances and problems for children's number learning are too numerous to discuss each of them here. The Field Guide to Whole Number Representations in Children's Books defines 45 pictorial representation traits, 45 written symbol traits, 10 tactile traits, two kinesthetic traits, and one auditory representation of number as *Onomatopoeia*. Thus, in light of prior research and to discuss future research across sensory categories, we selected a few to discuss: equation types, base-ten number pictures, objects arranged to form numerals, personified numerals, and relationships of several tactile traits.

Prior content analyses of number-related books (Nurnberger-Haag et al., 2020; Powell & Nurnberger-Haag, 2015; Ward et al., 2017) did not attend to the written symbols of the equals sign or inequalities (e.g., =, <, >). In light of research on textbook analyses and research on elementary student learning of the meaning of the equals sign (Knuth et al., 2006; Powell, 2012), we accounted for these written symbol traits in the field guide. It is crucial that from children's first exposure to the equals sign that the true productive meaning of the symbol as relational is communicated, including through varied equation formats (McNeil et al., 2006). Thus, equation formats used in books need to be considered for potential misconceptions and ensure that within a collection of books that a child would see varied equation formats. The field guide would assist with such endeavors because it specified five subtypes of Equations-Horizontal that included canonical formats (e.g., 3 + 4 = 7) as well as the varied formats to which children must be exposed such as *2 Term Expression Right Side* (e.g., 7 = 3 +4) and *2 Term Expression = 2 Term Expression Same Operation* (e.g., 2 + 5 = 3 + 4). The examples provided in this paragraph for example were mathematically accurate; however, using images in place of mathematical symbols of numerals to represent quantities in an equation are inaccurate although commonly found in resources for young children (Carpenter, et al., 1999). Thus, we accounted for these issues by encouraging a field guide user to document accurate and inaccurate uses of the equation format with a superscript A or I, respectively.

Petersen et al (2014) found in a laboratory study that pictures of objects better facilitated preschool children to become cardinality-knowers than counting the same objects. The field guide allowed us to recognize that similar studies should also be conducted for more advanced number concepts, such as base-ten number. For example, early elementary children often use physical objects to build the initial base-ten understanding of the teen numbers as 10 and x more. Might using children's whole number books that show pictorial representations of this meaning be more or less beneficial than when children physically form these representations with manipulatives? The development of base-ten number conceptions and operations within the base-ten number system is a major focus of elementary mathematics (CCSM, 2010; Fuson et al., 2015). Thus, the field guide includes this special case of base-ten composing/decomposing with pictorial codes of "Ten-Frame" and seven trait identifiers under the category Base-Ten Number Images. An entire study that used these seven traits as a framework for analysis would be warranted to analyze children's opportunity to learn about base-ten number through their books. Given the frequencies of decade numbers due to skip counting that Powell and Nurnberger-Haag (2015) reported, we suspect, for example, that when numbers greater than 10 are included in books, most of the pictorial representations children could see would be *Collections of Ten-Multiples of Ten*, yet this is the most basic and potentially misleading way of conceiving of quantities. Children need to experience representations with the trait *Collections of Ten-With Remainders*, meaning they show groups of ten with extras that have not formed a complete set of ten, such as the teen numbers or a number such as 47. It takes expert specialized content knowledge to know the kinds of representations children need to experience (Hill, et al., 2008) and even experts need frameworks and guides to specify what to look for in children's books (Nesmith & Cooper, 2010; van den Heuvel-Panhuizen & Elia, 2012). Moreover, when something is absent from an experience or a resource it is harder still to recognize the need to take a critical stance. Consequently, the base-ten number traits missing from prior content analyses of number books that we identified in this field guide should help researchers and teachers critically assess how particular books or an entire sample of books available in a school or class might support or limit what children have the opportunity to learn from these representations.

The typical framework for representations of early number using the categories of numeral, number word, and picture is too broad to capture the varied ways written numerals are related to quantity in children's books and the internal sense of number that children need to develop through such external representations. The approach in this study to inductively identify how and in what ways numbers could be represented in children's books yielded traits not accounted for in prior analyses (e.g., Powell, et al., 2015; Ward et al., 2017). For instance, we documented *Personified Numerals*. Personifying an inanimate symbol might perhaps enhance a child's interest or engagement to score better on these criteria specified in prior rubrics (e.g., Hellwig, et al., 2000). However, given the negative impact children's books that personify animals have had on children's science concepts (Ganea, et al., 2011), how personified numerals impact children's learning should be investigated. Another way illustrators conflated written symbols and pictorial representations was to arrange quantities of objects to form shapes (i.e., circle, triangle, square) or numerals. These quantities might represent the stated numeral (*Objects Arranged to Form Numeral-Match;* e.g., the numeral 6 is made by 6 objects arranged to write the numeral 6*).* In other instances, however, the quantity representation and numeral representation contradicted each other (*Objects Arranged to Form Numeral-Mismatch*). The field guide draws attention to nuances that warrant experimental studies to determine if a written symbol representation has the trait *Objects Arranged to Form Numeral-Match*, for example, meaning quantity is embedded with the written symbol promotes for the child reader a better association or translation between the written symbol that labels the quantity and the quantity itself or if *Numeral* should be typed in the text in a standard format in the context in which children most likely will see the numerals in the future. Prior research on the difficulties of learning to transfer between unfamiliar contexts (Barnett & Ceci, 2002) and optimal spacing between text and illustrations (Godwin et al., 2018) lead us to suggest comparisons of these traits would be an important study about children's whole number books as well other implications for informal and formal learning situations.

In a recent study of preschool number books in an urban library, 10% had a tactile element (Ward et al., 2017) and how well a tactile experience matches the intended quantity to be learned may be a crucial factor to investigate (Gaylord et al., 2020). This Field Guide to Whole Number Representations in Children's Books, rather than a priori grouping together any tactile element as simply tactile, we identified specific ways tactile elements could occur to provide a basis for future research to determine the relative positive or negative learning impact of these traits. For example, this field guide distinguished Lift the Flap traits from others such as Holes or Texture. Moreover, additional distinctions were made in terms of the mathematical ideas communicated: *Lift Flap/Pull Tab Related* or *Unrelated*. Both of these provide the opportunity for a child to interact with the book with the same tactile motions or sensory experience; however, the *Related* trait does so in a way that more visually reveals association with quantity by showing a quantity or numeral on one part of the flap and then the child reveals the associated representation that had been hidden. Yet, this *Lift Flap/Pull Tab Related* interaction does not provide the child a tactile experience of the quantity in the way *Raised* or *Holes* representations do (i.e., a reader can touch the total quantity, for example the concept of the number 6 by touching 6 raised objects or putting a finger in 6 holes). Our field guide accounts for these pop-up/lift-the-flap traits that are so popular in young children's books as well as other tactile traits that provide the child opportunities to touch the quantity. Experimental studies using each of these three types as a condition to compare learning affordances for example, would provide important insights for informal and formal learning of number. Future research should investigate if all such quantity representations such as holes, raised, texture, and so forth afford the same learning results, then each such trait could be collapsed into a single more manageable category of tactile experience represents the quantity. On the other hand, if the *Textured* trait, for example, provided a certain benefit to particular populations of children more so than others, then this would also be important to understand. Furthermore, the interactions between the pictorial representation trait and tactile trait are how children would experience the book, so future research should consider the interactions of these multiple traits of whole number representations. Thus, this initial field guide provides a starting point for multiple fields of mathematics education, early childhood education, autism education, visually impaired education, educationally psychology, and other fields to plan studies that carefully control variables of learning by selecting specifically identified traits.

Conclusions and Implications

Given a recent study verified that almost all preschool classrooms have mathematics-related books (Stites, et al., 2021) and with the increasing encouragement to incorporate children's books into elementary mathematics instruction and parental interactions (Hojnoski, et al., 2014; Hojnoski, et al., 2016; van de Walle, et al., 2019), The Field Guide to Whole Number Representations in Children's Books developed here is important for at least four reasons.

Recognition that the Representations of Number are the Opportunities to Learn

First, the field guide increases attention to the ways that number representations differ within and between children's books as a prerequisite to understanding children's learning with number-related books. The language that disciplinary science uses when naming field guides reflects this purpose and focus that it is a "field guide to" draw attention *to* ideas, rather than simply *of* or *about* information. This is a Field Guide to Whole Number Representations in Children's Books, that is to draw attention to that which might otherwise go unnoticed. As Nesmith and Cooper (2010) documented, what people notice varies tremendously due to mathematical and pedagogical knowledge. This field guide is now an open-source resource, researchers can use to consider which number-related books to use to investigate children's learning with books. Researchers can compare the numerical representations in potential books to each descriptor in the field guide to notice nuance that they might not have attended to without this guide. Although the portrayals of science in children's books have been critically assessed for potential learning implications for decades (Ford, 2006; Ganea, et al., 2014; Mayer, 1995; Rice, 2002; Trundle et al., 2008), this has been a "very under researched area of early childhood mathematics education" (Hachey, 2015, p. 316). In spite of this lack of critical analyses of the mathematics content, increasingly preschool and elementary teachers are being encouraged to use children's books as part of their formal mathematics instruction (e.g., Hojnoski et al., 2016; van den Heuvel-Panhuizen & Elia, 2011). Some of the reasons this approach is becoming more popular is the value of using books to promote positive feelings about mathematics, mathematizing experiences from students' lives, and integrating literacy and use of stories as a way to understand concepts (Hellwig et al., 2000; Hong, 1996; van den Heuvel-Panhuizen & Elia, 2011; Whitin & Whitin, 2004). Yet, more critical and nuanced attention to how specific representations influence learning is needed (Björklund & Palmér, 2020). The Field Guide to Whole Number Representations in Children's Books brings adult attention to traits of the representations that will influence children's concepts of number and aide researchers and teachers to recognize when their collection of books for a study or in practice relies only on particular or simplistic representations, thereby limiting children's opportunity to learn. This field guide could facilitate researchers and teachers to recognize the need to seek out additional books that would diversify the representations of whole numbers children experience in the books they encounter in homes, daycares, and schools.

Recognize that Number Books can and should be Conceptually Varied

Second, this field guide with 103 traits, inspires the field to recognize the extent of diversity of number representations that are possible. For although the numbers used in children's books have been shown to be overly simplistic, often focusing within 10 (Powell & Nurnberger-Haag, 2015; Ward et al., 2017), recent research on a specific kind of number-related book, counting books revealed that the ways sequences are portrayed in children's books are quite varied with eight counting sequence types (Nurnberger-Haag et al., 2021). In other words, children's number books are more varied and should be more varied than adults would expect for a topic they might view as "simple" (Lee & Ginsburg, 2009; Nurnberger-Haag, et al., 2021). The Field Guide to Whole Number Representations in Children's Books contributes evidence on a wider range of books than counting books to describe the complexity of representations in whole number-related books.

Foster Potential for Future Teaching Practice

Third, another implication is the potential for future expansion and refinement of the field guide for teaching practice. Future versions of this Field Guide to Whole Number Representations in Children's Books could include incorporating more traits or simplification if research on how traits' impact children's learning is sufficiently similar to warrant collapsing trait categories. Just as field guides in science disciplines such as biology are using technology to increase access and make illustrative examples easier to create and find (Farnsworth et al., 2013), this field guide could also be expanded and refined for teachers using an on-line format that contains clickable annotations of research summaries about the potential value or problems with each trait and links to children's books that already use such representations.

Serve as an Example of the Value of Adopting Methods from another Field

Fourth, in order to conceive of an appropriate way to organize the 103 traits found, we needed to look beyond approaches typical in mathematics education. Thus, we adopted a way considered foundational in disciplinary science, the use of field guides. We view this translation across disciplinary boundaries as another implication of the study. This field guide approach might prove valuable for other mathematics education studies, especially content analyses that document characteristics of resources. Systematic content analyses of the mathematics in children's books that categorize and provide frameworks for future research on learning with such books have only recently begun, with a rating scale framework for books about shapes (Nurnberger-Haag, 2018b), a Counting Book Typology (Nurnberger-Haag, et al., 2021) and this Field Guide to Whole Number Representations in Children's Books. These approaches to using long-standing structures in disciplinary science such as taxonomies, typologies, and field guides as foundations of knowledge generation should continue to be applied to mathematics education in order to advance our own field. These approaches would lead to a more thorough foundational understanding and organization of phenomena to better investigate claims about relative affordances for learning mathematics.

Final Thoughts

If children's books are used as part of a mathematics lesson or even informally with the intent to teach mathematical ideas, the mathematical representations should be a primary consideration of selection criteria and factor into reported interpretations of learning outcomes (Hachey, 2015; Nurnberger-Haag et al., 2021; Nurnberger-Haag, 2017; Nurnberger-Haag, 2018b; Yilmaz Genc, et al., 2017). Yet, even experts need guides to focus book selection decisions on crucial characteristics (Nesmith & Cooper, 2010; van den Heuvel-Panhuizen & Elia, 2012). Given that Nesmith and Cooper (2010) found that how an adult judges the mathematics of a children's book varies tremendously by their content knowledge, to meet the needs of the broader field who may not be experts in early number learning or in mathematics, it is even more crucial that the field develop tools to inform how to recognize the potential mathematical learning opportunities in children's books.

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Author Information

Julie Nurnberger-Haag
https://orcid.org/0000-0002-3083-8846
Kent State University
Kent, Ohio
United States
Contact e-mail: *jnurnber@kent.edu*

Amy Scheuermann
https://orcid.org/0000-0002-8321-2900
Minnesota State University Mankato
Mankato, Minnesota
United States

Janis S. McTeer
https://orcid.org/0000-0002-3250-5842
Ph.D.-Kent State University
United States

Appendix A. Pictorial

Trait Identifier	Descriptor
Ordinal	Images of people or objects shown in sequence labeled with ordinal numerals such as 1st, 2nd, 3rd, etc. to draw attention to the position of a person or object in order compared to others. Alternatively, no written ordinal numerals are labeled on the images, but the text asks the reader who or which is first, third, etc. in the image shown.
Continuous	
Number Line	Number line shown in any form, vertical or horizontal, or coordinate axes
Linear Measurement/ Distance	Some visual indication of distance (e.g., shows a bar between two objects, colored spacing, a measurement tool)
Weight	Since this is a pictorial category, this might be communicated with a photo or illustration of a balance scale, characters on a teeter totter, etc.
Volume/capacity	Some visual indication of capacity (container that could hold certain volume) or volume shown in the form of liquid or solids within a container.
Temperature	Some visual representing temperature or relative warmth.
Time (date, hour/min, etc)	Calendar, analog or digital clock, sand-timer, or any other visual display of the abstract concept of time (not elapsed time, see next code).
Elapsed Time	Visual represents two time points and in some way portrays the distance or space between those time points.
Mismatched Representation/ No Continuous Representation	When a number is used to represent a continuous amount either no pictorial representation is provided, or the pictorial representation cannot support understanding the stated amount.
Discrete	
Base-Ten Number Images	
Collections of Ten	
Collections of Ten-Multiple	**$10X_2 + X_1$ where $X_1 = 0$ & X_2 is an integer >0;** Multiples of ten images are shown with sufficient spacing to separate collections of ten [no borders or containers used]
Collections of Ten-Extras	**$10X_2 + X_1$ where X_2 and X_1 are integers >0, and X_1 <10;** Multiples of ten images are shown with sufficient spacing to separate collections of ten [no borders or containers used] and spacing to include the X_1 remainders that do not form a full collection of ten
Collections of Ten-Container	A dotted border or some other illustrated representation is used to imply grouping or containing into sets of ten singles or collections of ten are shown in containers or within borders so that a reader could refer to the composite unit by name

Base-Ten Place Value Images	Images to represent numbers larger than 100 with spacing, borders, or containers to show sets of hundreds, tens, etc.; 3 or more levels of base-ten images shown together in a representation
Non-proportional Base-Ten Number	Non-proportional representations of base-ten number or place values (e.g., colored chips to represent place value quantities)
Property Representations	Discrete images to pictorially represent mathematical properties of Circle all that apply [commutative, associative, distributive, multiplicative identity, etc.]. So 5 + 7 is the same as 7 + 5 or & 7 groups of 5 is the same as 5 groups of 7 etc. shown with images to show that these quantities are the same.
Counting & Cardinality	
No Cardinal Representations	No cardinal representations shown to focus on counting word or number sequences
Mismatched Cardinal Representations	Numerals or number words used, but pictures of contexts that correspond to the stated numeral or word do not represent those written symbols.
Linear Arrangement	A line could be drawn through objects or animals. Line may be in any direction (horizontal, vertical, diagonal, askew, etc.)
Aligned but not linear	Think of a follow the leader idea in which each object or animal/person is single file, but a straight line could not be drawn through all objects or animals.
Multiple Alignments	Think of a follow the leader idea as more than one "aligned but not linear" arrangement in which each object or animal/person is single file, but a single line could not be drawn through all objects or animals.
Objects arranged to form numeral - Match	Objects are arranged to form a numeral (no numeral is written or drawn, see distinction between this code and "numerals with touch points"). The number of objects used to form the numeral matches the stated numeral.
Objects arranged to form numeral - Mismatch	Objects are arranged to form a numeral (no numeral is written or drawn, see distinction between this code and "numerals with touch points"). The number of objects used to form the numeral is not the same as the stated numeral.
Numerals with Touch Points	A numeral 1 with 1 dot on it, a numeral 4 with four dots, a numeral 6 with either 6 separate dots or 3 dots with a ring around each to count each dot and ring to represent 6 and so forth. The distinction between this code and "objects arranged" code is that the numeral is written for this code and dots or objects are just placed on the numeral.
Circular Arrangement	Objects are arranged to form a circle.
Shape Outline Arrangement	Objects are arranged to form an outline of a shape (e.g., square, triangle, etc.). This may be for no other purpose than to try to also teach shapes.
Figural Numbers	Objects are arranged to form what are mathematically known as "figural numbers" such as square numbers, triangular numbers (e.g., how bowling pins are arranged), etc.

1-1 Images	An object is matched to another object (or person to person, or object to person, etc.)
Rearranged	"More than one representation of the same item set is shown at the same time. Sets are rearranged." Ward et al (2017, p.? Table 1). The idea is that the images could promote conservation of number.
Array	Arrays of discrete objects, animals, or peoples shown.
Tallies	Tallies shown.
Five-Frame	A 1 by 5 frame. If multiple five-frames are shown as part of a single representation, then notate using coefficient notation such as 20FF meaning 20 Five-frames were together to represent a number within 100.
Ten-Frame	A ten-frame, either a 2 by 5 frame or a 1 by 10 frame. If multiple ten frames are shown as part of a single representation, then notate using coefficient notation such as 2TF meaning two ten-frames were together to represent a number within 20.
Common Pattern	A common pattern that might be perceptually subitizable such as the dots on dice and dominoes.
Composable Sets	
Common Pattern Sets	Two or more common patterns that might be perceptually subitizable such as the dots on dice and dominoes (such as five dots and 3 dots, or five dots and five dots), so that together they might be conceptually subitized.
Repeated Groups	Two or more groups of the same quantity that may or may not be arranged in similar ways.
Repeated Groups with Remainders	Two or more groups of the same quantity that may or may not be arranged in similar ways as well as remainders.
Unequal Groups	Two or more groups of different quantities
Unsystematic Groups	Three or more quantities of objects, animals, or people such that sets or separations of the groups are unclear; it is only due to the different categories of objects that a reader can discern that multiple groups are shown.
Natural Fixed Discrete	Naturally occurring arrangements that can be counted and are always fixed; so 5 spirals in pine cone as example of Fibonacci sequence. This code should not be used for natural occurrences that are not always fixed (5 ducks by a pond that move around would not count for this code).
Common Manufactured Discrete – Arrangement	Common manufactured items with countable parts that always look the same. Note the difference between this code and real-life numerosity is that the reader can count these, but real-life numerosity can't.
Real-life numerosity	Image does not allow 1-1 counting, but arrangement or packaging is the way it would be seen or experienced in real-life meaning it is expected due to cultural or environmental typical arrangements, e.g., top view of a closed egg container (in US typically 12), roll of US stamps

Non-proportional Cardinal Value	Countable objects for which the value of each object is culturally assigned, for example, U.S. coins and bills [each bill or coin can be counted, but the value of a single bill is often more than a single dollar value; 3 quarters may be counted but the total value is 75 cents].
Impossible to Count	Impossible to count all or attend to all pictorial representations of a stated quantity due to size of illustrations, overlapping, or any other reason.
Unsystematic	If the objects are countable but there does not seem to be a discernable pattern, in other words another code cannot describe the arrangement, then these are unsystematic or ungrouped objects. The meaning may be what was intended when Ward et al. (2017) used the word "random," but random has a particular mathematical meaning of occurring by chance as opposed to an authors' illustration.
Number Track	Numerals presented in a sequence with each numeral surrounded by a border. This may look like a board game path of squares in which two numbers share a border of a square or there may be spaces between circles or other outlines with numerals within them. This is NOT the same as a number line in which each numeral is represented by a tickmark or point on a line.

Appendix B. Written Symbols

Trait Identifier	Descriptors
Numeral Labels	A numeral that is used to identify which person, thing, or object (e.g., soccer jersey)
Ordinal Numerals	These need not be presented in order, but ordinal numerals are written (e.g., 1st, 2nd, 3rd, etc.)
Ordinal Words	These need not be presented in order, but ordinal words are written (e.g., First, second, third, etc.)
Personified Numeral	A numeral is used as a character
Numeral	Numerals such as 5, 10, 100, etc. that imply a cardinal or continuous quantity meaning
Traceable Numeral	Numerals are drawn large enough and with arrows or other indications to encourage readers to trace how to write the numerals.
Number Words	Number words such as five, ten, one-hundred, etc.
Numeral Pattern	A number pattern is shown as a single written representation separated with commas or spaces (e.g., 1, 4, 7, 10; Fibonacci Sequence, Pascal's Triangle, growing patterns, etc.)
Number Word Pattern	A number pattern is shown as a single written representation (e.g., one, four, seven, ten; growing patterns, etc.).
Set Words	Words such as "group," "set," or other synonyms for a cardinal quantity. Circle all that apply: [group, set, duo, pair, couple, trio, quartet, quintet, hextet, octet, dozen, or write in others ___________________]
Cardinal Quantity Words	[e.g., a lot, any, few, need to remember term for this]
Cardinal Quantity Comparison Words	[e.g., fewer, less than, more than, the same value as]
Continuous Amount Words	[how much, how long, how big, etc.]
Continuous Comparison Words	[Circle all that apply] [same amount as, how much more, how much less]
Combining Like Units	
Basic Unit Explicit-Superordinate Explicit	Units explicitly stated in words at Basic Unit Level and also at a superordinate unit level (apples and fruit)
Basic Unit Explicit-Superordinate Implicit	Units explicitly stated in words at Basic Unit Level but not at superordinate unit level
Basic Unit Explicit with 3 or more levels of units explicit at once	See Trait Identifier Name

Basic Unit Explicit with 2 or more levels of units implicit at once	See Trait Identifier Name
Place-value unit vernacular	Words used to describe any collection of ten ideas (2 groups of ten balls & 3 balls) using the actual images shown [may or may not also use place value terms]
Place-value Terms	Mathematical place value terms used such as “thousands” “hundreds” “tens” and ones; “standard form”, “expanded form” [not all need to be used, circle all that apply or add in others]
Categories of Numbers	Circle all that apply [odd/even, prime/composite, palindrome…]
Property Terms	Circle all that apply [commutative property, associative property, distributive property, multiplicative identity, additive identity, etc.]
Operation Words	
Addition Words	Circle all that apply [add to, sum, plus]
Subtraction Words	Circle all that apply [take away, minus, subtract* (any version with base word), difference]
Multiplication Words	Circle all that apply [factors, multiples, product]
Division Words	Circle all that apply [factors, quotient, separate, divide, remainder]
Exponentiation Words	Circle all that apply [exponent, power, square, cube, etc.]
Operation Symbols	
Addition Sign	+
Subtraction Sign	-
Multiplication	Circle all that apply [*, x, ab, a(b)]
Division	Circle all that apply [/, ÷]
Charts	
Factor Trees	A factor tree is shown.
Number Grid	With or without color coding to reveal patterns, regardless of how used, a grid or array of numerals rather than objects is typically called a number grid or number chart.
Place Value Chart	Image of place value chart using words [List which place values are shown]
Addition Table	Typical grid of numerals across row and column and filled in with resulting sums.
Multiplication Table	Typical grid of numerals across row and column and filled in with resulting products.

Fact Family	Regardless of what this is called in a book, the visual representation is a triad of numbers such as 12, 5, and 7 [12-5=7, 5 + 7 = 12, 7 + 5=12, and 12-7 =5] often shown in a triangle.
Equations & Inequalities	*Note: For any equation or inequality code, these may be numerical terms, numbers and variables, some symbol for a missing value (variable, question mark, blank, symbol, shape, etc.)
Equations-Vertical (EV)	
EV-2 *Terms*	Vertical equations with two terms.
EV-Multiple Terms	Vertical equations with more than two terms.
Equations-Horizontal (EH)	
Canonical	2 Term Expression Left Side [e.g., superscript A would mean accurate, superscript I would be used for inaccurate examples such as found where mixes images with an equals sign as *Plus 2, Minus 2* TPL sample did]*
Canonical 3 Term Expression Left Side	3 Term Expression Left Side and "answer" on right side [e.g., superscript A would mean accurate, superscript I would be used for inaccurate examples such as found where mixes images with an equals sign as *Plus 2, Minus 2* TPL sample did]*
2 Term Expression Right Side	Canonical 2 term expression reversed so atypical in that "total" is on left with two terms on the right*
2 Term Expression = 2 Term Expression Same Operation	Both sides of an equation have two terms (one of these may have a blank or question mark to be filled in)* but the operation is the same on both sides either all addition or all subtraction.
2 Term Expression = 2 Term Expression Different Operation	Both sides of an equation have two terms (one of these may have a blank or question mark to be filled in)* but the operation on one side is different from the other side (e.g., one side addition the other subtraction).
Inequalities	Circle all that apply: [>, < , ≤, ≥ , ≠] *

Appendix C. Tactile, Kinesthetic, and Auditory

Trait Identifier	Descriptor
TACTILE	
Smooth	Picture so each number image is indistinguishable from the next by touch.
Holes	A page has holes that a reader can put their finger in to count or stick their fingers through.
Tool	A tool is included to accompany book to point, touch, etc. images or objects;
Manipulative	Money, objects, foldout ruler, etc. from a pocket to then use
Raised	Image of each element of a quantity is raised, probably also including some texture but does not have to
Textured	Image of each element of a quantity has fur, bumps, or some other textural element although a flat surface (if raised is checked for an image, texture would be assumed, don't check this as well)
Lift Flap/Pull Tab - Related	The top of flap has a numeral and/or number word and after a reader lifts the flap they see an image representation of the quantity, or vice-versa.
Lift Flap/Pull Tab - Unrelated	The flap lift does not serve to connect representations of numbers.
Pop Up	The reader can perform an action to make part of the page rise into a 3D space.
Numeral-Trace	Numerals are drawn large enough and with arrows or other indications to encourage readers to trace how to write the numerals. (also cross-listed in written symbols traits)
KINESTHETIC (MOTIONS)	
Character's Actions Represented Visually	Motions in story represented in some visual way due to limitations of static book. In other words the representation is of a particular number of physical motions
Reader Encouraged to Act	Story encourages reader to physically enact a quantity
AUDITORY/ORAL (SOUNDS)	
Onomatopoeia	Onomatopoeia that represents a number or quantity. Quantity represented with non-word sounds (e.g., three represented as "swish, swish, swish" or "knock, knock, knock")