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From Mathematical Micro- to Macro-identities: How Mathematical Authority Emerged from Moment-to-moment Positioning

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Abstract

This paper aims to better understand how students' moment-to-moment micro-identities thicken into macro-identities, focusing specifically on one student, Tia, as a mathematical authority. Our analysis involved identifying the ideas that students proposed, how they were responded to, and the mathematical micro-identities (Wood, 2013) at play during these interactions. Findings (1) reveal the relationship between how this teacher-researcher and her students proposed and responded to ideas and how they were positioned into mathematical micro-identities; (2) confirm Wood's (2013) findings regarding the nature of positioning into micro-identities; and (3) build on Wood's framework to describe how micro-identities can be nuanced, or specialized. Our findings help explain how moment-to-moment positioning contributes to the development of one student's macro-identity as a mathematical authority.

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Introduction

This study was inspired by Engle and Conant's (2002) seminal paper on productive disciplinary engagement that focused on one case study of a group of students' discussions about orcas. Engle and Conant's (2002) research uncovered how one student in particular was able to wield his authority among his peers during these discussions. We build upon their work to investigate the relationship between macro- and micro-identities. We use Wood's (2013) framing of micro-identities when she analyzed one student in one lesson to explore the various micro-identities they enacted. For our work specifically, we explored the following research question: *How does the thickening of moment-to-moment micro-identities contribute to the development of a macro-identity?*

Mathematics education literature has broadened from traditional research focusing solely on students' mathematical comprehension to include discussions about students' identities and their effect on how students navigate various experiences within the mathematics classroom (Cribbs et al., 2021; Philip & Gupta, 2020). More specifically, this paper considers how students' mathematical experiences contribute to the development of their mathematical identities. Froschl and Sprung (2016) defined mathematical identity as, "a person's beliefs, attitudes, emotions, and dispositions about mathematics and their resulting motivation and approach to learning and using mathematics knowledge" (p. 320). Many researchers have drawn attention to student narratives about their self-perception as mathematical learners and doers (e.g., Anderson et al., 2018; Black et al., 2010; Boaler et al., 2018, 2000), taking into account the fact that mathematical identity is, as McCarthey and Moje (2002) argued, entirely dependent on context. Student accounts of their mathematics experiences and subsequent dispositions towards the subject offer researchers insight into how students navigate doing mathematics and provide (or deprive) students agency in constructing mathematical identities. However, these constructions are not entirely determined by students alone. According to Langer-Osuna and Esmonde (2017), the stories students tell about themselves relative to their mathematics experiences help them reveal different mathematical identities that are based on the interactional relationship they have with others or the interpersonal connections one has "with others during interactions, in particular through talk" (p. 639).

Langer-Osuna and Esmonde (2017) not only focused on the stories people tell about themselves, they also discussed how others craft stories about each other. In this paper, we consider how a teacher-researcher and students responded to five students' mathematical contributions during one mathematics lesson of a Classroom Teaching Experiment (CTE) and how during these interactions students were positioned into particular mathematical micro-identities (Wood, 2013). Because mathematical identity is associated not just with how students position themselves, but how they are positioned by others (Langer-Osuna & Esmonde, 2017), this research needs to be comprehensive, including how mathematics teachers can also influence students' mathematics experiences.

In Brizuela et al.'s (2023) paper on the construction of students' mathematical authority, the authors examined first grade mathematics students' discourse to explore how they were positioned by their peers as mathematical authorities, and how this positioning was constructed over time. The fundamental purpose behind their research was to examine students as the authority bearers through analyses of students' mathematical contributions during whole-group discussions. With Engle et al.'s (2014) theory of influence undergirding the analysis, their focus was on positioning, and not identity, but the results reveal one particular student's, Tia, identity as a mathematical authority. We argue that Brizuela et al.'s (2023) results reveal Tia's macro-identity as a mathematical authority among her peers. Macro-identities are "relatively stable, long-term constructions of who a person is" (Wood, 2013, p. 776). While Tia's identity as a mathematical authority may not be as macro- and long-term because it does not span multiple years of schooling, for instance, it is evident, through Tia's role as a discussant and Tia's peers' responses to their contributions, that this is how Tia is thought of in this context.

In this paper, we respond to Wood's (2013) call to better understand how moment-to-moment micro-identities thicken into macro-identities by using the same dataset as Brizuela et al. (2023) and investigating a different dimension; namely, we consider students' moment-to-moment mathematical micro-identities (Wood, 2013) with an aim to shed light on the thickening of Tia's macro-identity as a mathematical authority among her peers. Figure 1 represents our conceptualization of micro-identities thickening into a macro-identity that changes shape over time.

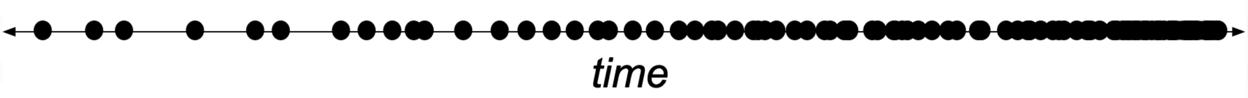

Figure 1. A Representation of the Thickening of Micro-identities to a Macro-identity

Conceptually speaking, identity is fluid and ever-changing, and it influences how both teachers and students relate to mathematics and to one another. Much of the research on mathematical identity centers on macro-identities. We use Wood's (2013) concept of mathematical micro-identity that suggests that mathematics learners can take up identities and be positioned into identities which are "enacted in a moment of time" (p. 778), and that fluctuate depending on the context, at times over the course of a lesson. In this paper we consider one lesson during the CTE to identify how the teacher-researcher positioned students into particular micro-identities and how students enacted those micro-identities. We focus on this lesson because it involved a critical moment when Tia's macro-identity as a mathematical authority was vocalized and enacted (see Brizuela et al., 2023). Moreover, we focus on the micro-identities of five focal students who were identified as key players in the thickening of Tia's macro-identity to address our research question.

Theoretical Framework

Mathematical Micro-Identities and Interactional Positioning Theory

To study how mathematical micro-identities are constructed, Wood used interactional positioning theory (van Langenhove & Harré, 1999), focusing on how people are labeled or identified through interactions with one another, as well as how these labels shift as interactions shift. While position has different connotations, such as being physical, or describing one's role in terms of practice, positioning here refers to the ways in which people interact through discourse and actions to structure social dynamics. Positioning and identity are not equivalent terms; rather, positioning involves the interactions that lead to a particular placement, or identity. van Langenhove and Harré (1999) described interactions as frequently occurring verbal or non-verbal communication between people. Further, the positions people take on as a result of their interactions may come about through their own labeling, or through that of others.

Similar to how interactions shift, Wood (2013) posited that mathematical micro-identities can shift frequently, depending on the context, making them variable instead of static. Wood (2013) observed three mathematical micro-identities among the student participants in her study: the mathematical explainer (a student explaining their reasoning in response to teacher inquiry), the mathematical student (a student capable of learning), and the mathematical menial worker (a less than mathematically capable student). In Wood's (2013) work mathematical explainer and mathematical student fall under a broader umbrella of mathematical capability. Moreover, Wood argued students shift between them at any given time.

Prior Research

Influence Theory

Engle et al. (2014) explored how influence was developed and applied through a case study of fifth grade science students' interactions during small group discussions from Engle and Conant's (2002) work investigating productive disciplinary engagement. Engle et al. (2014) defined influence as the "degree to which an individual becomes socially positioned as having convinced others to take up his or her ideas" (p. 250). Engle et al. (2014) observed how different students in the group gained and negotiated authority amongst their peers in a discussion about the classification of orcas as dolphins or whales. They focused specifically on one student, Brian, who gained undeserved influence despite the low quality of his contributions.

To probe how influence was negotiated in Engle and Conant's (2002) study, Engle et al. (2014) used proposal negotiation units (PNUs), a unit for modeling the components of influence. The first phase of a PNU is the proposal bid, where an idea is proposed. The second is when the proposal is either taken up, changed, ignored, or rejected. In analyzing the eight episodes described in Engle and Conant (2002), Engle et al. (2014) developed a five component framework for influence theory which Brizuela et al. (2023) used as a foundation for their analysis of student interactions. In the same way that Engle et al. (2014) built on Engle and Conant (2002), we build on Brizuela et al. (2023). Specifically, we analyze a subset of Brizuela et al.'s (2023) data, one lesson, which is deemed critical by the authors; we analyze it by building on Engle et al.'s approach (2014) to studying influence and use Wood's (2013) definitions to understand the teacher-researcher's positioning of students into mathematical micro-identities. Our goal is to investigate the link between the thickening of moment-to-moment micro-identities of Tia and her peers and Tia's macro-identity as a mathematical authority among her peers.

Teachers Positioning Students into Mathematical Identities

Yamakawa et al. (2009) explored the relationship between teacher positioning and mathematics identity construction. The authors collected ethnographic data in a classroom of 17 third grade students and their mathematics teacher over the course of four months. More specifically, Yamakawa et al. (2009) focused their attention on two students, Ophrah and Pulak, to examine which mathematical identities their teacher positioned them into based on interactions during whole-group discussions. The authors found that the teacher made specific commentaries on these students' mathematical contributions that reflected a particular type of mathematical identity she positioned them into. For instance, in her response to Ophrah's answer to a problem, the teacher remarked, "And this is something some of you might want to write down" (p. 10) to the entire class, positioning Ophrah as a student with "useful information" (p. 8) to her and her peers. The teacher frequently revoiced Ophrah's mathematical statements in order to help her communicate her ideas to her classmates. Consider also her response to Pulak employing a regrouping strategy as being "too difficult for most…students to understand" (p. 11); in this comment, the teacher positioned Pulak as an advanced mathematics student, a position she maintained for him across the data. The teacher rarely revoiced Pulak's strategies out loud, often speaking directly to him about his high level of mathematical proficiency. Yamakawa et al. (2009) concluded that in the teacher taking vastly different interactional approaches with Ophrah and Pulak, she was continuing a sort of storyline in which she positioned each student into the mathematical identity she felt was most appropriate based on their contributions during whole-group discussions.

We add to Yamakawa et al.'s (2009) study in also focusing on the interactions between a teacher-researcher and her students during one lesson in order to examine the relationship between interactions and the micro-identities that the students are positioned into. In doing so, we elaborate on the findings of Brizuela et al. (2023) who described how one student became a mathematical authority over time, and show how the positioning of micro-identities coupled with interactions that lead to positioning contributed to the thickening of that one student's macro-identity as a mathematical authority.

We explore the following research question: *how does the thickening of moment-to-moment micro-identities contribute to the development of a macro-identity?* To answer this question, we analyzed Lesson 12 from Brizuela et al. (2023), which included a "pivotal moment" (p. 7) in which one student's, Tia, mathematical authority became apparent through another student's, Azaiah, trust in Tia's (incorrect) mathematical idea.

Method

Participants

The participants in this study were the same as those in the Brizuela et al. (2023) study, mirroring the design in Engle et al. (2014) who built on Engle and Conant's (2002) study. Our participants include a teacher-researcher, a university professor in education at the time, and 22 first grade students who attended a public school in the Northeast United States. At the time of the study, 1% of the students at this school identified as White, 47.3% identified as African American, and 46.8% as Latine. The year that these data were collected, 52% of third grade students were considered "warning/failing" on a state mandated standardized test in mathematics, and an additional 40% were considered "needing improvement." We focus on a subset of five of these students who were identified as the focal students in Brizuela et al. (2023). These students were identified as focal students because they were part of a "pivotal moment" (p. 7) in which these students evaluated each others' mathematical ideas.

The Classroom Teaching Experiment

The CTE was designed to engage students in early algebraic thinking. The 16 CTE lessons involved reviewing geometric pattern sequences to introduce the idea of co-varying relationships, and then reasoning about functions in the form of $y=mx$ and in the form of $y=x+b$. Lesson 12 from the CTE is included in the Appendix.

All CTE lessons involved a problem scenario, which the teacher-researcher presented to the students before opening up a whole-group discussion to explore their understandings. When reasoning about functions, students were challenged to identify a verbal rule, or expression, to represent the relationship. In order to capture students' ideas, the teacher-researcher typically used chart paper or a white board. The CTE lessons were supplemental to students' mathematics instruction with their regular classroom teacher.

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Data Sources and Selection

The data analyzed in this study come from one lesson (#12) which was designated by Brizuela et al. (2023) as a significant lesson in terms of the distribution of mathematical authority. The goal of our data selection was "not representativeness but rather the…immersion in the details of a complex moment" (Nemirovsky et al., 2013, p. 385). We used microgenetic analysis to focus solely on Lesson 12 with an aim to understand the moment-to-moment micro-identities at play as one student, Tia, was established as a mathematical authority in this context. In this lesson, students discussed how to represent the ages of two sisters, Janice and Keisha (see Brizuela et al., 2015). In the situation being discussed, Keisha is three years older than Janice.

Data Analysis

The analysis involved identifying (1) PNUs according to Brizuela et al. (2023) and (2) mathematical micro-identities according to Wood (2013).

(1) Tracking students' mathematical ideas and responses to their ideas (PNUs). First we identified students' mathematical ideas as well as the teacher-researcher's responses to those ideas. Using Brizuela et al.'s (2023) definition, an idea is "a thought or suggestion which can be articulated orally or in writing, [and this is done using] the smallest possible unit" (p. 14), meaning that differing and/or conflicting ideas could be proposed by the same student, in the same moment, or among other children in the same classroom. The types of ideas proposed may vary, as they may be contextually dependent on the activities or tasks at hand. Further, ideas could also be used to create new ideas, forgotten about, or re-introduced later on. A speaker could also propose several ideas, but responses to these ideas could treat them as one single idea. To track ideas, the research team carried out a line-by-line review of the lesson transcript, keeping track of any interactions between students, specifically focusing on interactions where students proposed ideas.

Once ideas were flagged, the research team coded the various ways in which students' ideas were responded to using the codes from Brizuela et al. (2023) (see Table 2) that were based on Engle et al.'s (2014) PNUs.

Table 2. Codes developed by Brizuela et al. (2023)

PNU part	Code Description	Example
Bid	***An idea is proposed*** when a student or adult shares something (orally, gesturing, writing) they are thinking. It does not have to be an idea that has not previously been shared, nor does it have to be shared by students alone. It can also be stated as a question.	Student: Yeah there's one nose on a dog. *Lesson 9, line 39.* Nyah: If four dogs are there, that means there's gonna be four noses. *Lesson 9, line 46*
Response	***An idea is accepted*** after a child proposed an idea and someone responded indicating that they agree with that child's idea. Sometimes it is subtle, and other times it is lengthy and clearly articulated. The acceptance does not need to happen immediately after the child proposed the idea.	Nyah: 'Cause they [dogs] each have one [nose]. Teacher-researcher: Did you hear that? They each have one. *Lesson 9, lines 48-49*
	An idea is responded to (neutrally) after a child proposed an idea and someone responded to it without accepting or rejecting it. The response might simply be an acknowledgement, or it might be a push for more information (e.g., justification). The response does not need to happen immediately after the child proposed an idea. This is often done by the teacher-researcher. The code is typically not used in conjunction with other codes because, for instance, if an idea is accepted then it cannot also be responded to neutrally.	Teacher-researcher: So, Nyah says she does not agree, Tia. Why don't you agree Nyah? *Lesson 9, line 43*
	An idea is explicitly rejected after a child proposed an idea, and someone responds, stating that they disagree with the child's idea. The rejection does not need to happen immediately after the child proposed an idea.	Teacher-researcher: Oh. Ok. So Tia says if there are four dogs, there are eight noses. Teacher-researcher: Do you agree or not? Nyah: I don't agree. *Lesson 9, lines 35, 40, 42*
	An idea is not taken up after a child proposed an idea, and someone avoids taking up the child's idea by either not acknowledging it or making a statement that suggests they are questioning the accuracy of the child's idea. It often involves a transition towards talking about a different idea as well. We do not refer to this code as a child's idea is not accepted because that could suggest it	Tia: So, if we make…the y 23 and then z 28, it would, it would go with the dogs noses. Teacher-researcher: Now um… which of these, which of these would match up the dogs and ears story? I want more hands, I see very

PNU part	Code Description	Example
	was rejected, and we would be inferring that the child's idea was intentionally not taken up when that is not necessarily the case in these instances. This code includes possible implicit rejections.	few hands, I think Leo can put his hand up and Vana and Judy. *Lesson 9, lines 67-68*

It is important to note here that several important aspects of the exchanges between the students and teacher-researcher that were visible in the lesson video recordings are not as clear in the transcripts. For instance, inflections and tones of voice are not captured in the transcripts, which could potentially make a difference between coding an utterance as a neutral response, an implicit acceptance, or a subtle challenge to a student's proposed idea. This was addressed by having coders watch the videos while reading the transcripts. The teacher-researcher, who was the teacher in this lesson, also reviewed the analysis. Two team members coded the Lesson 12 transcript together and discussed when to apply the codes in Table 2 and used the agreement by consensus approach described in Syed and Nelson (2015).

(2) Coding of students' mathematical micro-identities. To continue with a microgenetic approach that details a complex moment, instead of representing many moments (Nemirovsky et al., 2013), we carried out another review of the lesson transcript, keeping track of any moments when students were positioned into or enacted mathematical micro-identities. Table 3 includes Wood's (2013) micro-identities: (1) *Mathematical explainer*; (2) *Mathematical student*; and (3) *Mathematical menial worker [Although we used Wood's (2013) original micro-identities, we believe it is important for researchers to consider taking an asset-based approach to categorize students' mathematical identities. For example, the "menial worker" label could be reconsidered from an asset-based perspective, e.g., "emerging learner" or "developing learner".]*.

Table 3. Mathematical Micro-identities

Mathematical Micro-identity	Definition
Mathematical explainer	This student is "responsible for articulating an argument for [their] solution to the task" (Wood, 2013, p. 789).
Mathematical student	This student is "mathematically capable" (p. 790).
Mathematical menial worker	This student is "less than mathematically capable" (p. 792).

To code for Wood's (2013) micro-identities, at each turn of talk, we asked, "who is being spoken to?" If it was clear that someone was being spoken to, we asked, "how is this person being positioned? Are they being asked to explain (i.e., *mathematical explainer*), to contribute mathematically without explaining (i.e., *mathematical student*), or to contribute in a non-mathematical way (i.e., *mathematical menial worker*)?" We then considered how this person responded. Did they enact the micro-identity in which they were positioned? In other words, did they explain a mathematical idea? Did they share a mathematical response without explaining it? Or did they contribute in a non-mathematical way?

To confirm the reliability of the coding of students' mathematical micro-identities, two members of the research team reviewed the analysis. If they disagreed on a code they discussed the code until agreement was met, and if needed they involved another member of the research team for third-party resolution (Syed & Nelson, 2015).

Results

We sought to investigate the relationship between macro- and micro-identities. In doing so, we identified a relationship between how ideas were proposed and responded to and how individuals were positioned into Wood's (2013) mathematical micro-identities. Through our analysis, we observed nuanced micro-identities and in turn built on Wood's (2013) framework by identifying specialized micro-identities, which are in Table 4.

Table 4. Mathematics Micro-identity Specializations

Mathematics micro-identity specializations	
Correct	When someone agrees, using words and/or gestures, with another person's mathematical idea or claim.
Incorrect	When someone disagrees, using words and/or gestures, with another person's mathematical idea or claim
Collaborating	When students work together on proposing or responding to mathematical ideas.
Discussing	When someone evaluates (agrees or disagrees with) another person's mathematical idea or claim.

We also confirmed Wood's (2013) findings regarding the nature of positioning of micro-identities. Here we elaborate on each of those findings.

The relationship between ideas and positioning. When students were positioned into *mathematical student* and *mathematical explainer* micro-identities, ideas were proposed and responded to. During these moments of positioning, ideas were a currency exchanged between the teacher-researcher and students. The typical exchange involved the teacher-researcher positioning a student or students as Wood's (2013) mathematical students or mathematical explainers by asking a question. The question was usually an open question, suggesting that she was seeking an explanation, and thus students were positioned as *mathematical explainers*. After the teacher-researcher posed a question, a student typically enacted Wood's (2013) micro-identity of *mathematical student* or *mathematical explainer*. This occurred through a bid or a proposal, which is the first part of a PNU. The teacher-researcher typically responded to the students' idea, which was the second part of the PNU, which often involved additional positioning into Wood's (2013) micro-identities. Sometimes the student was positioned into the same micro-identity, but sometimes the micro-identity fluctuated. We share an excerpt with several examples of these exchanges in Table 5.

In the following example the class was discussing how to represent the ages of two sisters, Janice and Keisha (see Brizuela et al., 2015; Brizuela et al., 2023). The teacher-researcher positioned Zyla as a *mathematical student* and

Tia as a *mathematical student* or *explainer* (i.e., based on the data we have, the teacher's intention is unclear). When doing so, the teacher-researcher was also responding to the students' ideas, in this instance, neutrally, until she accepted an idea at the end of the excerpt. That is, until the end, the teacher-researcher did not indicate to the student whether they were correct or incorrect, rather she simply moved the conversation forward, in many cases by requesting that the student "show" their idea.

We also observe a specialized *mathematical explainer* which we refer to as *collaborating mathematical explainer* (see rows 11 and 12), in this example.

Table 5. Lesson 12 Transcript

	Speaker	Utterance	Wood's Micro-identities (2013)	PNU (Brizuela et al., 2023)
1	Teacher-Researcher	Zyla, could you come up and show us some way of thinking of what you were saying, "she could be any age"? Or how did you say it?	positions Zyla as mathematical explainer	an idea is responded to (neutrally)
2	Zyla	I said "it could be any number because you didn't tell us yet"	enacts mathematical explainer	an idea is proposed
3	Teacher-Researcher	Could you show, then, a way of showing Janice's age right there?	continues to position Zyla as math explainer	an idea is responded to (neutrally)
4	Zyla	[draws on board]	enacts mathematical explainer	no code
5	Teacher-Researcher	So how could you—how would we know—oh, she could be four? That's what Ty said, right? Ok. So, other uh, other questions or ideas about the problem?	positions students, in general, as mathematical students or explainers	an idea is responded to (neutrally)
6	Teacher-Researcher	Questions or ideas about what the problem is telling us about Janice and Keisha. Umm, Tia. No, no, but stay sitting and tell us. Yeah. Listen to Tia.	positions Tia as a mathematical explainer and student	no code
7	Tia	She could be—she could be any number because—because Keisha is three—is three years older than Janice.	enacts mathematical explainer	an idea is proposed
8	Tia	If she's twenty and… she's twenty,	continues enactment	an idea is proposed

	Speaker	Utterance	Wood's Micro-identities (2013)	PNU (Brizuela et al., 2023)
		Janice can be—Janice can be… nineteen. And then—and if—and if… and if Keishas three—three years older than—than Janice, then… then she's twenty-three.		
9	Teacher-Researcher	If she's twenty, then she's twenty-three? If Janice is twenty, Keishas twenty-three?	positions Tia as mathematical student	an idea is responded to (neutrally)
10	Tia	If Janice is—if Janice is nineteen, then Keisha, if she's three years older than her—	enacts mathematical explainer	an idea is proposed
11	Teacher-Researcher	If she's nineteen, twenty…	positions Tia as a collaborating mathematical explainer and enacts collaborating mathematical explainer herself	an idea is responded to (neutrally)
12	Tia	Twenty-one, twenty-two, twenty…	enacts collaborating mathematical explainer	an idea is responded to (neutrally)
13	Teacher-Researcher	…twenty-two. She's—so if Keisha—if Janice is nineteen, Keishas twenty two? Ok. So I heard two times already, "she could be any number." What other ways of showing any number do you know?	positions Tia as mathematical explainer; positions students, in general, as mathematical students	an idea is accepted

The identification of specialized micro-identities. Throughout lesson 12, we observed the teacher-researcher positioning students into Wood's mathematical micro-identities and students enacting these micro-identities. We also noticed that these micro-identities became specialized in some instances. For example, in the excerpt above, there were moments when the teacher-researcher and student worked together to explain a mathematical idea, illustrating the *collaborating mathematical explainer* micro-identity. This typically involved the teacher-researcher leading the student to respond in specific, mathematical, ways.

In Lesson 12, we observed four instances of the teacher-researcher positioning a student as both a *mathematical student* and a *mathematical explainer*. Three of those instances involved Leo, and one involved Tia. We observed five instances of the teacher-researcher positioning a student as a *collaborating mathematical explainer* or the student enacting *collaborating mathematical explainer.* We only observed Tia and Zyla be positioned into or enact this micro-identity.

In addition to *collaborating*, we observed that when students were positioned as *mathematical students* or *mathematical explainers*, according to Wood's (2013) definitions, they were sometimes simultaneously positioned as *correct* or *incorrect*, or as *discussing* or *evaluating* another person's idea. Additionally, in this lesson, we observed students collaborating through what they described as copying; we consider this to be the *collaborating* specialization of *mathematical student* or *mathematical explainer*.

The following transcript (see Table 6) illustrates how the teacher positioned Zyla and Tia as *correct mathematical explainers* and *discussing mathematical explainers*. In this exchange, Zyla is referring to another lesson in which they had discussed representing an unknown number of candies in a box using a variable. She is using this idea in the girls' ages context.

Table 6. Lesson 12 Transcript

Row	Speaker	Utterance	Wood's Micro-identities (2013)	PNU (Brizuela et al., 2023)
1	Zyla	Because… it's just like the second grader, but it's a different letter.	enacts mathematical explainer	an idea is proposed
2	Teacher-Researcher	Oh! Could you remind us of the second grader? Yeah, I told you about Lisa a couple—last week, right?	positions Zyla as mathematical explainer	an idea is responded to (neutrally)
3	Zyla	Yeah, it was…	enacts mathematical explainer	no code
4	Teacher-Researcher	Z.	positions Zyla as collaborating mathematical explainer	an idea is proposed
5	Zyla	Z plus two.	enacts mathematical explainer	an idea is proposed
6	Teacher-Researcher	Mmm for the candy box.	positions Zyla as collaborating mathematical explainer	no code
7	Zyla	But this time—this time I—I just said J plus three, because… because she's three months—she's three years older.	enacts mathematical explainer	an idea is proposed

Row	Speaker	Utterance	Wood's Micro-identities (2013)	PNU (Brizuela et al., 2023)
8	Teacher-Researcher	Oh. Did you hear that? I—you know what I like about what Zyla is saying? Well, what all of you are saying. But, you're picking up on what other people said before, and thinking about how to use that new idea.	positions Zyla as correct mathematical explainer	an idea is responded to (neutrally)
9	Teacher-Researcher	So Zyla remembers that I shared with you last week about um, Lisa. Ms. Benson knows Lisa.		no code
10	Teacher-Researcher	I do.		no code
11	Teacher-Researcher	And Lisa said that the candy boxes would be Mary would have Z plus two. And so Zyla is saying, well I'm using that idea and I'm saying then Keisha has to be J plus three.	positions Zyla as correct mathematical explainer	an idea is proposed
12	Teacher-Researcher	Which I think is the same thing that—that you were saying uh Nyah. It's like what you were saying. A little differently. Right? [Nyah nods] Ok. [Tia shakes her head] You are disagreeing Tia.	Tia continues enacting discussing mathematical student; the teacher-researcher positions Tia, Zyla, and Nyah as a discussing mathematical explainer	an idea is responded to (neutrally)
13	Tia	Because I think it's—I think it's L because—because J, K, L.	enacts discussing mathematical explainer	an idea is proposed
14	Teacher-Researcher	Ok. I'll—I'll put it down here, as your idea. I think that um, I think you're counting off from different places, that's why you're getting different letters.	positions Tia as discussing mathematical student	an idea is responded to (neutrally)
15	Tia	J, K, L, M, N, O, P.	enacts mathematical explainer	an idea is proposed
16	Teacher-Researcher	I think, uh you're counting from J, K, L. And I think Nyah is doing	positions Tia, Zyla, and Nyah as discussing	an idea is responded to

Row	Speaker	Utterance	Wood's Micro-identities (2013)	PNU (Brizuela et al., 2023)
		K, L, M. You're just starting at different places. So I can't say that one is better than the other. Ok?	mathematical explainer	(neutrally)
17	Teacher-Researcher	Thank you for sharing all your ideas.		an idea is responded to (neutrally)
18	Student	I didn't get a chance/		no code
19	Teacher-Researcher	Are you—why didn't you get a—do you wanna get a chance now? Well you didn't get a chance now but you got a chance before, working at the tables.		no code
20	Teacher-Researcher	You were the first one who said four. Do you remember that? Yes. So you did get a chance.		no code
21	Azaiah	I—but I don't [unintelligible]. I disagree with Tia/	positions Tia as incorrect mathematical student and enacts discussing mathematical student	an idea is explicitly rejected
22	Teacher-Researcher	Ok.		no code
23	Azaiah	/because I think—I think you should—I—because Zyla said she—she—she's… no, I mean Nyah was saying about M and Tia—and Tia was saying about L/	enacts discussing mathematical explainer	an idea is responded to (neutrally)
24	Teacher	Mmhmm.		no code
25	Azaiah	/and—and… L, M is next to each other. So you can—so that means Tia was uh, less—that means Tia was the right one because she put three on there and that was—and Leo needed help.	positions Tia as correct mathematical explainer	an idea is accepted

In Lesson 12, we observed 11 instances of *correct mathematical student* or *mathematical explainer,* two instances of *incorrect mathematical student* or *mathematical explainer*. Of those 11 instances of *correct mathematical* *student* or *mathematical explainer*, four of them were enacted by students (three by Tia and one by Nyah). Of the remaining seven instances, six of them involved the teacher-researcher positioning a student as *correct* (three involved Nyah, two involved Zyla, and one involved Tia), and one instance involved a student positioning another student as *correct* (the aforementioned instance of Azaiah positioning Tia as *correct*). Of the two instances of students being positioned as *incorrect*, one involved Azaiah as positioning Tia as *incorrect* (again the aforementioned instance), and the other involved the teacher-researcher positioning Leo as being *incorrect* in a small group. We observed 20 instances of students being positioned as or enacting *discussing mathematical explainer*. In short, we observed these instances of specialized versions of Wood's micro-identities many times over the course of this one lesson.

Confirmation of Wood's (2013) findings regarding the nature of positioning of micro-identities. Lastly, we report results that confirm Wood's (2013) findings regarding the nature of positioning of micro-identities. Wood (2013) reports teacher and student positioning, and as noted we observed both as well. Given that Lesson 12 involved both whole class discussion and small group work, we are not surprised to see both types of positioning.

Another critical characteristic of Wood's (2013) framework is that it refers to moment-to-moment micro-identities. That is, these instances of positioning are brief and fluctuate often. We observed the same type of positioning in our analysis, and this is evident in the transcript excerpts. In the excerpts we present fluctuations in positioning and enactment of micro-identities even within one PNU. For example in rows 6 to 11 in the first excerpt, the teacher-researcher positioned Tia as *mathematical student*; Tia then enacted *mathematical explainer*. In response she is positioned as a *mathematical student,* but she again enacted *mathematical explainer*. Then in response the teacher-researcher positioned her as *collaborating mathematical explainer*. This all occurs over the course of about two PNUs.

One aspect of Wood's (2013) framework we did not observe was positioning of students as *menial mathematical workers*. To see if our observation about moment-to-moment fluctuates holds true in other lessons as well, and to observe the third mathematical micro-identity from Wood's framework, *menial mathematical worker*, we conducted ad hoc analysis, which we describe below.

After a review of our dataset, we observed the teacher-researched position both Nyah and Zyla into the *menial mathematical worker* micro-identity in Lesson 9 and 11, respectively. Thus, we analyzed how the teacher-researcher positioned Nyah in Lesson 9 and Zyla in Lesson 11 as we did with the five students in Lesson 12. That is, we conducted the same analysis as described above, determining PNUs and identifying Wood's (2013) mathematical micro-identities, but focused only on Nyah in Lesson 9 and Zyla in Lesson 11.

We found that the positioning of both Nyah and Zyla fluctuated throughout the lessons. Specifically, we observed that the teacher-researcher's positioning of Nyah shifted from *mathematical explainer* to *mathematical menial worker* to *mathematical student* and *mathematical explainer* within Lesson 9. In Lesson 9, the teacher-researcher spoke to Nyah the most (45 utterances), and Nyah proposed the highest number of ideas (27). She also had the highest number of accepted ideas (10), the lowest number of ideas responded to neutrally by the teacher-researcher

(3), and the lowest number of ideas not taken up by the teacher-researcher (1). Despite having many ideas accepted and few ideas rejected, the teacher-researcher's positioning of Nyah fluctuated significantly, even including positioning her as a *mathematical menial worker* at one point.

In terms of Zyla, we observed that the teacher-researcher fluctuated in her positioning of Zyla from *mathematical explainer* to a *mathematical menial worker* to a *mathematical student* and back to *mathematical explainer* within Lesson 11. Despite Zyla being absent from school for three of the 12 CTE lessons, the teacher-researcher spoke to Zyla the third most compared to the other seven focal students, with 133 utterances directed towards her. Zyla also proposed the third highest number of ideas (77), of which the teacher-researcher accepted approximately 26% of them. Similarly to Nyah in Lesson 9, Zyla ranked favorably for several categories in Lesson 11. She proposed the most ideas (21) and the teacher-researcher accepted more of hers than her peers (6). But the teacher-researcher also responded neutrally to nearly 67% of her ideas. Based on the ad hoc analysis of these two students in these two lessons, we conclude that the micro-identities attributed to these students shifted as many as three times in a single lesson.

Discussion and Conclusion

We sought to investigate the relationship between macro- and micro-identities. Specifically*, how does the thickening of moment-to-moment micro-identities contribute to the development of a macro-identity?*

Through our analysis in this paper we (1) identify a relationship between how first-grade students and a teacher-researcher propose and respond to ideas and how they position one another or themselves as mathematical authorities; (2) confirm Wood's (2013) findings regarding the nature of positioning of micro-identities; and (3) build on Wood's framework by describing how the micro-identities she observed can be nuanced or *specialized*. Specialized micro-identities are subsets of the larger micro-identities and are more nuanced, either describing a particular action the person is engaged in while enacting a type of micro-identity, or an action that someone expects them to enact. We argue that the exchange of ideas is the foundation of the thickening of mathematical micro-identities—the ideas are the currency that is exchanged as students are positioned and enact their positions. Consider the specialization of the mathematical explainer micro-identity, the mathematical explainer discussant, in that students are expected to evaluate their peers' ideas and explain their evaluation. These moments of evaluation are moments of valuation. That is, when ideas are evaluated as correct or incorrect, they are given more or less value. Additionally, the ideas are attributed to an individual who is simultaneously positioned into a micro-identity as their ideas, which now have value, are solicited, proposed and responded to.

We observe the culmination of the exchange of ideas and positioning into micro-identities when suddenly one student, Azaiah, evaluates his peers' ideas, and determines who is "the right one." It seems that the economy of ideas that have been exchanged prior to this moment has supported moment-to-moment positioning that overtime thickens into a moment in which one student reveals their trust in another student, positioning them as a mathematical authority among their peers. Our findings confirm Langer-Osuna's (2018) work on the role of student authority relations during collaborative work in mathematics classes. In her paper, Langer-Osuna posited that students' discursive influence were major contributors to their characterization as mathematical authorities relative to their classmates. Similarly to the students in Langer-Osuna's (2018) study, Tia proposing ideas during discussions advanced her micro-identity as a mathematical explainer. Although Brizuela et al. (2023) observed this critical moment between Azaiah and Tia, we question the authors' theoretical perspective in choosing the CTE as a research method.

In a standard mathematics classroom, the teacher faces a plethora of constraints that affect their teaching, student's learning, and ultimately the mathematics identity development of the students. These constraints can include curricular choice, student behavior, access to technology and other resources, etc. Conversely, for teacher-researchers conducting CTEs, there may be considerably fewer constraints, as the experiments are specially designed to allow the teacher-researcher more freedom and focused instruction to explore their research question. In entering the classroom as an outsider, teacher-researchers can come into the mathematics classroom with their own lessons, tools, and may experience less challenging student behavior, as the main teacher is there as an additional support. Without these constraints, the teacher-researcher in Brizuela et al. (2023) had more flexibility to execute their intentional design of the CTE. As such then, we argue that Azaiah's evaluation of his classmates' mathematical contributions was specific to the classroom teaching experiment, and does not tell the entire story about mathematical authority, and who is positioned into which mathematical micro-identities in that classroom. It is likely that we observed the mathematical explainer discussant micro-identity because of the teacher-researcher's teaching style and carefully planned CTE lessons.

Implications for Future Research

While this paper solely investigated how the teacher-researcher positioned Nyah and Zyla into different micro-identities, there are implications for teacher practitioners and researchers to consider how students interpret mathematics teachers' behaviors during whole-group discussions. Indeed, students can interpret how the teacher-researcher responds to them as a reinforcement of the micro-identity they've been positioned into. According to Ball (2018), just and equitable teaching can significantly transform the mathematics education field, especially for students, who behave in response to the moves their teachers make in their classrooms. The decisions mathematics teachers make regarding students' mathematical contributions suggest how their students can essentially play their roles, and they also convey important messaging about how they can behave in both the micro and macro mathematical communities. Regardless of how teachers use their mathematical authority, their actions will undoubtedly have a carryover effect in that students may use their roles in constructing a type of micro-identity.

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Appendix. The Classroom Teaching Experiment

Brizuela et al. (2023) used design-based research (Cobb et al., 2003; Collins, 1992; Kelly, 2003) to design and carry out a CTE. This experiment included eight weeks of classroom instruction which was separated into two 4-week cycles, and involved two 40-minute lessons taught per week. In addition, there were select student interviews before, between, and after the two cycles of instruction. The CTE tasks were created based on field-tested tasks which were developed in previous early algebra research by the project researchers. Early algebra research exists to better prepare students for formal algebra by introducing algebraic thinking practices into early mathematics education, and incorporating arithmetical and algebraic content and techniques. In this way, students can begin to engage in many of the fundamental practices inherent in formal algebra, such as representations and reasoning, as tools for success in algebra in later grades. This was not a primary focus of the Brizuela et al. (2023) study, however, it was the intention of the CTE. In particular, the classroom included students who were encouraged to utilize the different tools in early algebra, which the authors claimed may have lended to mathematical discourse which could influence authority dynamics amongst students.

In the focal lesson of Brizuela et al.'s (2023) study, Lesson 12, referenced above, students were discussing how they might use variable notation to depict a relationship between two quantities (two sisters' ages). Recent research has shown that students as young as those in Brizuela et al.'s (2023) study (first grade students), can use variable notation. Moreover, in doing so at an earlier age, it may also help students to represent relationships more generally (Blanton et al., 2015; Brizuela et al., 2015; Carpenter et al., 2003; Fujii & Stephens, 2008). Table provides a summary of the structure of the CTE.

Table. Sequence of Activities during the Classroom Teaching Experiment

Activity	Pre-interview	8 activities	Mid-interview	8 activities	Post-interview
Function type	$y = x$	$y = mx$	$y = x + b$	$y = x + b$	$y = mx$ $y = mx + b$