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Learning Quadratic Functions with ChatGPT: An Innovative Experience in High School Mathematics Education

Alexandre Trocado [1] *, **José Manuel dos Santos** [2] , **Mousa Saimon** [3] , **Zsolt Lavicza** [4]

[1] CIDMA-Campus Universitário de Santiago, Portugal, 0000-0001-5589-8100
[2] University of Coimbra, Portugal, 0000-0002-6830-6503
[3] Johannes Kepler University, Austria, 0000-0003-4758-8270
[4] Johannes Kepler University, Austria, 0000-0002-3701-5068
* Corresponding author: Alexandre Trocado (mail@alexandretrocado.com)

Article Info

Abstract

The present study explores the possibility of using ChatGPT as a tutor, to support the teacher's work in creating solid didactic and adidactic situations in the mathematics classroom, employing qualitative analysis methodologies. The study was based on a didactic experiment carried out in a 10th-grade class in Portugal, involving 26 students who were obtained through purposive sampling. Data sources included submitted student work in the classroom, responses to scripted questions, and answers to open-ended questionnaires. Data were analyzed thematically. Findings show that ChatGPT has a potential to enhance learners' development of critical thinking and mathematical reasoning when used as an assistant tutor in a mathematics classroom. Additionally, the study highlights the limitations of ChatGPT-3.5 when used as the assistant tutor in a mathematics classroom. The study underscores the need to balance pedagogical and technological approaches, addressing the constrains of tools like ChatGPT.

Introduction

Digital tools that facilitate the understanding and learning of mathematical concepts play a vital role in mathematics classrooms. GeoGebra is one of the dynamic mathematics software programs that integrates geometry, algebra, spreadsheets, graphs, statis- tics, and calculus into a single application. In addition, GeoGebra enables students to visualize abstract mathematics. GeoGebra is a dynamic mathematics software pro- gram that has gained widespread recognition for enhancing students' understanding of mathematical concepts, particularly through interactive visualization and exploration.

Recent studies demonstrate that GeoGebra significantly improves students' engagement, comprehension, and performance across various mathematical topics, such as geometry, algebra, and calculus (Uwurukundo, Maniraho, & Tusiime, 2020; Hernández, Perdomo-Díaz, & Camacho-Machín, 2019). Research also shows that GeoGebra facilitates higher- order thinking and problem-solving skills, making it a valuable tool for both teachers and students in secondary and higher education settings (Binti Misrom, Muhammad, Abdullah, Osman, Hamzah, & Fauzan, 2020; Yohannes & Chen, 2021; Bayaga, Mthethwa, Bossé, & Williams, 2019). Its capacity to provide visual feedback allows students to make connections between abstract concepts and real-world applications, thus fostering deeper conceptual understanding (Joshi & Singh, 2020).

Building on these findings, the present study explores the potential of integrating GeoGebra with ChatGPT, an AI-driven conversational tool, to enhance students' learning experiences. Specifically, this research investigates how the complementary use of these tools can support the development of mathematical concepts, such as quadratic functions, and addresses the challenges of using ChatGPT in isolation. By examining students' perspectives on this integration, the study seeks to answer key questions about the benefits and limitations of combining AI tools like ChatGPT with established educational technologies like GeoGebra.

ChatGPT is one of the digital tools in the form of a chatbot, a conversational artificial intelligence application. It understands natural language and responds to questions asked by users using the same language. Studies show that ChatGPT can transform education by providing students with opportunities for personalized teaching, automatic assessment and feedback, natural language processing, learning analytics, integration with adaptive systems (Chen, Xie, Zou, & Hwang, 2020; Lameras & Arnab, 2021), as well as support for teachers and collaborative learning (Chiu, Xia, Zhou, Chai, & Cheng, 2023; Lameras & Arnab, 2021). Therefore, it can be argued that combining the abilities of ChatGPT and GeoGebra can be a catalyst for students' engaging learning experiences in mathematics. As Canonigo (2024) posits, "Thus, technology, particularly GeoGebra and AI, has significant potential to enhance student engagement and cultivate conceptual understanding in mathematics. By providing interactive, personalized, and supportive learning environments, these tools can contribute to improved student outcomes." (Canonigo, 2024, p.3).

However, educators hesitate to integrate ChatGPT in teaching and learning because of concerns such as the privacy and security of student data, bias, and cheating in examinations (Chen et al., 2020). It should be noted that educators' reluctance to use ChatGPT in the classroom does not prevent students from using it beyond classroom

contexts. Von Garrel and Mayer (2023) observed that nearly two- thirds of students in Germany from mathematics, engineering and natural sciences use ChatGPT. This implies the need for research to explore successful practices for harnessing the potential of ChatGPT in class- rooms. Consequently, scholars call for educators to create a learning environment that balances between technology and human interaction (AlAfnan, Dishari, Jovic, & Koba Lomidze, 2023; Gouia-Zarrad & Gunn, 2024).

Research reports on using ChatGPT in mathematics classrooms could offer pedagogical insights to educators. However, there is limited research on experimenting with the use of ChatGPT in teaching and learning mathematics. This leaves educators less informed of how they could integrate ChatGPT to transform their classroom practices. In response, we experimented with the integration of ChatGPT in teaching and learning mathematics in one of the secondary schools in Portugal. Thus, in the present study, we aim to share lessons drawn from our experimentation on the usefulness of integrating ChatGPT in mathematics classrooms. In other words, we aim to answer the question: how useful is ChatGPT in enhancing the learning of mathematics concepts in secondary education?

Related Studies

The fact that ChatGPT could improve, or harm education has led researchers to put considerable effort into ensuring its integration in teaching and learning yields more positive than negative results. The efforts include examining educators' and students' perceptions (Ding, Li, Jiang, & Gapud, 2023; Egara, & Mosimege, 2024), and investigating students' interaction with ChatGPT (Yoon, Hwang, Lee, Roh & Kwon, 2024) in different contexts and disciplines. For instance, Gouia-Zarrad and Gunn (2024) observed that engineering students perceived ChatGPT as a useful tool in enhancing their application of numerical solutions in solving problems. However, they also noted its weaknesses, such as providing incorrect information.

Similarly, Alafnan et al. (2023) report that ChatGPT provides insights for improving practical aspects in business communication skills. However, the authors caution against using ChatGPT for theoretical aspects, as it enhances rote learning among students. Based on this, they advise teachers to avoid theoretical questions when students are permitted to use ChatGPT in the class- room. Instead, they propose using case-based assignment that requires personalized solutions. Moreover, Tong, Tao, Y., Zhang, Dong, Hu, Pan, & Liu (2023) noted that ChatGPT 4 can perform better than lower-level students in solving physics problems and helps in developing conceptual understanding among learners. However, the authors emphasize teachers' guidance on how students interact with ChatGPT and encourage avoiding over-reliance on it. Ding et al. (2023) report that students perceive ChatGPT as ideal when used as a virtual assistant tutor in physics. There- fore, the authors advocate for developing critical thinking skills among students when they are allowed to use ChatGPT.

The reported piloting of ChatGPT use in classrooms offers insights into best practices for integrating ChatGPT in teaching and learning. However, these insights stem from classrooms in disciplines other than mathematics, which is the target for the present study. Consequently, generalizing findings from these studies to mathematics classrooms may be risky. This makes studies that focus specifically on mathematics class- rooms significantly

valuable in informing mathematics educators. On the other hand, Egara and Mosimege (2024) observed that mathematics educators in Nigeria have a positive perception of ChatGPT in teaching mathematics. The authors report that these teachers, however, felt that the successful integration depends on curriculum alignment and teacher training. Despite focusing on mathematics, this study does not address students' perception. Excluding students' view may lead to technology integration that fails to harness learners' potential engagement with the technology appropriately. Furthermore, the findings from this study were not based on classroom experimentation with ChatGPT use, which limits researchers' ability to identify classroom-relevant challenges and needs. Given the unique nature of each classroom context, we argue that only insights from classroom experimentation can effectively inform practices for integrating ChatGPT in the classroom. Thus, there is the need for experiment-based research.

Antunes Ribeiro, Rosotti Navarro and Kalinke (2024) noted that grade 9 students in Brazil struggled with prompting when using ChatGPT for learning basic mathematics. Likewise, Yoon et al. (2024) developed a framework for university students' interactions with ChatGPT based on experiment involving undergraduates from Korea University who are majoring in mathematics. Yoon et al. (2024) found that students could identify the strengths and weaknesses of the AI during their interaction, and they proposed ways to facilitate successful integration of ChatGPT in the mathematics classroom. Despite focusing on classroom experimentation involving students, these studies (Antunes Ribeiro et al., 2024; Yoon et al., 2024) approached ChatGPT as a stand- alone tool in the classroom, excluding other tools such as GeoGebra. This approach aligns with the view of replacing existing technologies with ChatGPT. However, considering ChatGPT as a complementary tool to existing technologies requires classroom experiments that integrate both ChatGPT and other software like GeoGebra. Consequently, studies like the present one, focusing on integrating ChatGPT in teaching mathematics alongside other technologies, are needed.

Yunianto, Lavicza, Kastner-Hauler, & Houghton (2024) conducted an experiment using ChatGPT with GeoGebra to enhance computational thinking skills among university students. However, in this study ChatGPT was intended to help students develop knowledge in using GeoGebra for learning mathematics to develop computational thinking skills. This is different from the present study, which focuses on using ChatGPT and GeoGebra in a complementary manner to develop students' mathematics knowledge. Therefore, we cannot rely on Yunianto's insights if we aim to understand how best to use ChatGPT in teaching mathematics, highlighting the need for the present study.

In another study, Canonigo (2024) experimented with ChatGPT and GeoGebra to enhance collaborative learning of quadratic equations among grade 10 students in Indonesia. Although this study involved students and integrated both ChatGPT and GeoGebra, the focus was on enhancing a collaborative learning approach rather than mathematical concepts. This shows the need for the present study, which focuses on developing students' mathematical concepts. Therefore, we argue that our initiative to experiment with ChatGPT's usefulness in developing learners' mathematical concepts could contribute to the broader research efforts to inform practices for integrating ChatGPT in mathematics classrooms.

Methods

Study Context

The pedagogical experiment took place during a 90-minute lesson with a 10th-grade class at a private school in Portugal, situated in a socio-economic context above the national average. The participants were students with consistent access to technological resources, both at school and at home, including smartphones, tablets, computers, and high-speed internet. The study focused on the 10th-grade mathematics curriculum, which includes the study of functions, particularly polynomial functions. Specifically, the experiment targeted quadratic functions, as outlined in the official mathematics curriculum in Portugal. The pedagogical intervention was conducted using computers to explore key concepts related to these functions. The experiment was implemented in the classroom by one of the researchers, who did not intervene during the course of the lesson, allowing the students to engage with the material independently.

Implementation

The experiment was organized into two phases, each lasting 90 minutes. In the first phase, the teacher assigned tasks for students to complete in pairs using ChatGPT and GeoGebra. Each pair of students was given access to one computer with ChatGPT-3.5 and GeoGebra 6 available on the web (see Figure 1). Table 1 represents the tasks given to students.
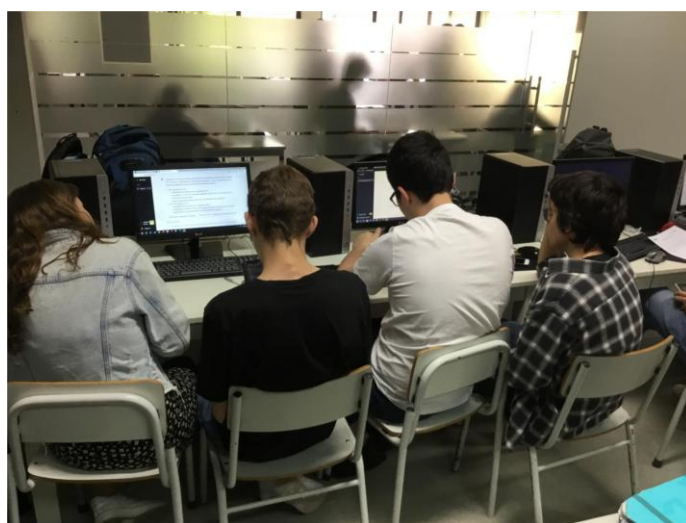

Figure 1. Students Working in Pairs

During this phase, the main role of the teacher was to assist students with technical issues. At the end of the session, the teacher collected the students' work and provided them with an open-ended questionnaire. The questionnaires give to the students are shown in Table 2.

In the second phase, the teacher guided students in reflecting on their experiences and observations from using ChatGPT and GeoGebra. The teacher clarified contradictory concepts related to functions to enhance students' understanding of the function $f(x) = ax^2$.

Table 1. Student Task Suggesting the Use of ChatGPT 3.5 and GeoGebra

Item	Task instruction
T1	Ask ChatGPT about the quadratic function. Then, in your own words, define what a quadratic function is.
T2	Refer to GeoGebra (http://www.geogebra.org/2d) to obtain examples of graphical representations of quadratic functions. Then, represent some of these examples.
T3	What are the analytical expressions that define quadratic functions?
T4	Does $f(x) = ax^2 + bx + c$ always represent a parabola?
T5	What is the influence of the parameter a on the graphical representation of f?
T6	What is the influence of the parameter b on the graphical representation of f?
T7	What is the influence of the parameter c on the graphical representation of f?
T8	Under what conditions does the quadratic function have zeros? Illustrate with examples both graphically and by indicating the analytical expression.
T9	How can we determine the maximum or minimum of the quadratic function?
T10	Refer to GeoGebra and the general case $f(x) = ax^2 + bx + c$ to illustrate the influence of the parameters on the representation of the vertex of the parabola.
T11	Explain step by step how to algebraically deduce the coordinates of the vertex of the parabola defined by the quadratic function.

Table 2. Open Questionary Applied at the End of The Class

Item	Task instruction
Q1	Do you know ChatGPT? If so, what is your opinion?
Q2	Have you used it? If so, in what way or what type of questions did you ask?
Q3	What is your opinion about the activity carried out on the computer using ChatGPT?
Q4	What is your opinion about the use of ChatGPT in future classes?

Data Collection and Participants

Data were collected from 26 students through surveys, small classroom conferences, and document analysis of 13 works submitted by students. Participants for this study were obtained through purposive sampling, which involves the selection of participants based on specific criteria. Our selection criterion was based on the members of the classroom who took part in the classroom experiment.

Data Analysis

This study analyzed the data thematically using Neuendorf (2018) framework of thematic data analysis. The framework is based on phases such as familiarization, initial coding, searching for themes, reviewing, defining and naming themes, and reporting. The familiarization phase involves reading the dataset repeatedly to gain a

comprehensive understanding. During initial coding, researchers assign agreed labels to the data to facilitate sorting and identifying patterns. In the theme-searching phase, researchers identify data patterns and establish their relationships. In the review phase, researchers exchange their identified data patterns for peer feedback and discuss any contradictions until reaching a consensus. In the defining and naming themes phase, researchers assign names to themes based on the content identified in the data under each classification. Finally, in the reporting phase, researchers summarize the key findings, providing examples from the dataset for each results category.

Results

The present study aims to examine the integration of ChatGPT alongside GeoGebra in the mathematics classroom from the students' point of view. In this section, we present the results based on our analysis of the data from students' work, reflections, and surveys.

From Students' Submitted works

Data analysis indicates that ChatGPT is useful for facilitating learners' understanding of mathematical concepts. This was demonstrated through the students' answers to the provided tasks. For instance, T1, is a direct question to which all students responded correctly. Given that the tasks involved new concepts for students, it follows that ChatGPT helped students to learn new concepts. Although the level of detail in the responses varied, as shown in Figure 2 and Figure 3, all answers were correctly provided.
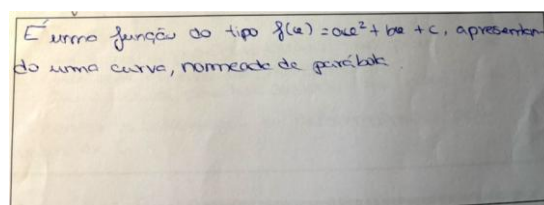

Figure 2. Low Detailed Answer

"It is a function of the form $f(x) = ax + bx + c$, presenting a curve known as a parabola."

Other questions, such as T5 and T7, were factually well answered by all students, with little variability in the details of the responses, indicating that ChatGPT played a valuable role as an assistant.
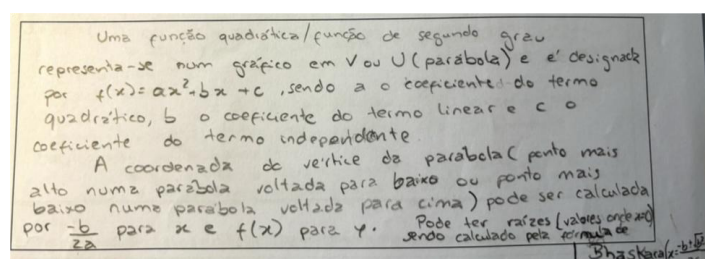

Figure 3. High Detailed Answer

"A quadratic function, or second-degree function, is represented on a graph in a V or U shape (parabola) and is designated by $f(x) = ax^2 + bx + c$, where a is the coefficient of the quadratic term, b is the coefficient of the linear term, and c is the constant term. The coordinates of the vertex of the parabola (the highest point in a downward-facing parabola or the lowest point in an upward-facing parabola) can be calculated by $-\frac{b}{2a}$ for x and $f(x)$ for y. It can have roots (values where x = 0) which are calculated using the Bashkara formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$,"

Additionally, data analysis indicates that integrating ChatGPT with GeoGebra is beneficial, as it allows students to compare the answers from the two applications or complement the answer from one of the applications. For example, in responses to questions T2 and T10, GeoGebra played a crucial complementary role, given the limitations of the ChatGPT version used, which lacked the ability to provide graphical representations or dynamic visualizations of a function's graph based on its defining parameters.

From Students' Reflections

Data analysis indicates that students perceived ChatGPT with some exceptions. For example, during reflection, some students stated that ChatGPT is beneficial because:

"I consider it an excellent platform that responds accurately and in a highly concise, explanatory, and professional manner to questions or requests, potentially even better than individuals with a degree in a specific field."

However, they reported to have been challenged by some ill-constructed questions. For instance, some students mentioned their unfamiliarity with the term "Bhaskara's formula" for the quadratic equation's resolvent formula. On the other hand, they view it as frustrating once it provides vague answers. For example, some students said when they asked ChatGPT, "how can we determine the maximum or minimum of the quadratic function", the response was sometimes just a set of steps for the user to follow without any rationale.

Survey Responses

The categories identified in the analysis of the survey responses are found in Table 3. Regarding the first survey item (Q1), it was noted that all students (26 responses) were familiar with this tool. Most expressed positive opinions, emphasizing its use- fulness and efficiency in research and accessing information (9 responses), as well as facilitating school and academic work (6 responses). Additionally, some students mentioned ChatGPT's potential to improve quality of life and education (3 responses) and its capacity to innovate and revolutionize learning and research (5 responses). However, there were also concerns about excessive student dependency (1 response) and ChatGPT's limitations and errors (5 responses).

Based on the analyzed responses, it can be observed that ChatGPT is widely regarded as a useful and innovative tool in education, with the potential to enhance research and facilitate school and academic work. However, there are concerns about students' excessive dependence on it and its limitations and errors. Despite recognizing some

limitations of this AI tool, students acknowledge its benefits and innovative potential compared to traditional internet search engines, as observed in one student's opinion:

> "Yes, I know it. It is very useful, as it answers questions that search engines do not understand, even if it takes more than 10 seconds."

Table 3. Analysis of Survey Responses

Item	Task instruction	Value
Q1: Are you familiar with ChatGPT? If yes, what's your opinion?	Familiar with it.	26
	Useful and efficient for research and information.	9
	Facilitates school and academic work.	6
	Improves quality of life and education.	3
	Innovative capacity and potential revolutionize learning and research.	5
	Concerns about excessive student dependency.	1
	Concerns about limitations and errors.	5
Q2: Have you used it? If yes, how and what kind of questions did you ask?	Never used it.	6
	Support for studies and clarification of doubts.	12
	Curiosities and general knowledge.	9
	Written assignments and book analysis.	5
	Games and recreational activities.	2
	Personal use and others.	1
Q3: What is your opinion about the computer activity using ChatGPT?	Found the activity interesting and innovative	22
	Mentioned limitations or technical issues with ChatGPT.	5
	Neutral or mixed opinions about the activity.	2
Q4: What is your opinion on using ChatGPT in future classes?	Clearly favorable opinions	7
	Favorable opinion only for complementing traditional classes.	9
	Negative opinions: concerns about the accuracy of information provided, lack of subjective understanding of doubts, and absence of the human factor in education.	6
	Neutral or mixed opinions: Recognized the potential benefits of using ChatGPT in future classes but also expressed concerns about its current limitations.	5

Analyzing the responses to the second survey question (Q2), it was noted that most students have used ChatGPT. They did so to clarify doubts and support their studies (12 students), demonstrating the tool's potential as an educational resource for this age group. The analysis also suggests that ChatGPT can be a valuable educational tool for students, particularly in helping clarify doubts and deepen their knowledge in various subjects. However, it is evident that among those most engaged in exploring its capabilities, some students already recognize the importance of ChatGPT's response profiles, its operating rules, and the significance of the user's prompt. For example, one highlighted response:

> "Yes. After making it become the DAN entity, which I saw on Reddit, breaking its guidelines, I asked it about my physics and chemistry homework."

Qualitative and quantitative analysis of students' responses to Q3 reveal a variety of opinions. Gener- ally, students found the activity innovative, interesting, and a departure from traditional teaching (22 students). Some students mentioned limitations related to the use of ChatGPT (5 students), specifically regarding the temporal restriction on the number of prompts a user can introduce within a short period, while others noted that the classroom dynamics differed from the traditional setup due to the shift from a single focus of attention (the teacher's explanation) to a type of class where the dynamics are distributed among several work groups. For example, one response highlighted:

> "I think it was an interesting and innovative approach that provided us with a more dynamic and autonomous class, increasing the class's interest in the subject"

In other words, the ChatGPT activity was considered an innovative and interesting experience by students, but there remains room for improvements in the precision and quality of the responses provided by the tool. Regarding the responses to Q4, it was noted that although students found the experience quite positive and innovative, they are not dazzled by its positive aspects, acknowledging that the current limitations of this technology make the exclusive use of this application in the classroom impractical. They mentioned that its positive aspects could serve as a useful complement to traditional classes (9 students) or even in situations where the human interaction is not accessible (6 students). Some students recognize its benefits but emphasize concerns regarding its limitations, concerns that are heightened when, at different times and with similar prompts, contradictory responses are provided. This emerged as a factor of insecurity in the information provided, which can be considered relevant in the educational trust relationship. Noteworthy responses include:

> "ChatGPT is a very impressive tool, but still far from being used in classes except in similar experiments; the lack of precision and time to respond means several gaps are quickly found. Answering the question, for now, no. In a few years, maybe."

Additionally, one student mentioned ChatGPT's inability to explain in different ways by altering the type of argumentation when the initial response is not understood:

> "In my opinion, we could use it in some classes, not always. It was interesting and a different experience, but it cannot explain in a different way if we did not understand the first time."

Other students, despite recognizing the application's benefits, emphasize the essential role of the teacher: "The use can be useful but will not replace the teacher's role, and sometimes ChatGPT makes some mistakes; it is still too recent."

Discussion

The integration of ChatGPT in this teaching experiment revealed a balance between didactic and adidactic situations, positioning AI as a complement to traditional teaching. The interaction of students with ChatGPT was examined, identifying benefits such as quick access to information and challenges such as the variability of

responses. Teacher mediation is crucial to ensure the ac- curacy and depth of learning, promoting a balanced approach that combines emerging technologies with traditional pedagogical practices, aiming to prepare students for an ever-evolving digital world.

Students' Views on Using ChatGPT in Combination with GeoGebra

The integration of ChatGPT with GeoGebra in these mathematics classrooms and within a mathematics topic demonstrated both pedagogical benefits and areas for improvement based on student feedback. In fact, some recent theoretical works align with this idea (Botana & Recio, 2024; Botana, Recio, & Pilar, 2024; Bagno, Noah, & Reches, 2024); however, to the best of our knowledge, we have not found studies that utilize this combination in class with students. Students widely recognized that combining the two tools allowed for a more dynamic approach to learning mathematical concepts, particularly quadratic functions. ChatGPT's capacity to offer immediate, textual explanations of complex ideas complemented GeoGebra's visual and interactive capabilities. This dual-tool approach enabled students to cross-reference outputs, thereby reinforcing their understanding of theoretical concepts through visual exploration. Students perceived this integration as beneficial for engaging with abstract mathematics, as it allowed them to compare analytical solutions with graphical representations. However, the study also highlighted the importance of teacher mediation to clarify concepts, particularly when discrepancies arose between the tools or when ChatGPT's responses lacked the depth or precision required for more advanced understanding. The implications of this view suggest that while ChatGPT can effectively support mathematical reasoning when used alongside GeoGebra, its role remains complementary and must be critically assessed to prevent over- reliance on AI-generated content.

ChatGPT's Weaknesses and Ill-Formed Answers

While students acknowledged the utility of ChatGPT, their feedback also reveal significant limitations, particularly concerning the generation of ill-formed or unclear responses. Several students found that ChatGPT occasionally provided vague or incomplete answers, which hindered their ability to fully grasp complex mathematical procedures, such as determining the maximum or minimum of a quadratic function. Specifically, students noted frustration when ChatGPT's explanations lacked logical progression or sufficient detail, resulting in confusion. Additionally, unfamiliar terminology, such as "Bhaskara's formula", further complicated their understanding, indicating the need for the AI tool to be more adaptive to regional variations in mathematical terminology and pedagogy. These issues emphasize the necessity for critical teacher oversight and intervention, ensuring that students are not misled by inaccuracies or gaps in the AI's responses. The implications of these challenges underscore the current limitations of ChatGPT in education, where its utility is undermined without clear guidance and precise questioning. Educators must address these shortcomings by fostering students' critical thinking and prompt-formulation skills, ensuring they can manage ChatGPT's limitations effectively.

Didactic and Adidactic Situation: Relevance in the Context of ChatGPT Integration

In educational literature, didactic situations involve the interaction between the student and knowledge mediated

by the teacher or prepared materials, while adidactic situations occur when the student interacts directly with knowledge, without explicit teacher mediation (Brousseau, 2021). ChatGPT can offer experiences that vary between the didactic and adidactic, depending on how it is used in the learning environment. The study indicated that students, when interacting with ChatGPT, of- ten found themselves in adidactic situations, exploring concepts and solving problems autonomously, which can foster self-efficacy but also requires critical skills to analyze the information provided by the tool Brousseau (1997) suggests that a balanced transition between didactic and adidactic situations can promote a deeper understanding of concepts. In this study, the teacher's intervention to correct incorrect information generated by ChatGPT illustrates the need for this balance. When used complementarily, the tool can enrich the learning environment, allowing autonomous exploration that is later structured by traditional didactic activities.

The Role of the Teaching Experience: Insights and Implications for Pedagogical Practice

The teaching experiment, as a research methodology, aims to understand and improve teaching and learning processes through specific and monitored interventions (Steffe & Thompson, 2000). In the present study, the experiment involved the integration of ChatGPT into classroom activities to investigate how students interact with the tool and how this interaction affects their understanding and learning of mathematical concepts. The data reveal that students used ChatGPT to explore mathematical concepts in different ways, comparing responses and discussing their findings with peers, allowing observation of how they deal with the variability in responses provided by the tool and how these influence their understanding and problem-solving abilities. It was found that, in the initial phase, situations occurring in the adidactic semi-space conditioned the students' actions, driven by their interactions with the chatbot. This supports our methodological hypothesis of viewing the adidactic semi-space as an object of investigation in itself and one worthy in- depth study.

The teaching experiment demonstrated that, although ChatGPT can facilitate the exploration and discovery of concepts, teacher guidance and support are essential to help students interpret and correctly use the information provided. These results align with the literature that emphasizes the importance of the teacher's mediating role in technology-supported learning contexts (Cobb, 2000; diSessa, 2000). The ability to adjust and adapt teaching based on observations made during the experiment is crucial to ensure that technology serves as an effective complement to traditional pedagogical practice.

Hypothetical Learning Trajectory: Evaluation and Reflections

The hypothetical learning trajectory (HLT) serves as a theoretical guide to understand how students might progress in understanding a specific concept through a series of planned activities (Simon, 1995). In the present study, the HLT was used to map students' progress in understanding mathematical concepts with the help of ChatGPT. The data reveal that the initial HLT, which anticipated a linear progression in concept assimilation, was not fully realized. Instead, students followed a more complex and non-linear learning trajectory, often deviating from the expected sequence and requiring reorientation and intervention by the teacher.

The divergence from the planned HLT can be at- tributed to the variability of the responses generated by ChatGPT and the students' ability to interpret and apply these responses in unexpected ways. The literature on HLT suggests that flexibility is essential to address these variations and that teachers must be prepared to adjust their pedagogical approaches in response to emerging student needs (Cobb, 2000; diSessa, 2000). The use of ChatGPT required continuous adaptation of the HLT, highlighting the need for dynamic and responsive pedagogical planning.

Integration of ChatGPT: Benefits and Limitations in Educational Practice

The benefits of ChatGPT, as pointed out by the students, include ease of access to information and sup- port in resolving doubts, facilitating self-directed and collaborative learning. However, the identified challenges, such as inconsistency in responses and the need for constant verification of information accuracy, underscore the importance of continuous supervision and a balanced pedagogical approach. The literature suggests that to effectively integrate AI tools like ChatGPT into the educational environment, it is crucial for teachers to develop specific skills to mediate the use of these technologies and guide students in critically interpreting the information (Fadel, & Bialik, 2019; Zawacki-Richter et al., 2019).

Students' perception that ChatGPT is a complement but not a replacement for traditional teaching reflects a balanced view that aligns with the recommendations in the literature. This study confirms the need for adequate preparation of educators to address the challenges presented by integrating these tools and maximizing their benefits. Combining emerging technologies with traditional pedagogical practices is seen as the most effective approach to promote comprehensive and meaningful learning, preparing students to face the challenges of an ever-evolving world (Mishra & Koehler, 2006; Selwyn, 2019).

Integration of AI in the Classroom

The literature on the integration of AI tools, such as ChatGPT, in the educational environment highlights several critical points essential for understanding the impact of this technology on learning. Studies indicate that AI can be a powerful ally in personalizing teaching, meeting the individual needs of students, and providing quick answers to frequent questions (Goel, & Joyner, 2017; Luckin & Holmes, 2016; Huang & Rust, 2020). However, Williamson (2017), Williamson and Eynon (2020), and Selwyn (2019) emphasize the need for a critical analysis of the growing dependence on digital technologies in education. These tools, despite being promising, present significant limitations, such as variability and inaccuracy in responses, which can result in fragmented or incorrect understanding of concepts by students.

In this study, it was observed that while ChatGPT-3.5 facilitated access to information and encouraged students' curiosity, it also presented inconsistencies and contradictory responses, as reported in the results section. This aligns with the concerns raised in the literature about the accuracy of AI-generated responses and the need for continuous supervision by educators (Fadel, & Bialik, 2019; Fischer, Lundin, & Ola, 2020). Indeed, maximizing the positive impact of AI in education requires a balance between the autonomy provided by AI and the supervision of educators to pre- vent the dissemination of erroneous information and ensure effective learning.

Nonetheless, more recent studies (Tong et al., 2023; Yoon et al., 2024; OpenAI, 2024) note that GPT-4 and GPT-4-turbo possess much stronger reasoning, contextualization, and dialogue management capabilities. Future research needs to explore how these new models are used in authentic classroom contexts. Such research can evaluate the extent to which the shortcomings seen with previous generations of these systems carry over and whether these more sophisticated AI systems have the ability to substantially redesign mathematics instructional experience instead of simply aiding with assignments. However, what we learn from this work will require much more than simply improving the current free ChatGPT text-based response system to one that combines text with static or animated images.

Interaction between Students and Technology

The interaction between groups of students and ChatGPT during classes, as described in the results section, revealed an interesting dynamic of collaboration and comparison. The physical proximity of the groups facilitated the exchange of information and the observation of responses obtained by other groups, which, in turn, encouraged experimentation with different prompts and the exploration of multiple answers. This underscores the need to discuss with students the probabilistic nature of responses generated by current AI systems and the importance of validating this information through other sources.

This behavior aligns with the principles of collaborative learning, widely promoted in the literature as an effective practice for developing critical and analytical skills (Johnson & Johnson, 1998). The exchange of experiences and the discussion of the responses obtained allowed students to critically evaluate the information and identify possible errors or inconsistencies, reinforcing the importance of a critical approach to the use of educational technologies. However, the variability of responses generated by ChatGPT, even with identical prompts, highlighted a significant limitation of the tool, as noted by Holmes et al. (2019). This inconsistency can lead to confusion and frustration among students, underscoring the need for constant supervision and guidance to ensure that the information provided is accurate and comprehensible.

Impact of Technical and Logistical Issues

Technical and logistical difficulties, such as connection issues with ChatGPT, negatively affected students' experience, leading to frustration and disrupting the flow of learning. This aspect underscores the importance of ensuring the stability and reliability of techno- logical tools used in the classroom, a point frequently highlighted in the literature (Mishra & Koehler, 2006; Selwyn, 2019). Additionally, the predominant use of ChatGPT over other tools, such as GeoGebra, suggests a possible tendency towards preferring solutions that offer quick and easy answers, which can limit deeper exploration and a more comprehensive understanding of mathematical concepts.

Analysis of Student Responses: Convergences and Divergences

The diversity in students' responses to the proposed questions, as detailed in the results section, reflects different interpretations and levels of understanding of the concepts discussed. This variability indicates the complexity of

learning and the need for a personalized approach that considers the individual skills and needs of each student (Fischer et al., 2020).The presence of detailed and more superficial responses, as well as the introduction of incorrect terms, such as "Bhaskara's formula", reinforces the importance of teacher mediation to correct misconceptions and ensure the accuracy of learned information. The literature emphasizes that teacher supervision is crucial for guiding students in the correct interpretation of responses and in developing a deep and meaningful understanding of concepts (Luckin & Holmes, 2016).

Students' Perceptions of ChatGPT: Benefits and Limitations

The results of the survey conducted with students revealed a predominantly positive view of ChatGPT, with many highlighting the tool's usefulness and efficiency in research and resolving queries. This positive feed- back is consistent with the literature, which points to the benefits of AI tools in education, including personalized teaching and support for self-directed learning (Luckin & Holmes, 2016; Zawacki-Richter, Marín, Bond, & Gouverneur, 2019). However, students' concerns about the accuracy of responses and the risk of excessive dependence on technology reflect the challenges and limitations identified in the literature.

Over-reliance on quick and automatically generated answers can lead to a superficial understanding of concepts and a lack of critical skills, as warned by authors like Selwyn (2019) and Williamson (2017). Students' perception that ChatGPT can complement but not replace traditional teaching is a balanced view that aligns with the literature's recommendations. Combining technological tools with traditional pedagogical practices is seen as the most effective approach to promote comprehensive and meaningful learning (Mishra & Koehler, 2016; Fadel, & Bialik, 2019), but attention must be paid to beliefs about learning and teaching mathematics (Dos Santos, Silva, & Lavicza, 2024).

Conclusions and Future Work

The experience with ChatGPT in the classroom, considering empirical evidence and a literature review, underscores the importance of a careful and critical integration of emerging technologies in teaching. Although ChatGPT offers significant benefits, such as quick access to information and support for collaborative and self-directed learning, its limitations, such as the variability in and accuracy of responses, necessitate rigorous supervision and continuous guidance from educators.

The future integration of ChatGPT and other AI tools in education must be approached with caution, ensuring that students develop critical skills to evaluate and interpret the information they receive. Teacher mediation is essential to correct potential errors and ensure that students acquire a deep and accurate understanding of concepts. In summary, while ChatGPT has the potential to enrich the learning experience, it should be used as a complement to traditional teaching, with an emphasis on promoting critical and analytical skills. The combination of emerging technologies with effective pedagogical practices can offer a balanced approach that prepares students to navigate the ever-evolving digital world confidently and competently.

Ethical Statements

Written informed consent was obtained from all study participants to conduct the research and publish the results. All data were processed to maintain the strict anonymity of participants. The pedagogical management of the school where the study took place authorized the research to be carried out and holds a copy of the Informed Consent Form signed by the representatives/parents/guardians of the students involved. The data were collected through researchers' diaries and participants' responses to the questionnaire about their experience and are securely held by the researchers. More information about these can be made available according to relevance of the request. The Authors declares that there is no conflict of interest in carrying out the study, with all contributing equally to the preparation of this article. This research did not receive a specific grant from any funding agency in the public, commercial, or non-profit sectors.

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