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Exploring Beginning Teachers' Ability to Design Mathematical Games

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Abstract

This paper investigates how novice secondary-school mathematics teachers develop problem-posing skills through the design of game-based mathematical tasks. Conducted within a graduate-level teacher education course, the study engaged 14 early-career teachers working in pairs to explore mathematical games, analyze their underlying structures, and create original classroom-ready tasks. Using a qualitative case study approach, we focus on two representative pairs: one that undertook a reflective inquiry into the classical NIM game and another that designed two novel, parity-based strategy games. The findings address how teachers engage with mathematical games as tools for creative task design, how the design process supports pedagogical insight and the development of teacher identity, and what forms of mathematical and instructional reasoning emerge during task creation. Participants reported heightened creativity, deeper conceptual understanding, strategic reasoning, and an evolving sense of pedagogical agency. The study underscores the value of integrating structured opportunities for problem design into mathematics teacher education programs as a means of cultivating reflective, innovative, and student-centered educators.

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Introduction

Problem-posing is increasingly recognized as a core competence in mathematics teaching and a catalyst for professional growth (Silver, 1994; Singer et al., 2011; Cai et al., 2015). The ability to formulate, adapt, and refine mathematical tasks supports curriculum development, classroom responsiveness, creative thinking, and deeper student engagement (Leikin & Levav-Waynberg, 2007; Paolucci & Stepp, 2021). Yet novice teachers often face challenges transitioning from solving predetermined textbook problems to creating rich, open-ended tasks. While many feel confident applying familiar procedures, designing meaningful experiences that provoke reasoning and curiosity remains an elusive early-career skill.

Recent research underscores that well-designed mathematical games can foster not only procedural fluency but also conceptual understanding, reasoning, and motivation when integrated purposefully into instruction (Pan et al., 2022; Vankúš, 2021). Games and puzzles can serve as a bridge for novice teachers, making the shift from solving to designing problems. As hybrid forms blending problem-solving with problem-posing, they invite teachers to explore mathematical structures, test strategies, and construct rules, supporting authentic mathematical thinking (Movshovitz-Hadar, 2011; Kontorovich, 2020).

Studies show that engaging pre-service teachers in designing or adapting games promotes deeper inquiry, flexible problem-solving, and learner-centered design skills (Applebaum, 2025; Botes, 2022; McColgan et al., 2018). Such creative activities help teachers move beyond being consumers of pre-made tasks to reflective educators capable of crafting engaging, conceptually rich learning experiences (Applebaum & Freiman, 2014). Prior research confirms that in-service teachers already value games for supporting reasoning, engagement, fluency, and problem-solving (Bragg et al., 2021), highlighting the need to help novices move beyond using existing games to create their own, student-centered tasks.

Beyond final products, understanding the process of problem-posing is essential for teacher development. Research describes how teachers move through distinct phases, situation analysis, variation, generation, problem-solving, and evaluation, when creating new problems (Baumanns & Rott, 2022). Structured cycles that combine problem-solving and problem-posing have been shown to strengthen pre-service teachers' fluency, flexibility, and awareness of task structures, supporting more effective instructional design (Kar et al., 2025). Similarly, structured problem-posing tasks can foster mathematical creativity, moving prospective teachers beyond algorithmic reasoning toward conceptual understanding (Emre-Akdoğan, 2023).

Yet challenges in problem-posing are well-documented internationally. Mallart et al. (2018) found that even mathematically competent pre-service teachers struggled to create problems aligned with curricular goals, realistic contexts, and appropriate levels of difficulty. This underscores the need for targeted support in teacher preparation programs to help novice teachers move from task consumers to confident task designers. The purpose of this paper is to examine how early-career mathematics teachers engage in creating game-based or inquiry-driven tasks and to consider the pedagogical and developmental implications of these experiences. Drawing on two in-depth case studies, we trace how novice teachers navigated the transition from solving to designing tasks through analyzing classical games, modifying structures, and inventing new ones. We argue that such creative task design experiences are powerful for supporting teacher identity, building pedagogical agency, and nurturing the reflective habits essential for effective, student-centered instruction.

To guide this inquiry, we ask the following research questions:

1. How do early-career mathematics teachers engage with mathematical games as tools for creative task design?
2. In what ways does the process of adapting or creating game-based problems support the development of pedagogical insight and teacher identity?
3. What types of mathematical and instructional reasoning emerge when novice teachers are invited to design tasks rooted in strategic or structural principles like parity?

Theoretical Background

Problem posing, the ability to formulate new or reformulated mathematical problems, is widely recognized as a key element of mathematical thinking and creativity (Silver, 1994; Stoyanova & Ellerton, 1996). Silver (1994) described problem posing as both generating new problems and reformulating existing ones, highlighting its role before, during, and after problem-solving. Stoyanova and Ellerton (1996) emphasized students' interpretation of situations to construct meaningful problems, promoting engagement and higher-order thinking.

Recent research reinforces that problem posing is not only a measure of mathematical creativity (Singer & Voica, 2015; Yuan & Sriraman, 2011) but also a means to develop and assess teachers' mathematical knowledge for teaching (Cai et al., 2021). Emre-Akdoğan (2023) further highlights that problem-posing tasks can help prospective teachers progress through different levels of mathematical creativity, from domain-specific algorithms to theoretical reasoning, by engaging them in rich, contextualized design challenges. Specifically, problem-posing competence relates to both specialized content knowledge and the broader framework of mathematical tasks of teaching (Ball et al., 2008; Paolucci & Stepp, 2021). However, studies show that this competence involves more than creating mathematically correct tasks; it requires framing them with pedagogical intent. For example, Mallart et al. (2018) found that even mathematically strong pre-service teachers struggled to adapt problem difficulty, connect tasks to real-life contexts, and anticipate student needs.

Novice Teachers and the Challenge of Problem Posing

While experienced teachers regularly use games in practice (Bragg et al., 2021), novice teachers often need explicit support to transform games from motivating activities into vehicles for developing deep mathematical understanding and reasoning. Although problem posing is promoted as a valuable skill, early-career and pre-service teachers frequently encounter difficulties creating well-structured, pedagogically meaningful problems. These challenges include aligning problems to curriculum goals, anticipating student responses, maintaining cognitive demand, and designing authentic contexts (Mallart et al., 2018; Crespo & Sinclair, 2008).

Novice teachers may also lack confidence in their own mathematical creativity and struggle to transition from problem solvers to problem posers. Kontorovich (2020) notes that even expert problem posers often rely on authentic triggers, surprising phenomena, real-life dilemmas, or deliberate variation to generate quality problems. In contrast, novice teachers are frequently expected to create problems "on demand," which can inhibit creative ideation. Beyond purely cognitive perspectives, recent research highlights the catalytic role of playful redesign in helping novice teachers move from solver to author of mathematical tasks.

Math Games as Engines of Teacher Creativity

A growing body of work suggests that strategically structured games can stimulate both creative and critical mathematical thinking in teachers. Applebaum (2025) documents how pre-service teachers who redesigned the Nim game, a classic subtraction strategy puzzle, progressed from solving to generatively posing tasks while articulating rule variations, win-strategies, and student-facing questions. Each rule modification required analyzing parity structures, identifying decisive positions, and anticipating student responses, making game redesign an authentic problem-posing laboratory. Such activities foster divergent thinking, reduce mathematics anxiety, and promote collaborative reasoning. These findings complement research showing that problem posing flourishes when teachers engage with open, rule-based environments (Baumanns & Rott, 2021; Mallart et al., 2018), reinforcing the rationale for using games as a bridge between problem solving and problem posing in teacher education. Pan et al. (2022) emphasize that effective learning games embed content through contextualization, representation, and simulation, supporting meaningful learning rather than rote practice. This underscores the need for careful design choices in teacher preparation. Drawing on this evidence, the present study positions novice teachers' own game-based designs as a core vehicle for analyzing their emerging problem-posing processes.

From Problem Solving to Problem Posing: A Pedagogical Shift

There is a well-documented symbiosis between problem solving and problem posing (Silver & Cai, 1996). Encouraging teachers to modify or extend problems they solve promotes metacognitive reflection and deeper conceptual understanding. Cifarelli and Cai (2005) suggest that this recursive process is particularly effective in open-ended, exploratory tasks. More recent work has advanced phase models of problem posing. Baumanns and Rott (2021) propose a five-phase model: situation analysis, variation, generation, problem-solving, and evaluation, highlighting the dynamic and recursive nature of problem posing as a cognitive process. Complementing this process-oriented view, Kar et al. (2025) demonstrate that structured "pose–solve–pose" cycles can scaffold novice teachers' ability to generate more varied, flexible, and conceptually rich problems. This process-oriented perspective provides a valuable analytical framework for examining the strategies used by novice teachers when engaging with mathematical games and puzzles.

Integrating games into teacher education serves not only as a motivational tool but also as a bridge between creativity, inquiry, and pedagogy. Serious games, in particular, can help novice teachers develop pedagogical content knowledge and instructional design skills (McColgan et al., 2018). Similarly, Botes (2022) found that designing board games encouraged pre-service teachers to reflect on learning goals, assessment strategies, and content integration. When embedded in inquiry-based learning contexts, games support the formulation of open-ended questions, strategic thinking, and collaborative reasoning. For novice teachers, these experiences can scaffold the development of confidence and competence needed to pose meaningful, student-centered mathematical problems.

Methodology

This study adopted a qualitative, exploratory case study design (Merriam, 1998) to investigate the capacity of early-career secondary-school mathematics teachers to pose original problems rooted in mathematical games and inquiry-based contexts. The goal was to understand not only the final products of teachers' problem-posing activity but also the underlying processes, intentions, and pedagogical considerations. A case study approach was particularly suitable for examining a small, information-rich sample in depth and for highlighting individual learning trajectories and creative approaches (Stake, 1995).

Participants and Context

The participants were fourteen novice secondary-school mathematics teachers (all in their first year of teaching), who were recent graduates of a teacher education program and enrolled in a graduate-level course on mathematical creativity and problem design. They worked in pairs, forming seven groups. The course was designed and facilitated by the author and centered on using puzzles, mathematical games, and inquiry-based tasks as tools for instructional innovation. Drawing on evidence that game design features can shape learning outcomes (Pan et al., 2022), this study invited participants to deliberately analyze and redesign game structures to strengthen conceptual depth and pedagogical relevance. Participants were selected as a purposive sample due to their shared context of early-career teaching and their enrollment in a course focused on mathematical game-based teaching strategies. As Crespo and Sinclair (2008) emphasize, structured opportunities are critical for developing teachers' problem-posing capacity, especially in contexts that balance creativity with curricular constraints.

Course Design and Data Collection

The semester-long academic course comprised 13 weekly sessions combining mathematical content, didactical reflection, and creative assignments. Participants were introduced to a variety of games (e.g., strategic puzzles, parity-based tasks, mathematical games), explored their underlying structures, and collaborated in building solutions.

As a capstone task, each pair of teachers was asked to design an original game suitable for classroom use, including an explanation of the mathematical goal, student challenge, and pedagogical rationale. This mirrors research showing that asking pre-service teachers to adapt or create games fosters critical and creative thinking while preparing them to use such strategies in their own classrooms (Applebaum, 2025).

Data sources included:

- Problem design artifacts: the final games created by participants.
- Written reflections: narratives in which teachers explained their design choices, anticipated student interactions, and reflected on their learning.
- Field notes and observations: instructor notes during in-class presentations and discussions of the designed tasks.

These multiple sources allowed for triangulation, strengthening the validity of the findings (Creswell, 2013).
To illustrate the findings, this paper focuses on two representative case studies. One pair (Nina and Visal) engaged deeply with an existing game, extending its strategic structure through reflective modelling. The other pair (Abed and Michael) designed entirely new games grounded in parity strategy. These two cases were selected for their pedagogical richness and diversity: they exemplify complementary approaches to problem posing, elaborative adaptation versus original construction, and reflect the range of mathematical creativity, metacognitive reflection, and pedagogical insight observed across the seven participating pairs. Their work provides a coherent yet contrasting lens for examining the development of problem-posing competence in early-career teachers.

Data Analysis

A thematic analysis approach (Braun & Clarke, 2006) was applied to all textual data. Each artifact and reflection were first read closely to identify initial codes related to mathematical content, game structure, creativity, and pedagogical intent. These codes were then categorized into higher-level themes such as mathematical richness, problem clarity, engagement potential, and student-centeredness. Informed by Baumanns and Rott's (2021) five-phase model of problem posing, situation analysis, variation, generation, problem-solving, and evaluation, the analysis also sought evidence of recursive thinking and metacognitive engagement. This approach aligns with research showing that combining problem solving and problem posing helps pre-service teachers improve the originality and structural quality of their tasks (Kar et al., 2025). Furthermore, teachers' work was further interpreted through Ball et al.'s (2008) framework of mathematical knowledge for teaching (MKT), particularly the domains of specialized content knowledge and knowledge of content and students. Research on problem posing emphasizes the importance of connecting mathematical content with pedagogical goals and anticipating student reasoning (Paolucci & Stepp, 2021).

Ethical Considerations

All participants were informed of the study's aims and gave written consent for their work and reflections to be used for research purposes. Anonymity and confidentiality were assured, and participants were free to withdraw at any time. The study followed institutional ethical guidelines for research with human subjects.

Findings

To illustrate the development of problem-posing competence in early-career teachers, we present two complementary cases. The first group, Nina and Visal, modified the rules of a classic game of Nim and then analyzed the resulting gameplay to deepen their understanding, demonstrating both analytical and pedagogical growth. The second pair, Abed and Michael, designed two novel games rooted in the Parity Principle. Together, these cases exemplify how game-based learning and task creation can nurture mathematical insight, instructional creativity, and reflective teaching practices.

Case Study 1: Nina and Visal's Journey from Play to Pedagogy

To highlight the depth of learning novice teachers can experience through problem posing in game-based contexts, we present the case of Nina and Visal, two first-year mathematics teachers who explored the game of Nim. In its standard form, Nim is a two-player game with several rows of tokens, such as 3, 5, and 7. Players take turns removing any number of tokens from a single row on their turn. The typical goal is to avoid taking the last token or to force the opponent into taking it, depending on the agreed-upon rule. Nim is well-known for its underlying mathematical structure: optimal strategies can be derived using binary representations and nim-sums, which let players force symmetrical or balanced positions that guarantee a win under perfect play.

"In our version of the game, we introduced a twist to the classic Nim rules: on each turn, a player could take 1, 2, or 3 matches, but only from a single row, which forced us to rethink our strategies and explore new patterns of balance and advantage." This rule change, as Nina and Visal explain, deliberately increased the game's complexity while keeping it accessible. By limiting the number of matches that could be taken per move, they made planning more nuanced, shifting players' attention toward maintaining balance across piles and anticipating the opponent's constrained choices. Their design choice also highlights an important pedagogical principle: small rule modifications can transform a familiar game into a new problem-posing opportunity, requiring players (and students) to analyze altered conditions, develop fresh strategies, and articulate reasoning. For novice teachers, this move from simply solving a known game to creating and analyzing a variant demonstrates emerging problem-posing competence and supports their growth as curriculum designers. Nina and Visal's written reflections reveal a significant shift in perception, the development of a research-oriented stance, growing pedagogical awareness, and the formation of a professional teaching identity. Below, selected excerpts from their work appear alongside analytical commentary.

From Puzzle to Platform: Shifting Perceptions of Mathematical Games

> "In the beginning, we saw NIM as a favorite puzzle game, but the more we delved, the more we realized that it is a rich platform for mathematical thinking, the development of analytical skills, and a deep understanding of reactivity."

This excerpt marks a critical moment of conceptual transformation. What began as a recreational puzzle became, through sustained inquiry, a vehicle for engaging deeply with strategic thinking and abstraction. Their experience supports the idea of games as epistemic playgrounds, fostering both mathematical insight and metacognitive awareness (Applebaum, 2025; Kontorovich, 2020). Crucially, this shift reflects movement from consuming mathematics to constructing it, an essential step in teacher identity development.

Inquiry Emerges: From Solving to Questioning

> "These questions did not arise at the beginning but grew naturally out of the deepening and transition from theoretical to applied thinking."

As they progressed, Nina and Visal moved from solving a puzzle to posing authentic, research-worthy questions about generalizability, accessibility, and pedagogical use:

> "Is it possible to translate the model we developed into a simple algorithm that can be taught to high school students...?"
>
> "What is the right balance between strategic planning and intuition...?"
>
> "How can we check whether the use of our model contributes to the development of high mathematical thinking...?"

These questions reveal a growing awareness of learners' needs and reflect what Cai et al. (2021) identify as a hallmark of pedagogical problem posing: intertwining mathematical reasoning with didactic reflection.

Social Learning and Productive Struggle

> "Moments of frustration, confusion, or disagreement later became engines of learning."

Their reflections show how collaboration, debate, and cognitive dissonance spurred growth. Their teamwork, from designing scenarios to analyzing results, created a dynamic environment for the co-construction of knowledge. These experiences resonate with Botes's (2022) findings on the role of peer negotiation and social learning in game-based teacher development.

> "True teamwork – cooperation in choosing the directions of the investigation, building documentation tables, and analyzing the results…"

Such collaboration reflects the value of co-inquiry models where teachers learn with and from one another through authentic tasks.

From Reflection to Mission: The Emergence of Pedagogical Agency

> "We have a sense of educational mission – to take what we have developed and turn it into a real educational tool... For us, the work on the game of Nim was more than a seminar paper; it was an experience in exploration, creativity, and learning."

This powerful conclusion signals the development of a broader professional vision. Nina and Visal see themselves not just as learners but as contributors to mathematics education, ready to test, adapt, and share their model with students and colleagues. Their plans, implementing, comparing, and enhancing the model, demonstrate a clear trajectory toward educational design and classroom innovation.

Overall, Nina and Visal's reflections illustrate the potential of mathematical game-based inquiry to promote not only content knowledge and problem-posing skill but also metacognitive growth, emotional resilience, and pedagogical ambition. When novice teachers are positioned as creators rather than mere users of tasks, their engagement deepens and their identity as educators solidifies. This case shows how a structured design of mathematical games supports the broader goals of teacher education: fostering strategic thinking, collaboration, reflection, and pedagogical imagination. Their full reflections are provided in Appendix 1.

Case Study 2: Abed and Michael - From Strategic Play to Game Innovation

Inspired by a classic two-player strategy game presented at the International Olympiad in Informatics (IOI) in 1996 (https://ioinformatics.org/page/ioi-1996/22), Abed and Michael undertook a creative process of modifying and analyzing game structures. Original game premise: A sequence of positive integers is placed on the board. Players alternate picking numbers from either end. Once chosen, numbers are removed. The first player wins if their sum is at least as large as the second player's, with both playing optimally. Building on this foundation, Abed and Michael designed and analyzed two original variations (Game 1 and Game 2) that significantly expanded the game's pedagogical potential, promoting fairness, optimization, and strategic reasoning. They deliberately focused on odd-length rows of integers because, in the classic parity-based strategy, an even-length row admits a well-known deterministic solution that strongly favors the first player. By enforcing an odd length, they ensured richer, less predictable strategic reasoning, where parity analysis requires closer attention and more subtle choices.

Game 1: Two-Round, Role-Reversal Parity Game

- The First Player declares an odd row length (e.g., 9–19).
- The Second Player records exactly n integers (e.g., 13).
- The First Player chooses who moves first.
- The Second Player may swap the places of two integers.
- Players alternate picking from ends.
- Roles swap for Round 2.
- The highest combined score wins.

Game 2: Alternating-Placement Parity Game

- The First Player declares an odd row length (e.g., 9–19) and range (e.g., [1,20]).
- The Second Player chooses who moves first
- Players alternate placing integers (that belong to the declared range);
- Players alternate picking from ends.
- Roles swap for Round 2.
- The highest combined score wins.

Commentary on Abed and Michael's Pedagogical Reflection

Abed and Michael's reflections reveal thoughtful attention to mathematical structure, fairness, and instructional purpose, demonstrating a level of pedagogical maturity uncommon among novice teachers. They deliberately chose to work with odd-length rows of integers, recognizing that classic parity-based strategies for even-length sequences yield deterministic solutions favoring the first player. By enforcing odd-length sequences, they created richer, less predictable scenarios where parity analysis demanded closer attention and subtle, adaptive reasoning.

In describing their Two-Round, Role-Reversal Parity Game, they wrote:

> "We wanted to transform the classic take-from-ends puzzle into a fairer, more strategic contest by making both players actively involved. Giving one player the power to choose the length and the other the chance to set and adjust the sequence kept both of us engaged. The picker's choice about who moves first forced us to analyze odd-versus-even sums, while the single swap was our last chance to tip the balance."

This highlights how they designed the game to emphasize invariant structures and parity reasoning as an organizing principle for strategy, supporting Polya's (1945) notion of heuristic problem solving. Their decision to include role reversal shows a deliberate pedagogical move to ensure symmetrical engagement, aligning with Vygotsky's (1978) emphasis on learning through social interaction and perspective-taking:

> "Swapping roles for the second round made us think about both sides of the strategy, it wasn't just about building a trap, but learning to see and avoid it."

In their Alternating-Placement Parity Game, they wrote:

> "We wanted to push shared control even further by alternating the placement of numbers from a given range. Every move changed the parity sums in real time. It wasn't enough to have a plan, we had to adapt to what the other player did."

This approach introduced authentic collaborative reasoning and real-time analysis. By alternating number placement, they designed an environment that demands sustained parity tracking and flexible strategy testing, exactly the kind of dynamic decision-making that can foster students' mathematical thinking:

> "It made us monitor the sums of odd- and even-indexed positions constantly. The chooser's first-move decision became a direct test of parity analysis. Alternating placement also meant both of us felt ownership over the sequence we created."

Their reflections show recognition that designing constraints and roles intentionally can support learning goals such as fairness, reasoning, and argumentation. They summarized the pedagogical benefits of their games as:

- Transforming passive problem-solving into active strategic design
- Deepening understanding of parity through sustained engagement
- Developing flexible thinking about structural change
- Promoting balanced exposure to multiple cognitive roles
- Encouraging progression from manipulation to early proof

Such detailed, intentional design thinking is uncommon for novice teachers, suggesting the power of well-scaffolded game-creation tasks to build transferable teaching skills. Their work aligns with research emphasizing that structured problem-posing fosters mathematical creativity (Silver, 1994; Emre-Akdoğan, 2023) and that designing learning tasks requires anticipating student reasoning and embedding learning goals (Paolucci & Stepp, 2021).

Overall, Abed and Michael's journey illustrates a shift from rule-following to rule-creating, a hallmark of teacher professional development. Their deliberate choices around parity balancing, alternating placement, and role reversals mirror best practices in classroom instruction that promote student reasoning, reflection, and justification. This intentional process not only advances their own understanding but also serves as a model for cultivating teacher identity, pedagogical agency, and the ability to design rich, student-centered mathematical experiences. Their full reflections and game designs are provided in Appendix 2 and Appendix 3.

Discussion

This study examined how engaging early-career mathematics teachers in the design of games and inquiry-based tasks supported their professional development, conceptual understanding, and pedagogical awareness. Through two detailed case studies, Nina and Visal's exploration of the Nim game and Abed and Michael's creative extension of the parity game structure, we uncovered patterns of reflective practice, strategic insight, and emerging teacher identity. These findings address our research questions about (1) how novice teachers engage with mathematical games as tools for creative task design, (2) what types of reasoning emerge during design, and (3) how these activities support the development of pedagogical insight and teacher identity.

From Solvers to Designers: Teacher Identity through Problem Creation

Addressing Research Question 2, both cases demonstrate how creating new mathematical games catalyzed a shift in role perception, from passive implementers to active designers. Nina and Visal moved beyond analyzing a familiar structure to generating and evaluating a new mathematical model, while Abed and Michael transformed a classic puzzle into two custom-designed, pedagogically rich games. In both instances, the teachers demonstrated epistemic agency, taking ownership of mathematical ideas and framing them with learners in mind. Such experiences are formative in building a professional teaching identity, marking a transition from task-following to curriculum innovation and pedagogical authorship.

The Role of Playful Creativity in Deepening Mathematical Insight

Responding to Research Question 1, the creative, game-based format provided teachers with an emotionally engaging and intellectually generative context for exploring mathematical ideas. Nina and Visal's inquiry into Nim prompted questions about strategy, abstraction, and generalization. Abed and Michael engaged in structural play that illuminated principles like invariance, symmetry, and fairness, and articulated these insights in explicitly pedagogical terms.

In both cases, playful yet purposeful design encouraged experimentation, pattern recognition, and structural reasoning, practices essential not only for doing mathematics but for teaching it effectively. These findings align with research showing the cognitive and affective benefits of mathematical play in teacher education (Silver, 1994) and reinforce systematic reviews highlighting that game-based learning enhances students' motivation and engagement (Vankúš, 2021). Practicing teachers also value games for precisely these qualities, engagement, reasoning, and fluency, which our novice teachers purposefully embedded in their designs (Bragg et al., 2021).

Moreover, as Pan et al. (2022) argue, well-designed learning games go beyond rote practice to promote reasoning, strategic thinking, and content integration. The strategic richness of games like Nim is well documented even in computational research: for example, Peres et al. (2024) show how small rule changes can dramatically alter optimal strategies, requiring adaptive, pattern-based reasoning. Such findings reinforce the value of inviting novice teachers to experiment with game modification, as this work demands precisely the creative, reflective thinking needed to design meaningful learning experiences. Similarly, research demonstrates that pre-service teachers who adapt or create games themselves engage more deeply with mathematical inquiry and develop flexible, innovative problem-solving skills (Applebaum, 2025).

Designing Tasks as a Bridge Between Problem-Solving and Teaching

Directly addressing Research Question 3, the design of games served as a conceptual bridge between problem-solving and instructional planning. Nina and Visal's model-building emerged organically from their attempt to solve the Nim game, showing how sustained problem-solving can lead to authentic problem-posing. Meanwhile, Abed and Michael's symmetric, parity-based gameplay positioned them as architects of learning experiences rather than consumers of pre-made tasks.

In both cases, task design became a rehearsal space for pedagogical reasoning, an opportunity to align mathematical structure with student engagement, challenge, and accessibility. This aligns with research emphasizing that effective problem-posing requires teachers to anticipate student understanding, select representations, and embed learning goals in meaningful contexts (Paolucci & Stepp, 2021). It also resonates with calls to integrate problem-posing into teacher preparation to cultivate autonomy, creativity, and instructional flexibility (Leavy & Hourigan, 2020; Chapman, 2012).

Supporting Novice Teachers as Mathematical Designers

These findings underscore the importance of providing novice teachers with structured opportunities to design original mathematical tasks, effectively addressing all three research questions. Through their game creation, Abed and Michael demonstrated how task design involves balancing constraints, anticipating learner responses, and embedding clear conceptual goals. Nina and Visal, in contrast, illustrated how inquiry into familiar problems can evolve into actionable pedagogical plans supporting student learning.

Prior research suggests that carefully structured game-design experiences can strengthen pre-service teachers' critical and creative thinking and help them envision how to integrate such activities in their future classrooms

(Applebaum, 2025). Just as in-service teachers use games flexibly for diverse pedagogical goals (Bragg et al., 2021), novice teachers benefit from learning to design tasks that deliberately support reasoning, problem-solving, and student engagement.

Both cases highlight the need for supportive environments that promote collaborative reflection, conceptual framing, and iterative feedback, helping early-career teachers bridge their own mathematical reasoning with effective classroom implementation. This is consistent with findings that structured, context-rich problem-posing tasks can help prospective teachers develop mathematical creativity at increasingly sophisticated levels, from rule-based algorithms to conceptual and theoretical reasoning (Emre-Akdoğan, 2023). Our findings resonate with Kar et al. (2025), who show that iterative cycles of posing, solving, and re-posing tasks can support pre-service teachers in developing flexibility, fluency, and pedagogical insight. This aligns with evidence from other fields; for example, science education research has shown that game design can deepen pedagogical content knowledge and encourage reflective practice (Botes, 2022). As Pan et al. (2022) recommend, embedding authentic content and conceptual goals into game design is critical, and teacher education should scaffold these choices to help novice teachers align mathematical reasoning with pedagogical intent.

These findings also echo Mallart et al. (2018), who found that pre-service teachers need explicit support to create problems that are realistic, appropriately challenging, and pedagogically sound. Structured opportunities for design, reflection, and feedback are essential for building this capacity. Furthermore, research shows that immersive game-design projects can enhance early-career teachers' confidence with technology integration, lesson planning, and strategies for engaging students (McColgan et al., 2018). Integrating these kinds of creative design opportunities into teacher education curricula can empower novice teachers to become not only skilled deliverers of content but also thoughtful designers of rich, meaningful learning environments.

Conclusion

This study investigated how engaging novice teachers in the creation and analysis of mathematical games can support the development of conceptual understanding, strategic reasoning, and pedagogical agency. Through two illustrative cases, Nina and Visal's reflective inquiry into the Nim game and Abed and Michael's creative construction of parity-based games, we addressed our research questions by demonstrating how early-career teachers move from passive problem-solving to intentional, student-centered task design.

Several key findings emerged. First, problem and game creation acted as a catalyst for developing teacher identity (Research Question 2). Both cases showed a shift from executing pre-designed tasks to authoring original mathematical experiences, marked by growing ownership, agency, and pedagogical vision. Second, the playful and exploratory nature of game design supported deep engagement with mathematical ideas as tools for creative task design (Research Question 1). Participants demonstrated curiosity, collaboration, and persistence, exploring strategies, structures, and student perspectives in meaningful ways. Third, the design process itself served as a bridge between problem-solving and teaching (Research Question 3). By reasoning structurally, anticipating student thinking, and aligning mathematical principles with pedagogical goals, teachers practiced essential instructional design skills.

These insights carry important implications for mathematics teacher education. Programs should integrate structured opportunities for novice teachers to create, adapt, and reflect on game-based or inquiry-based mathematical tasks, scaffolding the full process of problem posing, from initial situation analysis to final evaluation, to support reflective and strategic teaching practice (Baumanns & Rott, 2022). This recommendation aligns with evidence that iterative "pose–solve–pose" cycles can strengthen novice teachers' ability to design varied and conceptually meaningful problems (Kar et al., 2025). Such experiences help teachers connect mathematical content with pedagogical goals and anticipate student reasoning (Paolucci & Stepp, 2021). They can also scaffold teachers' progression through multiple levels of mathematical creativity, from procedural fluency to conceptual understanding and theoretical reasoning (Emre-Akdoğan, 2023). This aligns with calls to incorporate structured game-design tasks into teacher preparation (Botes, 2022), not merely as engaging activities but as vehicles for fostering reflective habits of mind, instructional innovation, and student-centered learning environments. Indeed, game-based learning has demonstrated positive impacts on students' motivation, engagement, and attitudes toward mathematics (Vankúš, 2021), reinforcing its value as a pedagogically rich context for developing teachers' design and reflection skills.

Similarly, research underscores the importance of supporting pre-service teachers in creating relevant, level-appropriate, and engaging tasks to build their confidence and competence as thoughtful, responsive educators (Mallart et al., 2018). By moving beyond drill-and-practice uses of games and emphasizing design that integrates mathematical content and reasoning, teacher education can promote more effective, conceptually rich teaching practices (Pan et al., 2022).Further research is warranted to expand and deepen these findings. Comparative studies could examine differences between novice and experienced teachers in task creation. Classroom-based research might investigate how students engage with teacher-designed games, while longitudinal studies could trace how early experiences with creative task design shape teachers' professional trajectories over time.

Ultimately, this study affirms that game design is not a peripheral enrichment activity but a meaningful pedagogical practice. By encouraging teachers to explore mathematics through playful invention, we help them deepen their own understanding while building the foundation for crafting engaging, conceptually rich learning experiences for their students. This transition, from solver to creator, from learner to designer, is at the heart of growing into a thoughtful, reflective, and empowered mathematics educator (Applebaum, 2025).

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Appendix 1. Nina and Visal: Reflection on NIM Game Analysis and Model Development

At the start, we carefully recorded the outcome of each game and identified patterns such as balance, symmetry, and repeatable advantages. We noticed that when we managed to maintain a balanced situation, where piles were similar or symmetrical, we consistently won. In contrast, entering an asymmetrical position without being able to correct it often led to a loss.

Game setup:
The game begins with three rows containing 3, 5, and 7 matches, respectively. Players take turns removing 1, 2, or 3 matches from a single chosen row on their turn. The objective is to force the opponent to be the one who takes the last remaining match.
Initial setup:
Row 1: 3 matches; Row 2: 5 matches; Row 3: 7 matches.
Player A: Takes 3 from Row 3 → (3, 5, 4)
Player B: Takes 2 from Row 2 → (3, 3, 4)
Player A: Takes 1 from Row 3 → (3, 3, 3)
This move sequence creates a completely symmetrical state, a key point in Nim. The next player is forced to break the symmetry, which the opponent can then restore for a strategic advantage.
Player B: Takes 1 from Row 1 → (2, 3, 3)
Player A: Takes 2 from Row 1 → (0, 3, 3)
Player B: Takes 1 from Row 3 → (0, 3, 2)
Player A: Takes 1 from Row 2 → (0, 2, 2)
Now, it does not matter what Player B will take; he will lose the game.
Through our simulations, we formulated four simple yet effective principles:
Keep a balance. Ensure that after each turn, the stacks remain in a state where the opponent cannot easily create an advantage.
Weaken the dominant row. Reduce larger rows to approach the size of others, while avoiding giving the opponent an advantage.
Dictate the opponent's moves. Aim to leave positions with only one “correct” move for the opponent, making their response predictable.
Master the final stages. When a few rows remain, maintaining balance becomes critical to ensure victory.
One of the most meaningful parts of our work was how our perspective on mathematical games changed.
In the beginning, we saw NIM as just a favorite puzzle game, but as we dug deeper, we realized it is a rich platform for mathematical thinking, analytical skills, and understanding strategy.
Our model not only improved our in-game success but also served as an exercise in abstract thinking and theory-building, boosting our confidence in formulating ideas from real experience.
As we developed our model, new questions arose naturally from experience and analysis:

- Can we translate our model into a simple algorithm for high school students without needing formal binary explanations?
- How can we adapt the model for more complex setups, like larger or inconsistent piles?

- What is the right balance between strategic planning and intuition, especially for beginners?
- How can we measure whether our model fosters high-level mathematical thinking, such as reflection, planning, and self-regulation?
- Could this approach, observation, trial and error, pattern recognition, be applied to other math topics like geometry, functions, or open problems?

These questions highlight future directions for teaching and research and deepen our understanding of the link between play, mathematical thinking, and active learning.

The entire process, especially writing this work, offered us a meaningful experience in independent, collaborative, and exploratory learning.

True collaboration in choosing research directions, building documentation tables, and analyzing results helped us combine perspectives and share responsibilities. Instead of solving exercises mechanically, we faced open-ended questions, uncertainty, and the challenge of concluding an evolving process. We had to ask new questions at each step, test assumptions, and experiment with variations. Moments of frustration and disagreement became drivers of learning.

We also faced challenges:

- Making strategic principles intuitive without binary calculations required us to develop new explanations.
- Balancing accuracy and accessibility: Ensuring the model was both correct and understandable was not easy.
- Generalizing beyond one example: We worked to test many variations to make our model robust and flexible.
- Reflecting on this journey, we realize we learned much more than simply how to play Nim.

We learned to:

- Analyze mathematical phenomena qualitatively, not just numerically.
- Think strategically and dynamically about responses and influence.
- Turn personal research into a model that can benefit students, teachers, and researchers.

We didn't just "read about the model", we built it ourselves, step by step, through experience, mistakes, and understanding.

Looking Ahead

This work feels like a beginning rather than an end. We're left with ideas to:

- Integrate computer graphics to visualize strategy.
- Teach the model to students and test its impact.
- Compare our model with existing ones from research.

We feel a sense of educational mission: to turn what we developed into a real teaching tool that helps students see mathematics as alive, dynamic, and accessible. For us, this project was much more than a seminar paper; it was an experience in exploration, creativity, and learning. We're proud of the model, the questions it generated, and the path we traveled together.

Appendix 2. Abed and Michael: The Two-Round, Role-Reversal Parity Game

We designed the Two-Round, Role-Reversal Parity Game to turn the familiar "take-from-ends" puzzle into a more dynamic and engaging strategic challenge. In our version, one of us chooses the length of the row (always an odd number to keep the parity sums meaningful), while the other composes the entire sequence of numbers but is allowed a single swap at the end. This setup makes both of us think carefully about how the numbers are arranged and how to influence the final sums in odd- and even-indexed positions.

The rule about selecting who goes first after reviewing the sequence adds another layer of strategy. It forces the picker to analyze the parity sums before deciding, while the chooser must anticipate this when composing the row. The single allowed swap is a critical move; it creates a moment of tension and requires us to judge which change will have the biggest impact on the parity balance.

We also wanted to make sure the game was fair and balanced, so we included a second round with reversed roles. That way, each of us experiences both perspectives, planning and reacting, composing and adjusting. This role-reversal structure helped us reflect on different strategies and understand parity reasoning more deeply. Overall, we found that alternating control over setup and play forced us to think in new ways, stay engaged, and appreciate the subtleties of invariant structures in a playful but challenging setting.

Game 1: Two-Round, Role-Reversal Parity Game

- The First Player declares an odd row length (e.g., 9–19).
- The Second Player records exactly n integers (e.g., 13).
- The First Player chooses who moves first.
- The Second Player may swap the places of two integers.
- Players alternate picking from ends.
- Roles swap for Round 2.
- The highest combined score wins.

Example Playthrough

Round 1

Alice chooses n=11.

Bob writes the following integers: 4, 9, 1, 7, 2, 8, 3, 6, 5, 10, 11.

Alice chooses Bob to move first.

Bob swaps the positions of the numbers 1 and 7: 4, 9, 7, 1, 2, 8, 3, 6, 5, 10, 11.

Play result: Bob wins 43–23.

Round 2 (Roles swap)

Bob chooses n=9.

Alice writes the following integers: 2, 5, 3, 7, 10, 4, 1, 12, 6.

Bob chooses Alice to move first.

Alice swaps the positions of the numbers: 10 and 4: 2, 5, 3, 7, 4, 10, 1, 12, 6.

Play result: Alice wins 36–14.

Final Outcome: Alice's total = 23 + 36 = 59. Bob’s total = 43 + 14 = 57. Alice wins by 2 points.

This example shows how parity analysis and the chooser’s single swap become central to the strategic depth of the game.

Appendix 3. Abed and Michael: The Alternating-Placement Parity Game

For our Alternating-Placement Parity Game, we wanted to push the idea of shared control even further by making both players actively responsible for building the number sequence. Instead of one person writing the entire row, we took turns placing numbers from a declared range until we had an odd-length sequence. This meant that every placement became a strategic decision that could shift the parity sums in real time.

We found that this alternating placement made the game much more interactive and collaborative. Each move required us to track how the sums of odd- and even-indexed positions were evolving, forcing us to think ahead and adapt our strategy on the fly. It wasn't enough to have a plan at the start; we had to respond to every choice the other player made.

The chooser's role in deciding who goes first added another level of parity analysis. We had to figure out which sum was stronger and predict how the take-from-ends phase would play out under optimal moves. Swapping roles for the second round ensured that both of us got to experience the challenge from both perspectives, planning and reacting, constructing and analyzing.

Overall, this design helped us see how important parity reasoning is in strategic play and gave us insight into how such a task could engage students in thinking flexibly, anticipating consequences, and explaining their choices. We believe this version encourages ownership, creativity, and reflection in a way that makes the mathematical principles behind the game much more tangible and accessible.

Game 2: Alternating-Placement Parity Game

- The First Player declares an odd row length (e.g., 9–19) and range (e.g., [1,20]).
- The Second Player chooses who moves first
- Players alternate placing integers (that belong to the declared range);
- Players alternate picking from ends.
- Roles swap for Round 2.
- The highest combined score wins.

Example Playthrough

Round 1

Alice chooses n=9, range [1–20].

Alternating placement: Alice: 5, Bob: 17, Alice: 3, Bob: 14, Alice: 7, Bob: 12, Alice: 9, Bob: 2, Alice: 11.

Final row: [5, 17, 3, 14, 7, 12, 9, 2, 11].

Bob chooses to go first and wins 48–32.

Round 2 (Roles swap)

Bob chooses n=11, range [1–15].

Alternating placement: Bob: 4, Alice: 8, Bob: 2, Alice: 13, Bob: 1, Alice: 7, Bob: 10, Alice: 5, Bob: 3, Alice: 11, Bob: 6.

Final row: [4, 8, 2, 13, 1, 7, 10, 5, 3, 11, 6].

Alice chooses to go second and wins 44–26.

Final Scores: Alice = 32 + 44 = 76, Bob = 48 + 26 = 74. Alice wins by 2 points.

We also found that if either end of the row (in this case, a_1 or a_{11}) is larger than the difference between the sums of the odd- and even-indexed positions, then the chooser's parity-based strategy for selecting turn order cannot fully offset that imbalance.