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Competition, Collaboration, and Learning Mathematics through Games

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Abstract

This study investigates how gameplay format shapes mathematical reasoning and social dynamics in primary classrooms. We compared one-against-one (1v1) and paired (2v2) versions of five non-digital games implemented by five Year 3/4 teachers (≈90 students). A mixed-methods design triangulated teacher interviews, student reflections (n≈40), and video-recorded gameplay from 20 focus students, coded for reasoning (generating, evaluating, justifying, clarifying, predicting, reflecting, prompting/helping, and connecting) and interactional moves (game management, emotional tone, self-talk, and off-topic talk). Across classes, 2v2 play elicited more visible reasoning than 1v1. Frequencies rose for Generating (104 vs. 42), Evaluating (217 vs. 136), and Justifying (68 vs. 30), as students proposed ideas, weighed alternatives, and negotiated shared decisions. Prompting/Helping was also higher, reflecting peer teaching. Importantly, mathematical connections occurred more often in 2v2 (26 vs. 5), with students applying concepts such as primes, factors, and multiples to guide play. Teachers reported richer discussion and conceptual talk during 2v2. However, 2v2 also introduced challenges: Game Management (311 vs. 223), Emotional Tone (70 vs. 37), and Off-Topic Talk (47 vs. 16) all increased, signaling coordination demands and uneven participation. Student reflections preferred 2v2 (≈65%), citing teamwork, while 1v1 supporters valued autonomy. We conclude that 2v2 competition fosters co-constructed reasoning, yet its success depends on intentional facilitation.

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Introduction

Mathematical games are increasingly recognised as a valuable tool in primary classrooms, not just for engagement but also for deepening learning. Surveys consistently show high rates of game usage among teachers. For instance, in a large-scale Australian study (Russo et al., 2021), an overwhelming majority of teachers (nearly 80%) reported using games multiple times per week, with an even higher percentage (92%) agreeing that games are effective for generating rich mathematical discussions. Approximately nine in ten teachers endorsed the role of games in supporting all four mathematical proficiencies (Understanding, Fluency, Problem Solving, and Reasoning), which together describe how students make sense of mathematical ideas, develop efficient skills, apply mathematics to unfamiliar situations, and justify their thinking (ACARA, 2025). This indicates that teachers perceive games as more than just tools for practising basic facts but potent vehicles for fostering deeper mathematical understanding and discourse. Teachers frequently use games to make learning enjoyable, motivate students, and cultivate critical thinking (Russo et al., 2021).

Indeed research shows that using games in mathematics classrooms, beyond their use for developing computational fluency, can improve mathematical thinking, reasoning, problem-solving, and understanding (Cramer, 2019; McFeetors & Palfy, 2018; Pintér, 2010; Russo & Russo, 2020, 2025). While the general benefits of integrating games into the curriculum are widely acknowledged, the specific impact of different game formats on mathematical reasoning and student interaction within the classroom remains an area requiring further investigation. Investigating how game formats influence mathematical discourse and conceptual understanding is crucial for informing pedagogical practice. This paper explores the differential impacts of one-against-one (1v1) and paired (2v2) mathematical game formats on student reasoning and social dynamics in primary school settings. Drawing on triangulated data from teacher perspectives, detailed analysis of student-to-student interactions, and student reflections, we aim to provide insights into how game format shapes mathematical discourse, peer support, and classroom management. This paper first reviews relevant literature, followed by methods, findings, discussion, implications, and limitations.

Literature Review

Theoretical Framing: Social Constructivism

This study is grounded in social constructivism, where knowledge is co-constructed through social interaction and discourse (Clements, 1997; Fosnot, 2005). Within this paradigm, the mathematics classroom functions as a "mathematics-talk community" (Fosnot, 2005), where dialogue clarifies and extends thinking, and ideas are "objects of reflection, refinement, discussion and amendment" (NCTM, 2000, p. 60). This view of learning positions "talk "as central to conceptual understanding (Clements, 1997) as students externalise thinking by reasoning aloud and justifying choices (Fosnot, 2005); the quality and nature of student discussion become key indicators of learning.

Mathematical games offer a promising context for engaging in mathematical discussion. Gameplay inherently involves turn-taking, decision-making, negotiation, and crucially, opportunities for students to make their thinking public. However, not all games, and not all gameplay structures, promote reasoning in the same way. This study examines how different gameplay formats, one-against-one and paired games, shape students' opportunities to reason, communicate, and construct mathematical understanding. The study explores how the game format mediates reasoning by analysing student-student interactions during gameplay and drawing on teacher insights.

This theoretical framing shapes the study's design and the findings' interpretation. It views reasoning not as a private, individual process, but as something that happens through social interaction. In this view, reasoning is expressed through talk, shaped by peer interactions, and influenced by the game format (playing one-on-one or working with a partner against another pair).

Mathematical Reasoning and Discourse in Games

Mathematical reasoning is a cornerstone of mathematical proficiency, defined in the Australian Curriculum: Mathematics (ACARA, 2025) as the ability to explain thinking, deduce and justify strategies, and adapt learning across contexts. This dynamic process of refining ideas toward conceptual understanding is reflected in the "verbs of mathematical reasoning" (McFeetors & Palfy, 2018, p. 106), such as generalising, conjecturing, and justifying.

Games have the potential to uniquely promote purposeful dialogue, often requiring players to articulate and justify their reasoning (Russo et al., 2023). Indeed, there is research showing that game-based lessons prompt significantly more on-task mathematical talk compared to non-game lessons (Bragg, 2012b), fostering active communication as students build and refine convincing mathematical arguments, thereby co-constructing understanding (McFeetors & Palfy, 2018; Putra et al., 2020). Games inherently create opportunities for interaction and discussion due to their multi-player nature (Ernest, 1986; Gough, 1999). As Ernest (1986) noted, "Children cannot play games passively; they must be actively involved, making the concepts and skills of mathematics their own" (p. 3). Similarly, Skemp (1986) emphasised that games allow students to get their thinking "out on the table" (see Stacey & MacGregor, 2022, p. 219). Stacey and MacGregor (2022) explain that this was a key insight, as Skemp demonstrated that games provide a window into children's thinking, making their thought processes externally visible and accessible. The "games" thus provide a structured context for the underlying mathematical ideas to be explored discursively. Ultimately, the enduring appeal of games in mathematics education is seen in both earlier and recent research, with the work of Russo et al. (2023) reaffirming that their unique ability to get students' thinking "out on the table" and into the open for discussion is fundamental to their pedagogical value.

However, the quality of this discourse is not always uniformly positive. Heshmati et al. (2018) report lower quality teacher-student interactions during gameplay, compared with non-game lesson components, often focusing on the progress of gameplay (e.g., who is winning and whose turn it is) rather than on rich mathematical reasoning. Furthermore, Bragg (2006a), observed that while students learned to play the game and followed the rules, their inability to explain their gameplay actions indicated a procedural, rather than conceptual, engagement with the mathematics. This highlights a critical point: for games to effectively support mathematics, the discourse, both between teachers and students and between students, should extend beyond basic procedural dialogue. It needs to encompass a deeper mathematical discourse that includes explicit reasoning, explaining, justifying moves, and negotiating strategies. This articulation of thinking ultimately allows students to construct and refine ideas, thereby developing a robust mathematical understanding (Oldfield, 1991; Stigler & Hiebert, 2004).

These findings suggest that the benefits of discussion depend not only on the game chosen but also on how the discourse is facilitated to promote purposeful dialogue. While the quality of teacher-student interactions during gameplay warrants careful attention and indeed was examined as part of our larger research project (forming the focus of a separate paper; see Klooger et al., 2026), this current study investigates how different gameplay formats shape mathematical reasoning by examining student-to-student discourse during gameplay.

Researchers have developed various tools to assess mathematical reasoning. Trakulphadetkrai's (2022) "Student-to-Student Mathematical Talk" (SSMT) framework, for instance, captures reasoning types through categories of verbal interactions. Loong et al. (2013) developed a rubric for primary classrooms, assessing reasoning actions like analysing, generalising, and justifying. These frameworks aim to capture the dynamic and iterative reasoning process, aligning with McFeetors and Palfy's (2018) concept of reasoning being a "work in progress" (p. 104). In this study, we used these frameworks as sensitising references: they seeded our initial code list and later served as a comparative lens to check construct coverage. However, we did not apply either instrument verbatim; instead, we developed a concise, gameplay-specific scheme. Appendix 1 includes an alignment table linking their constructs to our reasoning verbs.

Well-designed Mathematical Games: Design Principles and Cognitive Conflict

Well-designed mathematical games integrate mathematical content and strategic gameplay to support deep learning (Gough, 1999; Russo et al., 2023; Russo & Russo, 2025), requiring students to apply reasoning, explore patterns, and make purposeful decisions. Building on recent work highlighting that instructionally rich games embed strategic agency, mathematical representations, and opportunities for reflection (Russo & Russo, 2025), this study employed five games deliberately designed to keep mathematics at the centre of play, encourage meaningful decision-making, prompt opportunities for reflection and discussion, and balance skill, strategy, and chance to support optimal cognitive demand (see Appendix 2).

For effective discussions to build understanding, they must often be centred around a cognitive conflict. This mental tension happens when new or contradictory information challenges a learner's thinking (Onslow, 1990). From a constructivist perspective, experiencing disequilibrium is vital when learning mathematics (Fosnot & Perry, 1996). Gameplay is a prime context where cognitive conflict can arise in several ways: when rules or outcomes contradict expectations, when teammates disagree on a move, when an opponent presents an unexpected strategy, or when gameplay reveals a gap in conceptual understanding (see Table 1 for an example of cognitive conflict in the games used in this study).

When students are challenged by unexpected outcomes, need to justify complex procedures, or reflect on why a strategy worked or failed, it naturally leads to articulation, debate, and resolution of misconceptions, the very mechanisms through which mathematical discourse and reasoning builds conceptual understanding. As others such as Bragg (2007) have observed, "students' aspirations to win encourages them to wrestle with mathematical concepts" (p. 39), constructing meaning from the problem-solving process and highlighting the crucial role of challenge in fostering mathematical thinking. However, cognitive demand alone is insufficient; games may not support deeper understanding unless students actively engage in reasoning and reflection (Stein & Smith, 1998), which requires deliberate teacher facilitation (Heshmati et al., 2018).

Table 1. Examples of Cognitive Conflict in the Study's Games

Type of conflict	Description and Example
Rules or outcomes contradict expectations	Occurs when a student's existing assumptions are challenged by how the game rules or results unfold Example (*Reverse Landgrab*): A student rolls a prime number (e.g., 17) and attempts to form multiple arrays, only to realise it can only be a 1x17 array, which does not fit the gameboard grid. This directly contradicts their assumption about number factorisation (e.g., "multi-digit numbers have many factors/ belong to many counting patterns"), highlighting the unique properties of prime numbers.
Teammates disagree on the correct move	Arises in collaborative play when partners hold differing mathematical strategies or understandings of the optimal move Example (*Three-in-a-Row Lucky Numbers*): Partners disagree on marking '18' after landing on a lucky number. Partner A argues for choosing 18, while Partner B argues for choosing a number with fewer factors. This leads to a discussion about probability, prime numbers, and strategic placement.
Opponent presents a better or unexpected strategy	Happens when a student observes an opponent's more efficient or mathematically sophisticated strategy Example (*Choc-Chip Cookies Game*): Player 1 uses repeated addition to calculate the total choc chips (e.g., "For 19 × 5: 19 plus 19 is 38, plus another 19 is 57…"). Player 2 quickly applies the distributive property (e.g., "20 × 5 minus 5"), prompting Player 1 to recognise, and potentially adopt, a more efficient mathematical strategy.
Gameplay reveals a gap in conceptual understanding	Occurs when the mechanics or outcomes of the game directly expose a student's mathematical misconception Example (*Skip Counting Bingo*): A student consistently chooses only even numbers as Bingo numbers (e.g., 26, 32). This reveals a misunderstanding that when skip counting, you will always be more likely to land on an even number. It highlights a gap in their understanding of factors and multiples of even/ odd numbers.

In a study of 32 Year 7 students who played three different integer games (Go-High/Go-Low, Integer Product, and Integer 24), Nurnberger-Haag and colleagues (2023) found that gameplay could demand complex reasoning, spark cognitive conflict, and provide the very conditions for rich mathematical discourse. Students often judged such games to be more than entertainment, valuing them as meaningful opportunities for learning. Importantly, game features such as turn-taking (Go-High/Go-Low, Integer Product), requirements to justify and verify solutions (Go-High/Go-Low, Integers 24), the role of chance elements (Go-High/Go-Low), and strategic blocking

(Integer Product) were observed to prompt discussion between players, thereby elevating the cognitive demand of the tasks. Strikingly, these discussions arose in the context of one-against-one play (i.e., 1v1 games), indicating that opportunities for mathematical dialogue and heightened cognitive demand can emerge even within 1v1 gameplay without requiring explicit collaborative structures.

Gameplay Formats: Competition and Collaboration

The gameplay format is a critical design element that significantly influences student interaction and learning outcomes in educational contexts. Research on its effects in educational settings indicates mixed findings, and its effectiveness often depends on how it is implemented and the specific conditions of use. Prensky (2001) distinguishes between competitive play, in which success is defined in relation to outperforming others, and collaborative play, in which participants work together toward a shared goal.

A review of 32 studies on non-digital mathematical games by Russo et al. (2024) found competitive formats to be predominant (15 studies), while purely collaborative games were rare (4 studies). The remaining studies incorporated a blend of competitive and collaborative elements (2 studies) or featured solo gameplay/unspecified structures (11 studies). Russo et al. (2024) note the scarcity of research on collaborative gameplay formats with non-digital games. It is worth noting that in their operationalization, collaboration referred to players working together within a pair or team toward a shared outcome, even where teams then competed against one another, whereas competition described formats in which individuals sought to outperform others without intra-team cooperation.

This differs somewhat from how we operationalized competition and collaboration in the current study, where gameplay format (1v1 vs 2v2) provided the structural context, but collaboration was examined as an interactional process that could also emerge within competitive settings through moments of shared reasoning, peer support, or cross-team dialogue.

Competition in Gameplay

Competition is a common design element in games often assumed to foster motivation, engagement, and persistence (Delahunty & Roche, 2024; Nurnberger-Haag et al., 2023; ter Vrugte et al., 2015). Students often express a strong preference for competitive gameplay, citing enjoyment of the competition and winning, with some explicitly preferring to work individually (Delahunty & Roche, 2024; Nurnberger-Haag et al., 2023). In Delahunty and Roche's (2024) study, 85 Year 3–4 students each played three mathematical card games (Nearest to the Gnarly Number, RowCo and The Same As) in competitive, cooperative, and collaborative forms before indicating which version they enjoyed and learned from the most. The authors defined *competitive* gameplay as players striving to outperform others without supporting one another, *cooperative* gameplay as players supporting each other's progress while still pursuing individual goals, and *collaborative* gameplay as players working together as one team toward a shared outcome. Notably, those who preferred one-against-one gameplay often reported learning effectively in that mode, indicating that for some learners, the autonomy of playing individually may contribute to both enjoyment and opportunities for learning.

The broader impact of competition in educational games is often mixed with studies revealing varying outcomes on learning. Plass et al. (2013) found that adding a competitive element to an arithmetic game motivated players to solve more problems than those who played alone. Yet, these gains did not translate into improved out-of-game fluency, suggesting that the motivation to win may not necessarily support deeper or transferable mathematical learning. Similarly, Çelik's (2017) compared game-based learning for third-grade students in geometry with other instructional methods, such as creating physical models of geometric shapes and traditional teacher-led instruction. While the game-based group reported some positive experiences, their learning gains were not statistically significant compared to the modelling-based approach. This study highlighted that the effectiveness of game-based learning depends less on the inclusion of a game element alone, and more on alignment with learning goals and the quality of implementation. However, other studies present more positive outcomes. Cagiltay et al. (2015), for instance, found that competition in games significantly boosted learners' motivation and improved post-test performance, suggesting a role for competitive play in fostering conceptual understanding and fluency when games are well aligned with instructional goals, thoughtfully designed and implemented.

Critically, competition also carries potential drawbacks, particularly for less secure learners. It can induce tension, anxiety, and feelings of frustration or inferiority, which may diminish working memory capacity, lead to avoidance, and reduce performance (Dondio et al., 2023; Nurnberger-Haag et al., 2023; Plass et al., 2013). To mitigate these pitfalls, especially those related to stress from speed-based competition, incorporating asynchronous play (turn-taking) and balancing mathematical skill with elements of chance (e.g., dice, cards) can create more equitable and less stressful experiences for diverse learners, given they have a reasonable chance of winning (Nurnberger-Haag et al., 2023; ter Vrugte et al., 2015). Teachers can mitigate negative effects by rebalancing teams, rotating partners, or varying the task structure so that success is distributed more equitably. Importantly, the focus can be shifted from *winning* to *strategic improvement or personal growth*, for instance by prompting students to articulate what they might try differently next round. Such moves sustain emotional engagement and preserve self-efficacy while maintaining the motivational benefits of competition. Furthermore, high student engagement in competitive games can be problematic if driven primarily by a desire to win rather than engagement with mathematical objectives (Heshmati et al., 2018). As Gough (1999) warned, an excessive focus on competition can distract students from educational content, leading them to be "so distracted by their natural interest in playing to win, that they fail to focus on the mathematics" (p. 14). Therefore, student behaviors must be directly related to the learning process, with energy expended on understanding complex ideas beyond merely winning.

Collaboration in Gameplay

Collaboration, viewed as a social process of knowledge construction, generally benefits learning by fostering shared meaning and problem-solving (Fosnot, 2005). In educational games, collaboration can provide continuous support, encourage knowledge externalisation, and lead to positive affective outcomes, such as improved attitudes towards mathematics (Ke & Grabowski, 2007; McFeetors & Palfy, 2018). Students favouring cooperative or collaborative formats value working with partners and opportunities to help or be helped (Delahunty & Roche, 2024; Dondio et al., 2023).

However, research findings on combining collaboration with competition in games remain complex and mixed. While some studies, such as Cichy et al. (2020), found collaborative gameplay resulted in faster acquisition of mathematical skills and knowledge, others report decreased performance or no significant effect on learning outcomes (e.g., Plass et al., 2013). The impact can also vary significantly based on individual student characteristics. For example, ter Vrugte et al. (2015) studied pairs of students collaborating on mathematics tasks within a digital game, with some pairs additionally placed in competition against other pairs. They found that for below-average students, competition undermined the benefits of collaboration, whereas for above-average students, competition in a collaborative setting showed a positive trend. Collaborative games also present challenges like uneven participation or social friction (Plass et al., 2013).

As Dillenbourg (1999) cautions, collaborative learning is not inherently productive; rather, its success depends on carefully designed situations that increase the likelihood of specific, productive interactions that facilitate learning. Peers do not learn simply because they are grouped together, but because they engage in particular activities such as explanation, disagreement, and mutual regulation, that activate learning mechanisms. To foster these processes, group members must share a valued common goal, and the activity's success must depend on each member's contribution and individual accountability (Dillenbourg, 1999). This highlights the importance of deliberate design, including thoughtful group composition, clear task structure, defined roles, interaction prompts, and teacher facilitation (Dillenbourg, 1999).

Measuring the Impact of Mathematical Games

The presence of conflicting findings on the impact of playing games on mathematical learning reported in meta-analyses and systematic reviews may in part stem from limited or incongruent outcome measures, with many studies frequently neglecting broader aspects of mathematics learning beyond procedural knowledge (e.g., Abdul Jabbar & Felicia, 2015; Kacmaz & Dube, 2022; Russo et al., 2024). Measuring the impact of mathematical games on learning requires assessing not just knowledge of procedures but also the development of a variety of mathematical proficiencies, which encompass:

- Understanding: making connections and explaining concepts.
- Fluency: recalling facts and applying procedures.
- Problem-solving: formulating, representing, and solving problems.
- Reasoning: explaining thinking and justifying strategies (ACARA, 2025).

Russo et al. (2024) found that achievement tests were used in roughly three-quarters of the studies reviewed. However, they also highlighted the importance of complementing such narrow, easily quantifiable measures with interviews, observations, student-produced artefacts, and reflections to capture outcomes more comprehensively. Building on this, the present study employs probing assessment methods that focus on student gameplay as well as teacher and student reflections. This multi-faceted approach is designed to capture the rich and diverse learning outcomes fostered by mathematical games, that might otherwise be overlooked if more traditional, reductive measures of achievement are employed exclusively.

Aims of the Current Study

Taken together, the literature highlights the need for closer examination of how different mathematical game formats influence students' reasoning and discourse, supported by richer forms of evidence. Addressing this gap, the present study set out to investigate how one-against-one (1v1) and paired (2v2) gameplay formats shape students' opportunities to reason and construct mathematical understanding. Specifically, the aims were to:

- Explore teachers' perceptions of mathematical reasoning during gameplay.
- Investigate students' preferences for different gameplay formats.
- Analyse student-to-student interactions during gameplay sessions to understand how reasoning is manifested and influenced by game format.

Method

This study employed a mixed-methods design, collecting data from teacher interviews, student reflections, and video-recorded gameplay transcripts, analysed through an interpretivist lens focused on mathematical discourse.

Research Context and Participants

The study involved five Year 3/4 teachers and approximately 90 students from two Victorian (Australia) state schools. Two classes ($n \approx 40$) completed written reflections, and 20 focus students (four from each class) were video-recorded during gameplay. Teacher interviews provided additional insights. Each teacher selected and used two multiplication games from a curated bank of carefully designed, non-digital games (see Appendix 2).

For each selected game, teachers taught two lessons: Lesson 1 in a one-against-one (1v1) format and Lesson 2 in a pair-based (2v2) format. These five games, adapted for both one-against-one (1v1) and paired-based (2v2) formats, were specifically designed to align with educationally rich game principles (see Russo et al., 2023; Russo & Russo, 2025). This structure supported comparisons of gameplay format through drawing on a common design framework. Table 2 summarises the games used.

Table 2. Games played by teachers across Lesson 1 and Lesson 2

Teacher	Lesson 1 (One-against-one)	Lesson 2 (Team-based)
Ashleigh	Three-in-a-Row Lucky Numbers	Reverse Landgrab
Emily	Choc-chip Cookies Game	Reverse Landgrab
Joel	Choc-chip Cookies Game	Reverse Landgrab
Morgan	Reverse Landgrab	Three-in-a-Row Lucky Numbers
Ryan	Skip Counting Bingo	Three-in-a-Row Lucky Numbers

Data Collection and Analysis

Data were collected from three primary sources:

1. Teacher Interviews: Semi-structured interviews were conducted with the five teachers at the beginning of the project, following each lesson, and again at the end, probing their perceptions of the 1v1 and 2v2 game formats.
2. Student Reflections: Written reflections were collected from approximately 40 students after each lesson, detailing their format preferences and affective responses as to why they preferred one game format over the other.
3. Video-recorded Gameplay Transcripts: Video recordings of the 20 focus students during gameplay sessions were transcribed.

Following data collection, teacher interview transcripts and student reflections were analysed through an open-ended process aimed at identifying recurring patterns in teachers' perceptions and students' experiences and preferences for game format. Quantitative preferences from students were tallied. Video-recorded gameplay transcripts were analysed using an iteratively developed coding framework. Initial inspiration was drawn from Trakulphadetkrai's (2022) Student-to-Student Mathematical Talk (SSMT) framework. However, early attempts to apply the extensive SSMT framework to our game-based video data proved too cumbersome. Recognizing the dynamic nature of gameplay, the coding process was refined through multiple rounds of viewing and discussion until a concise and contextually relevant set of categories emerged**.** These included categories for mathematical reasoning (e.g., Generating Ideas, Evaluating, Justifying, Predicting, Reflecting, Helping/Prompting, and Connecting) and broader interaction dynamics (e.g., Game Management, Emotional Tone, Student Talk, and Off-Topic Talk). The final code book retained shared "reasoning verbs" (e.g., generating, evaluating, justifying) and added interaction codes suited to gameplay (e.g., game management, emotional tone), and we cross-checked code coverage against the SSMT framework and Loong et al. (2013) to support validity. All codes and their application were systematically discussed and refined to ensure rigor. This process involved collaboration and refinement with the third author. This framework served as the analytical lens to systematically examine the nature and quality of mathematical reasoning in student-to-student discourse across game format. For details of the full coding framework (see Appendix 3).

Finally, triangulation was performed through validating emerging patterns and deepening interpretations by comprehensively comparing findings from all three data sources (teacher interviews, student reflections, and gameplay transcripts). This multi-perspective analysis aimed to provide a robust account of the study's key aims: teachers' perceptions of mathematical reasoning across different gameplay formats, students' preferences for different gameplay formats, and how gameplay formats shaped student mathematical reasoning and discourse.

Findings

Teachers' Perceptions of Mathematical Reasoning across Different Gameplay Formats

This study investigated teachers' perceptions of mathematical reasoning across one-against-one (1v1) and paired

(2v2) game formats. Teachers consistently expressed enthusiasm for the richer discussions and novel opportunities for mathematical reasoning afforded by the paired games, a format new to all five participating teachers. This perception emerged with Morgan explicitly noting, "for the paired one the discussion was far richer than the other one," a sentiment echoed by Joel, who found the collaborative aspect a "real eye opener." These observations align with the general teacher perception that games are potent vehicles for fostering deeper mathematical understanding and discourse (Russo et al., 2021).

Teachers observed students engaging in mathematical reasoning, particularly in justifying their mathematical ideas and evaluating others' ideas. For instance, Ashleigh recounted students exploring prime numbers in *Reverse Landgrab*, where "one suggested it wouldn't work because 19 is a prime number. You can only make long skinny arrays," demonstrating students connecting prior knowledge and justifying their reasoning. Similarly, Morgan described students exceeding expectations with discussions on factors for "39," in *Three-In-A-Row Lucky Numbers*, recounting: "Why did you choose that number? What other factors does it have? A student then asked, Is 39 prime? With another child saying no, you can divide it by three, but you just can't make it with these dice." She added, "I've never heard these kids more on task, when playing a game." These anecdotes illustrate students' engagement in the "verbs of mathematical reasoning" (McFeetors & Palfy, 2018) and directly correspond to types of cognitive conflict inherent in the games such as when rules contradict expectations, thereby creating conditions for rich mathematical discourse (Nurnberger-Haag et al., 2023; Onslow, 1990).

This enthusiasm for the 2v2 format also marked a notable shift in teacher perceptions regarding collaborative gameplay. Morgan acknowledged that the richer discussion emerging through this format was not something she had anticipated, stating:

> "I think, you know, in classrooms, sometimes we try and avoid, you know, too many kids working together, and it gets loud and messy and oh, my goodness. But I honestly thought that that was the better lesson."

This sentiment underscores a notable shift in her perception, valuing the depth of interaction over potential management concerns, and indicating her intent to "use this structure again" when using mathematical games. Joel concurred, noting that he had "never done it this way" with his students, who would typically engage in playing games one-on-one. Ryan also explained that normally he would play games where students would play "against another person, whereas with the paired game you have to work as a team." This novelty in teachers' experience with structured paired gameplay aligns with Russo et al.'s (2024) observation on the scarcity of studies on collaborative non-digital game formats, potentially explaining the limited research in this area, as teachers' prior practice has not often extended to such formats.

Teachers also highlighted significant peer support and emotional benefits in the paired format. Joel observed students naturally taking on "teaching roles," explaining concepts, while Ryan noted how peer support strengthened connections between peers, recalling one student who offered, "Let me show you how I figured that out." Morgan further emphasised the relational benefits, such as fluent speakers assisting English language learners. These observations corroborate literature on collaborative learning benefits, including knowledge externalisation (McFeetors & Palfy, 2018) and positive affective outcomes (Dondio et al., 2023; Ke & Grabowski, 2007).

Despite these benefits, teachers identified significant management and social challenges in managing four players in the 2v2 setup. These issues revolved around coordination and equitable participation, rather than fundamental problems with the games' mathematical content. Ashleigh's reflection vividly illustrates these complexities. She noted that while her class was "very good at that [working with anybody]," some students still "struggled in that situation." She recounted a specific incident during *Reverse Landgrab* where a grade three girl, "after rolling a 19 and being unable to place it, didn't want to listen to her partner," leading to significant interpersonal friction. "She sat on the floor and pulled apart a dice mat, and she just could not join back in." Joel further underscored these social challenges, noting that while "most of the students did okay with the collaborative side of it," there were "more than a handful of glaring problems working collaboratively." He also recounted an incident with a student who refused to cooperate with her partner and was asked to leave the room. These anecdotes highlight how the dynamics of a four-player setup, particularly social-emotional factors and student relationships, could significantly impede the flow of mathematical reasoning and game engagement.

Ashleigh also observed that "the paired games took longer to play because there were more people to negotiate," indicating that coordinating actions and turn-taking in a larger group added complexity. Morgan similarly commented that "some students did not view their participation as essential," suggesting uneven contributions sometimes emerged within a team of four. Emily echoed these management concerns, finding the one-against-one game "quicker to run compared to the pair game." She observed that "some pairs were focused more on the rules than the actual collaboration of talking and working together in a pair and completely dismissed their partner altogether and just took the reins." This directly impacted the perceived effectiveness of the paired format for "actual learning content," as students were sidetracked or dominant players overshadowed others. These findings align with research indicating that collaborative activities present challenges like uneven participation or social friction (Plass et al., 2013). Teacher reflections on these incidents highlight the importance of considering student groupings for effective collaborative learning (Dillenbourg, 1999).

To further address game management and social challenges, Ashleigh also used "built-in prompts" suggested by Dillenbourg (1999) to enhance collaboration between students, intervening during lessons with questions like, "Did you talk to him before you made that decision?" She later reflected, "I've never thought to play games in pairs before. I think I need to teach them how to work together when playing a game." Ashleigh noted she "would definitely like to try that again," but "with the focus of, well, now you're working with a partner, what does that mean?" Joel extended this idea, stating that "doing more doesn't necessarily mean it's going to get any better" and advocated for "breaking it down and modelling how you work collaboratively" would be valuable. In practice, this meant that teacher facilitation played a central role in keeping gameplay productive, with teachers stepping in to redirect unproductive dynamics, prompt inclusive decision-making, and model effective turn-taking. This proactive role aligns with Dillenbourg's (1999) recommendations for monitoring, guiding, and providing interaction prompts, underscoring that while the games themselves created potential for rich discourse, successful implementation of the 2v2 format required intentional pedagogical facilitation to overcome social and game management issues, nudging groups back towards constructive dialogue when necessary (Heshmati et al., 2018).

Students' Preferences for Different Gameplay Formats

Written reflections collected after gameplay reveal students' one-against-one or paired gameplay preferences. As shown in Figure 3, nearly two-thirds (65%) preferred the paired format, while about one-third (35%) favoured the one-against-one format. This finding contrasts with Delahunty and Roche (2024), who reported stronger preferences for one-against-one play. This discrepancy might be due to the paired format in this study retaining competitive elements, whereas their study removed them entirely. Maintaining the excitement and challenge typically associated with gameplay while fostering cooperation, this hybrid paired competition offered a middle ground, combining opportunities for discussion with the motivational benefits of competition.

Students who preferred one-against-one play often cited reasons related to autonomy and streamlined decision-making. Comments such as, "I don't need to discuss things. I have my own opinion," and "You don't need to decide together, which takes up time," highlight a value for independent action, quick decisions, and full control over gameplay. This aligns with prior research suggesting that competitive formats appeal to individuals who prefer independent strategy and self-reliance (Delahunty & Roche, 2024; Nurnberger-Haag et al., 2023). Some students also described one-against-one play as calmer and more predictable, allowing them to focus solely on their own strategy. This reduced the need for social negotiation, enabling a more focused individual strategy (Plass et al., 2013).

Conversely, students who preferred the paired format often emphasised the enjoyment of playing with a partner and relational benefits. Common responses included: "Because I have someone else's opinion," "You get to tell your partner what you think," "I love teamwork," and "Because you have a friend to help you." These findings strongly corroborate literature on collaborative play, consistently highlighting positive affective outcomes such as increased enjoyment for learning mathematics, mutual responsibility, and strengthened social connections (Dondio et al., 2023; Ke & Grabowski, 2007;). Some also noted that "more people have a chance to win or lose," suggesting that having a partner increased perceived chances of success. However, not all experiences were positive; several students mentioned frustration, such as, "I didn't like the 2v2 game because my partner didn't let me have a turn." In some cases, unequal participation or dominance by one player reduced the enjoyment, underscoring the challenges of collaboration also seen in teacher observations and the literature (Plass et al., 2013).

While most students who preferred team-based play emphasised enjoyment, some also described it as "more challenging" due to the need to agree on moves and justify choices. For others, the one-against-one format provided a clearer space to work independently on strategies, with comments like, "I get less help, which means I can improve better," and "Because I'm very competitive, so it's more fun to win on your own." This highlights that competitive formats can be perceived as conducive to self-directed learning and the development of individual strategic thinking. A few students, however, specifically valued the opportunity in team-based play to "discuss things with others," pointing to the role of peer discussion in refining strategy and problem-solving, which is a key benefit of collaborative learning contexts and a core tenet of social constructivism (Clements, 1997; Fosnot, 2005).

The one-against-one format appealed to those seeking independence and efficiency, while paired gameplay required negotiation and shared control. Team-based competition fostered belonging and enjoyment when peer dynamics were positive but caused frustration when peer dynamics when were not, making one-against-one play more appealing to students seeking predictability. Overall, these findings indicate that the preferred gameplay format often aligns with students' individual learning preferences and characteristics (ter Vrugte et al., 2015), competitive spirit, and ability to navigate group dynamics, demonstrating that different formats offer distinct advantages for personal growth and mathematical learning.

How Gameplay Formats Shaped Student Mathematical Reasoning and Discourse

Analysis of student-to-student interactions reveals how game formats fundamentally reshaped students' mathematical reasoning and discourse. While both formats indicate that an outcome of playing games is that they can stimulate rich mathematical reasoning, prompting students to externalise their thinking, debate possibilities, and evaluate their own and their opponent (s) strategies, the data indicates that all reasoning behaviours were notably less frequent in one-against-one games compared to the paired format (see Figure 1). While paired games prompted more reasoning overall, the purpose of that reasoning differed. Essentially, a 2v2 game transforms the activity from an individual problem-solving task into a collaborative one, demanding more explicit communication, shared planning, and consideration of multiple perspectives, all manifesting as increased external "reasoning". This profound transformation in the nature of mathematical discourse aligns with social constructivist principles, where knowledge is co-constructed through social interaction and dialogue (Clements, 1997; Fosnot, 2005).

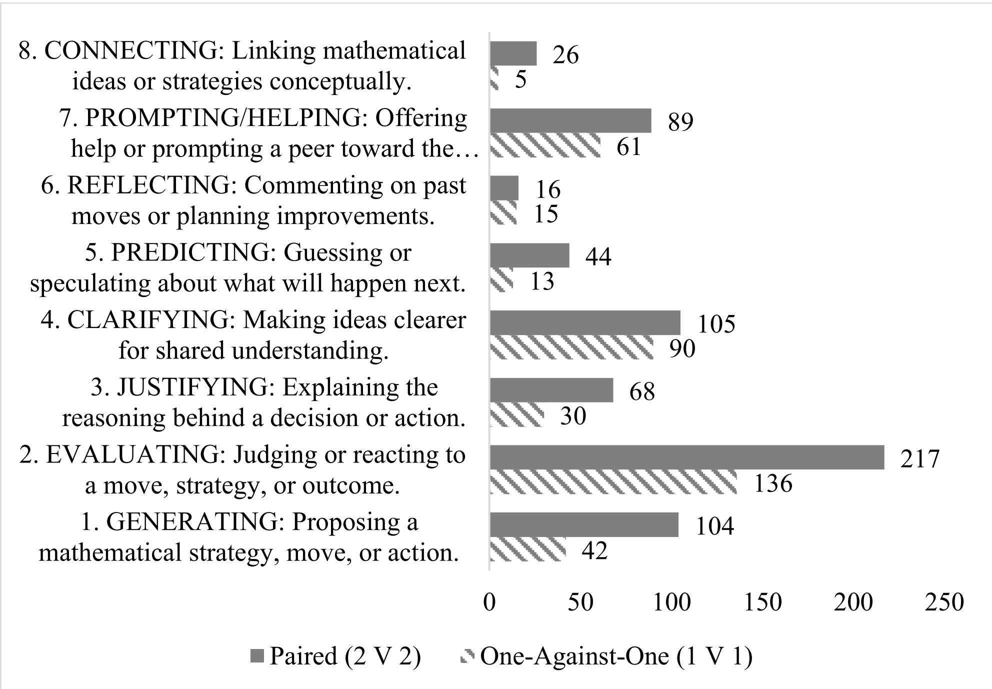

Figure 1. Frequency of Student Interaction Codes Observed during One-against-one (1v1) and Paired (2v2) Gameplay (Codes reflect distinct types of reasoning-related moves, including (e.g., Generating, Evaluating, Justifying) identified in gameplay transcripts.)

Generating Ideas: From Individual Strategy to Collective Problem-Solving

The “Generating” code (proposing a mathematical strategy or move) appeared significantly more often in 2v2 games (104 times) compared to 1v1 games (42 times). This increase signals a shift from individual strategy to collaborative problem-solving. In 2v2, students generated ideas for their team, brainstorming aloud and proposing a wider range of possibilities in *Reverse Landgrab*: “What we could do is block them so they can’t get in?”; “Should we do 5 × 2 or 1 × 10?”; and “We can fill the gap with smaller numbers. Is that right?” This contrasts with 1v1 gameplay, where students typically generated ideas for their strategy and mostly did not share these openly with their opponent.

The collaborative format encouraged students to make their thinking visible, with generating ideas notably increasing for Joel’s class (from 3 to 35), Ashleigh’s (3 to 21), and Emily’s (4 to 15). However, Morgan’s students showed high generation in both formats (28 in 1v1 and 25 in 2v2), suggesting her facilitation (“We need to hear your ideas”) encouraged explicit reasoning regardless of structure. Conversely, Ryan’s more modest increase (from 4 to 8) indicates that while 2v2 offers more opportunities, teacher practices remain crucial.

In 1v1 gameplay, discussion was more individualistic and often used to outmanoeuvre an opponent, as seen in *Reverse Landgrab* with comments like: “That was stupid, why did you go over there? You should have gone on one of the edges.” Although students in both formats often presented something they had noticed or a strategic idea, in 1v1, these ideas were more likely to be ignored by the opponent, a pattern not seen as readily in 2v2 games.

The *Choc-chip Cookies Game* (1v1) vignette vividly illustrates this lack of collaborative engagement:

> Player 1: “Okay, so I’m thinking my strategy is right because we’ve got a 10-sided dice and we’ve got 5 different rows. I divided 10 by 5, which is 2. Because we rolled a 3, it would go in the 2 column. Either 3 or 4 would go in here, which is times 2, which is 4. What’s yours?”
>
> Player 2: “I just risked it. I put it there hoping we rolled a 1 or a 2.”
>
> Player 1: “Okay go ahead, what’s our next number?”
>
> Player 2: “Five. If you roll a number higher than 5, my risk is worthless. Next number? (Rolls) No… I can still win.”
>
> Player 1: “Alright, our last number? (Rolls) My strategy of dividing won me.”
>
> Player 2: “Let’s just add up, just to check.”

Here, Player 1 uses proportional reasoning to explain their strategy, but their partner does not engage with it. Instead, Player 2 offers a risk-based approach and misses the opportunity to evaluate or build on Player 1’s explanation. The interaction remains parallel rather than dialogic, with each player working independently, with little collaborative interrogation or mathematical negotiation. At the game’s completion, Player 1 again justified their winning strategy, saying, “Okay, I think my strategy won me,” with Player 2 responding with “Let’s just add it up and check.” This highlights how reasoning in 1v1 was often for self-validation rather than shared construction of understanding. Notably, however, if an opponent made a suggestion, it was likely to benefit both players, for example, in *Reverse Landgrab*, “Let’s just save that spot [so that if either of us roll a 17, we would be able to have a turn].”

Evaluating and Justifying: From Self-Validation to Negotiated Reasoning

“Evaluating” (judging a move or outcome) occurred 217 times in 2v2 games versus 136 in 1v1 games, and “Justifying” (explaining a decision) more than doubled in 2v2 (68) compared to 1v1 (30). In 1v1 games, reasoning primarily served individual validation or defence. Evaluations were often internal or competitive, and justifications rarely invited further discussion, appearing as a series of publicly voiced individual assessments. An example from *Three-in-a-Row Lucky Numbers* illustrates this:

Player 1: “Oh my gosh, there are so many 24s.”

Player 1: “We are getting high numbers now.”

Player 1: “Oh my gosh, 24 is a good number.”

Player 1: “I did a partial array.”

In this dialogue, Player 1 voices a series of individual evaluations. The observation that 24 has many factors is integral to playing *Three-in-a-Row Lucky Numbers*; when rolling a product previously rolled, players can choose any number on the board, making it strategic to select numbers with fewer factors (like primes). However, this key observation is presented as an internal thought voiced publicly, not an invitation for collaborative discussion.

Conversely, in 2v2 collaborative games, reasoning was distributed, dialogic, and responsive. Ideas were collaboratively generated, immediately evaluated by peers, and justified in ways that invited agreement, critique, or revision. For example, a suggested strategic placement in *Reverse Landgrab* was met with: “That won’t work because you can’t fit it” or “This works better because it blocks them.” These justifications were typically offered in response to a teammate’s suggestion and grounded in the shared goals of the team. In this way, students explained why a choice made sense given the game’s constraints, collectively weighing multiple possibilities during gameplay. The sharp increase in evaluation instances and more than doubling of justification utterances in 2v2 reflects a clear need for externalised and negotiated reasoning in team settings, where the aim was often to reach consensus. However, at times, players would unilaterally make decisions without full agreement.

These collaborative reasoning sequences were often embedded within larger chains of interactions, blending evaluations, suggestions, justifications, and clarifications. For example, in *Reverse Landgrab*, the following discussion where teams debated prime numbers, array placement, and strategic consequences across team boundaries unfolded:

Team 2 rolls 17

Player 2 (Team 2): Okay 1 times 17 is the best option (Codes: 1 – Generating; 2 – Evaluation)

Player 2 (Team 1) “You have to do 1 times 17. You have to do an array.” (Codes: 4- Clarifying, 1-Generating)

Player 1 (Team 2) “It’s impossible to do that kind of array though (gets up to speak to the teacher).” (Codes: 2 – Evaluation, 3 – Justification)

Player 2 (Team 2): “12, 15, 17 times 1... I think it’s the best option. It’s the only one we have.” (Codes: 1 – Generating, 2 – Evaluation, 3 – Justification)

Player 2 (Team 1) "Go this way or the other way" (Pointing to vertical columns or horizontal rows)
(Codes: 1 – Generating, 7 – Helping)
Player 2 (Team 2): "You can't really use them going this way but actually let's try."
(Codes: 2 – Evaluation, 1 – Generating)
Player 1 (Team 2) Counts the squares "1,2,3,4…"
(Code: ST- Self Talk)
Player 2 (Team 2): "So only one block could go there (pointing to the horizontal row), which is not a good idea."
(Codes: 3 – Justification, 2 – Evaluation)
Player 1 (Team 2): " What we could do is also risk it and block them for going there."
(Codes: 2 – Evaluation, 3 – Justification)
Player 2 (Team 2): "But still, we are early in the game, so going up is still the best way to win for now." (Codes: 2 – Evaluation, 3 – Justification, 1 – Generating)

This pattern highlights a fundamental shift: in 1v1 games, reasoning was a tool of strategic persuasion or defence**,** while in 2v2 games, it became a tool of co-construction; a means of negotiating shared mathematical understanding.

Clarifying: Coordination and Shared Understanding

Clarifying (clarifying ideas) saw a modest increase from 90 instances in 1v1 games to 105 in 2v2 games. However, a significant portion of 2v2 clarifying interactions were procedural rather than conceptual. Teammates were not always simultaneously focused on the task; players often needed to restate or confirm dice rolls, rules, results, or defend intended moves before progressing. This procedural clarification reflects the coordination demands of multi-person play, where turn-taking, attention shifts, and shared decision-making require constant realignment. As noted earlier by the teachers about game management and pacing, 2v2 games introduced more negotiation and waiting time, which sometimes enhanced dialogue but also created moments where clarification served to simply re-synchronise the group rather than deepen mathematical reasoning.

Prompting and Helping: Fluency Support and Strategic Input

Prompting and helping were notably more frequent in 2v2 games (89 times compared with 61 times), indicating that this format consistently promotes peer teaching, mutual support, and collaborative problem-solving. This aligns with Joel's and Morgan's observation of students "helping one another" and suggests that the paired format naturally creates opportunities for "fluency support". This pattern aligns with teachers' observations and literature suggesting that mutual help is a key benefit of collaborative learning (Dondio et al., 2023; Ke & Grabowski, 2007; Mc Feetors & Palfy, 2018). Collectively, these findings imply that group-based competition, particularly with mixed-ability pairs, may provide a productive balance between challenge and collaboration, effectively mediating the anxiety often associated with competitive environments, especially where speed is a factor (Dondio et al., 2023; Nurnberger-Haag et al., 2023).

In 1v1 format, games often prompted a competitive or corrective tone, framed as "telling" rather than "working out together." Support was usually triggered by noticing a peer's mistake or inaction and keeping the game moving but was often met with resistance ("I know what I'm doing, okay!"). Players were largely expected to be independent, make individual decisions without prolonged discussion, and therefore showed little investment in genuinely supporting their opponent. For example: "I only helped you; I don't really care." An exchange from the *Choc-Chip Cookies Game* further demonstrates how help was offered as a challenge rather than collaboration:

Player 1: "I've still got a strategy."

Player 2: "You should have gone for the highest one," she challenged. "You can't do it now, though."

The tone and reception of prompting in 1v1 games varied widely. It was openly competitive and even provocative in some games: "I have a feeling I'm smarter than you." Offers of help were frequently resisted, as seen in this example from *Reverse Landgrab*:

Player 1: "Twenty-one."

Player 2: "Oh, actually, I know one. It's 3 times… I'll tell you; it's 3 times something."

Player 1: "3×7, you dumb, dumb."

Player 2: "What?"

Player 1: "3×7."

Player 2: "I was just trying to give you a clue."

Player 1: "I know my timetables."

Player 2: "19+4 is 23"

Player 1: "No, you can do another one and 19 + 14 is 33, 3 X …. (giving a hint)."

Player 2: "I can do ones, alright?"

Player 1: "I'm just telling you. You can also do 3 X 11."

Player 2: "I count quicker than you now."

Player 1: "Oh, you're doing what I told you."

Player 2: "Yeah, I was just counting by ones, but [then I saw] you blocked the way."

Player 1: "I'm not going to do it (help) again."

At other times, prompting in 1v1 games was more collaborative, with players offering scaffolded help to keep the game moving: "Okay, I'll help you make sixes" [counting by sixes together]. In these cases, assistance was welcomed and often essential for progressing in the game, rather than resisted.

In 2v2 play, prompting and helping occurred in two distinct ways. First, within teams, players encouraged contributions and discussed strategy together: "What should we do?" and "Go on, you say your idea." These interactions focused on building a shared approach and supporting teammates. Second, prompting and helping sometimes extended across team boundaries, with players voicing comments or questions that invited responses from anyone, teammate or opponent. For example: "Anyone know what 6 sixes is?" was asked aloud to the whole group, prompting others to supply the answer. In other cases, cross-team interactions involved offering strategic suggestions or warnings, such as advising an opponent to choose a certain move:

Player 1 (Team 1): "Let's choose 45."

Player 2 (Team 2): "You should choose 36. I'm saying this because of this. I think we can't block you if you do that."

Player 1 (Team 1): “Oh yes that’s true. Thank you for that.”

Player 2 (Team 1): “Yeh, but there’s still these two areas.”

Player 2 (Team 2): “But you have this one, and if you get that one, then this is blocked and so is that. They are all blocked.”

These exchanges showed opponents sharing reasoning, weighing trade-offs, and negotiating possible moves, even on different teams. At times, these strategic prompts were aimed at preventing a move that could shift the balance of the game:

Player 1 (Team 1): “Should we do 5 × 2 or 1 × 10?”

Player 2 (Team 1): “Do it this way (pointing to a horizontal line). We’re winning so far.”

Player 1 (Team 2): “Do not do 1 × 10.”

Player 2 (Team 1): “Why?”

Player 1 (Team 2): “Only if it’s up and down, because then it will block if someone has 17 again.”

Reasoning could also become a shared group process, with players discussing the implications of a move for everyone:

Player 1 (Team 1)— to teammate: “But now we can’t do 17.”

Player 2 (Team 1): “Okay, there won’t be 17 anymore.”

Player 1 (Team 2): “What do you mean?”

Player 2 (Team 1)— to all players: “Because then you can’t do 17 because she took the whole bottom.”

Player 1 (Team 1)— to all players: “Making it hard for all of us.”

Player 2 (Team 1)— to all players: “If you get anything above 17 or 17, you are missing a turn.”

Player 1 (Team 1)— to all players: “It will have to be under 13, now.”

Here, the opportunity to notice aspects of the game was not confined to one’s turn or team; reasoning was readily voiced by any player from any team. These across-team exchanges blended competition and collaboration, sometimes helping the opposition, but also showing awareness of the broader game state and inviting shared reasoning about strategic consequences.

Notably, as in 1v1 games, a competitive edge could surface even in team format:

Player 1 (Team 1): “24, I’ll block them.”

Player 2 (Team 2): “You should have done 34, you know.”

Player 1 (Team 1): “Where’s the eraser?”

Player 1 (Team 2): “No, no, no, you can’t change your mind.”

Here, competition completely overrides any collaborative intent built into the 2v2 structure. These findings suggest that in competitive contexts, the drive to win can, at times, override collaborative reasoning. This “override effect” reminds us that while team formats may be designed to encourage cooperation, the competitive framing of the game can still dominate student behaviour, and teachers may need to actively scaffold interactions to maintain a balance between collaboration and competition (Dillenbourg, 1999; Heshmati et al., 2018).

Across both formats, prompting and helping served dual purposes: sometimes it offered fluency support, while at other times it involved extended reasoning or strategic debate that slowed the pace but deepened mathematical discussion. Teachers may need to scaffold interactions, for example, through post-game reasoning discussions or targeted prompts, to balance collaboration and competition (Klooger et al., 2025). Structuring these opportunities, as Heshmati et al. (2018) highlight, ensures that rich exchanges between students become a deliberate and expected part of gameplay.

Reflecting and Connecting: Surface Noticing and Strategic Use

The data shows that "reflecting" was relatively rare across both formats, occurring with similar frequency in paired and one-against-one games. By contrast, "evaluating" and "generating" were far more common, suggesting that students were more engaged in immediate reactions and proposals than in stepping back to reconsider their actions or strategies. This may reflect the fast-paced nature of games or the developmental stage of the students, who may be more focused on playing.

By contrast, although still comparatively unusual relative to the other reasoning categories, students were far more likely to make mathematical connections in 2v2 formats (26 instances) compared with 1v1 formats (5 instances). This suggests that a collaborative environment, with its shared goals, provides more opportunities and incentives for students to engage in deeper discussions and explicitly link mathematical concepts.

In paired games, students more regularly tied number properties to their strategies. For instance, in *Reverse Landgrab*, one student observed, "You got a square number," while another reasoned, "If we make it a square, we can complete this row." Students also used concepts to block opponents, such as "We want a prime number so they can't make a square," or to justify properties, such as "The number 1 is not a prime number." They sometimes defined concepts for their peers: "A square number is a number multiplied by itself." The dialogic nature of paired games appeared to foster these richer explanations, with students questioning "What's a prime number?" and clarifying one another's reasoning. By contrast, one-against-one games yielded few connecting comments, and those that did appear tended to be individual reflections rather than shared elaborations. For example, in the *Choc-Chip Cookies Game*, a student commented on the likelihood of rolling a large number, saying, "Maybe it's a 50 or 60% chance." The collaborative demands of paired play more readily prompted students to articulate and apply mathematical connections within the flow of the game.

A closer look at the "connecting" comments reveals two distinct functions. Many were directly tied to the immediate action of a turn, where a student noticed a mathematical connection, such as "It's 2 × 2, which is a square number" or "It's a multiple of 4." These comments demonstrate awareness of mathematical ideas but remain descriptive, limited to explaining why a move is possible or constrained, as in "You can't do anything else because seven is prime." Other comments where students made mathematical connections, operated strategically, using mathematics to guide and influence play. For example, "We want a prime number" in *Three-in-a-Row Luck Numbers* was not simply a statement of fact but a deliberate, goal-oriented move. In such moments, students verbalised their reasoning as part of a collective game plan, leveraging mathematical properties to pursue advantageous outcomes. This dual role, sometimes descriptive, sometimes strategic, highlights the richness of particularly paired games where students' collaboration created both the need and the space for such connections. These discussions demonstrate the potential of games to serve as powerful contexts for teaching mathematics and fostering deeper conceptual understanding of mathematical ideas.

Game Management and Social Dynamics of Paired Play: Implications for Reasoning

Figure 2 illustrates the distinct frequencies of Game Management (GM), Self-Talk/Metacognition (ST), Emotional Tone (ET), and Off-Topic Talk (OTT) observed in paired versus one-against-one game formats.

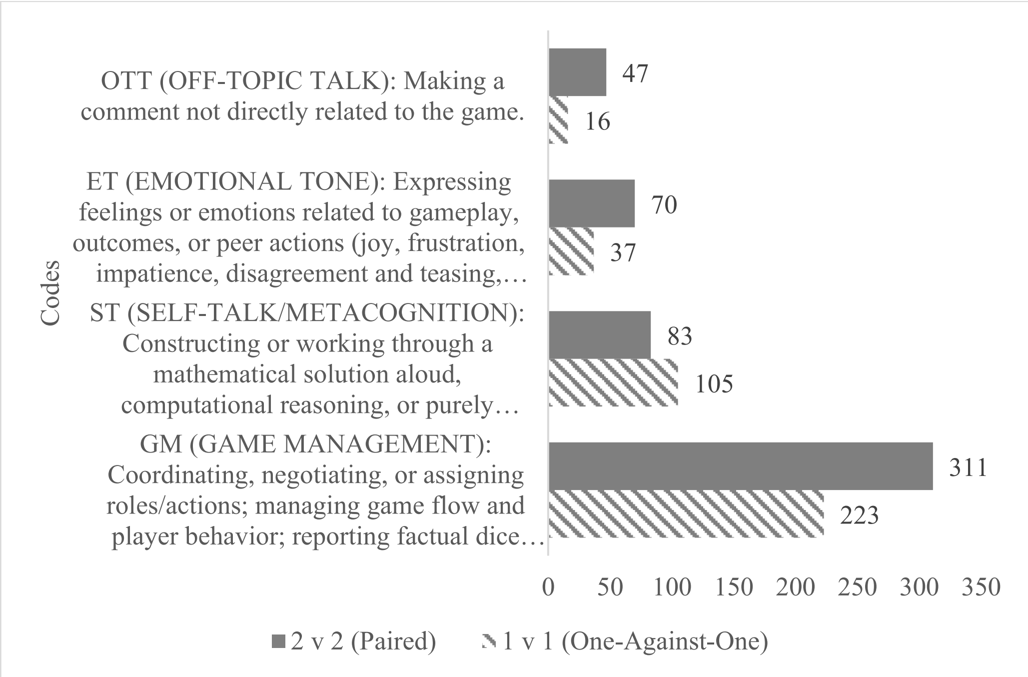

Figure 2. Frequencies of Game Management (GM), Self-Talk/Metacognition (ST), Emotional Tone (ET), and Off-Topic Talk (OTT) in Paired vs One-against-one Games

Game Management (GM) interactions were markedly higher in 2v2 play (311 vs. 223), reflecting the greater need for coordination and negotiation in a four-person structure. While increased GM reflects active engagement, it does not always equate to productive reasoning. Teachers directly linked this to the game management challenges; Ashleigh noted that managing interpersonal dynamics could override mathematical focus, including silent refusals or emotional outbursts. Joel observed “glaring problems working collaboratively,” and Emily saw pairs sidetracked by rules or dominant players. Student reflections corroborated this, with several students expressing similar views. One student preferred 1v1 precisely because “you don’t need to decide together, which takes up time,” while another described frustration in 2v2 when “my partner didn’t let me have a turn.” This highlights how the negotiation required in 2v2 could be perceived as challenging or lead to unequal participation (Plass et al., 2013).

Emotional Tone (ET) also more than doubled in 2v2 play (70 vs. 37), echoing teacher accounts of heightened social-emotional dynamics, both positive and negative. The paired format amplified celebrations, encouragement, and playful banter, but also increased frustration, teasing, and disengagement. Ashleigh’s and Joel’s examples show that these emotions could spill over into behaviour that disrupted reasoning and required teacher intervention. While some students valued the relational benefits of 2v2, stating “I love teamwork” or “Because you have a friend to help you,” the increased negative tone also aligns with the frustrations reported by students regarding partner dynamics, such as feeling sidelined when a partner dominated play or took control of the turns. Self-Talk (ST) was higher in 1v1 games (105 vs. 83), consistent with individual formats allowing players to think aloud independently without constant coordination. Student preferences for 1v1, such as “I don’t need to discuss things. I have my own opinion,” underscore this value for independent thought and streamlined decision-making, which could improve game flow but limit opportunities for co-constructed reasoning. Off-topic talk (OTT) was also far more common in 2v2 games (47 vs. 16), underscoring the challenge of redirecting attention in larger groups; some digressions aided social bonding, but others detracted from mathematical engagement.

Collectively, this data and teacher and student reflections suggest that while 2v2 games foster more visible coordination and emotional engagement, they also introduce greater management complexity and a higher risk of non-mathematical talk. Teachers recognised these were not game flaws, but a natural consequence of the larger-group format. As Ashleigh noted, explicitly teaching and modelling “how to work together” is crucial, ensuring increased Game Management (GM) and Emotional Tone (ET) become vehicles for collaboration and reasoning rather than barriers (Dillenbourg, 1999).

Discussion

Synthesising findings across the three data sources, we see that game format significantly influenced the nature and depth of mathematical reasoning, peer support, and socio-emotional dynamics during gameplay. While novel for teachers, 2v2 (paired) formats emerged as powerful stimulus for richer collaborative reasoning, despite presenting distinct management and social challenges.

Teachers unanimously expressed enthusiasm for the richer discussions and opportunities for mathematical reasoning in 2v2 games. They observed students engaging in explicit reasoning, like discussing prime numbers and factors, justifying ideas, and evaluating peers’ strategies. This qualitative observation is strongly supported by the student interaction data, with all reasoning behaviours notably more frequent in 2v2 games than 1v1 (see Figure 1). In 2v2 games, reasoning became distributed, dialogic, and responsive, with ideas immediately evaluated and justified, inviting critique and negotiation, grounded in shared team goals. This contrasts with 1v1, where reasoning was primarily for self-validation or outmanoeuvring an opponent, with justifications rarely inviting discussion. This convergence of teacher perceptions and interaction data demonstrates that 2v2 transforms the activity from an individual task into a collaborative one, demanding more explicit communication, shared planning, and consideration of multiple perspectives. What is crucial is that, in a 1v1 setting, neither player needs to say anything.

Typically, one-against-one play can be completed silently, with each player thinking privately, much like doing mental arithmetic or silent reading. By contrast, 2v2 requires overt verbalizing between partners, meaning that thinking becomes socially visible and linguistically expressed. This shift from silent, internal reasoning to spoken, shared reasoning adds a vital dimension: language becomes both the *medium* and the *evidence* of mathematical thinking. Talking not only facilitates coordination but also deepens conceptualization, engaging students in articulating, negotiating, and refining ideas that might otherwise remain unspoken.

Teachers frequently observed students actively clarifying mathematical ideas and offering peer support in 2v2 games, with some naturally taking on “teaching roles” and others strengthening emotional connections through offering and accepting assistance (Dondio et al., 2023; Ke & Grabowski, 2007; McFeetors & Palfy, 2018). This aligns closely with the student data regarding the reasoning category “Prompting and Helping”. The nature of “helping” differed by format: In 1v1 games, help often carried a competitive or corrective tone, framed as “telling” and frequently resisted, with interactions sometimes openly provocative. In 2v2 games, “prompting and helping” occurred both within teams (encouraging contributions and discussing strategy) and across team boundaries (voicing questions to the group and offering strategic suggestions to opponents). These cross-team exchanges blended competition and collaboration, showing awareness of the broader game state and inviting shared reasoning. However, a competitive edge could still surface in team formats, occasionally overriding opportunities for collaboration.

Student reflections further support these social dynamics. While a minority preferred 1v1 for autonomy, nearly two-thirds preferred the 2v2 team-based format, often citing relational reasons like “having someone else’s opinion,” “teamwork,” and “having a friend to help.” This preference for collaboration and shared experience underscores the social benefits of 2v2.

Despite the many benefits, teachers and students identified significant management challenges with the 2v2 format, impacting game flow and mathematical reasoning (Dillenbourg, 1999). These issues included managing four players, coordination, equitable participation, and interpersonal issues. Game Management (GM) interactions were markedly higher in 2v2 (311 vs. 223), reflecting the greater need to coordinate moves and manage player behaviour. Emotional Tone (ET) more than doubled in 2v2 play (70 vs. 37), indicating amplified social-emotional dynamics, both positive and negative (e.g., celebrations, but also frustration and disengagement). Off-Topic Talk (OTT) was also far more common in 2v2 games (47 vs. 16), suggesting that managing more players often required redirecting attention. Conversely, Self-Talk (ST) was higher in 1v1 games (105 vs. 83), allowing for independent thought. Teachers noted that 2v2 games “took longer to play” due to increased negotiation and observed communication breakdowns or dominant players. These observations are echoed in student reflections, where some preferred 1v1 for “streamlined decision-making” and “calmness,” while some 2v2 players expressed frustration due to unequal participation. Crucially, teachers recognised that these challenges were not inherent flaws in the games but a consequence of the larger group size and the novelty of paired play. They highlighted the need to explicitly teach and model “how to work together when playing a game,” advocating for scaffolding collaborative skills to maximise the games’ potential for rich mathematical reasoning.

Finally, the fact that connecting interactions were far less common than other reasoning categories indicates that such moments are not automatic outcomes of play, but rather valuable openings where games can be harnessed to help students make meaningful links between gameplay and mathematical ideas.

Practical Implications

The collaborative efficacy of 2v2 games stems from their intentional design and facilitation. Effective 2v2 play, often unfamiliar to students, requires explicit teacher modelling and instruction in teamwork. Teachers should demonstrate how to:

- discuss and compare strategies
- justify and explain choices
- offer and accept help
- navigate disagreements constructively.

Drawing on Dillenbourg (1999), enhancing collaborative potential in 2v2 games involves:

- setting initial conditions: pair students carefully and assist them in allocating roles (e.g., dice roller, recorder);
- scaffolding interaction rules: use prompts (e.g., requiring partner agreement before moves) to maintain focus and reduce dominance;
- monitoring and regulating interactions: circulate to deliver targeted prompts, encourage mathematical noticing, ensure contributions are equally distributed, and redirect off-task groups.

While the games provide a rich context for students to connect with mathematical concepts, this type of reasoning may not happen spontaneously. The data suggests that teacher facilitation is necessary to help students move from simply noticing mathematical properties to using them strategically. Teachers may need to act as a bridge, guiding their students toward deeper connections by asking targeted questions. For example, after a student remarks in *Three-in-a-Row Lucky Numbers,* "We seem to be rolling 24 a lot," it is the teacher's role to extend that noticing into a mathematical connection. The teacher could prompt, "Why do you think this is?" or "Why do you think rolling 24 is good for you in this game?" This type of intervention can shift the focus from just noticing something to actively using the mathematical ideas to inform strategy, leading to a broader and more purposeful engagement with mathematics.

A structured post-game discussion can further support this shift. The relative rarity of "reflecting" in the data suggests that students are not naturally inclined to analyse their own play. A debriefing session provides space to do so, allowing students to explicitly link their decisions to underlying mathematical concepts. This not only reinforces understanding but also develops their capacity for strategic, long-term mathematical thinking. Replaying the game after reflection strengthens this process: by applying new insights in a second round, students move from playing simply to win toward playing to learn (Klooger et al., 2025).

To keep reasoning central during intense competition, teachers can integrate "reasoning prompts," such as providing each player or team with prompt cards to be used before or after a move. This encourages deliberate reasoning during gameplay, regardless of the format. Table 3 provides a series of suggested prompts for supporting strategic thinking during gameplay. These prompts align with the eight identified reasoning codes, actively targeting mathematical reasoning by asking players to propose moves, forecast outcomes, convince others, explain choices, clarify actions, seek/offer help, reflect, predict an opponent's move and connect mathematical ideas. This alignment ensures that the prompts do not simply add conversation but actively target the forms of mathematical reasoning most associated with rich gameplay in the findings. Combining intentional game format selection with explicit teaching of collaborative skills and active facilitation of reasoning allows teachers to make use of both 1v1 and 2v2 structures, fostering individual reasoning alongside teamwork, dialogue, and the co-construction of mathematical ideas.

Table 3. Suggested Teacher Prompts for Supporting Strategic Thinking in Gameplay

Game Format	Example Prompt	Reasoning Code (s)	Purpose
1v1 (One-Against-One Game)	"What would you do if you were me?"	Generating, Predicting	Encourages perspective-taking and anticipatory reasoning
	"What do you think I'm going to do?"	Predicting	Highlights opponent's strategy and anticipation
	"I'll explain my thinking before deciding."	Justifying, Clarifying	Models reasoning and transparency
	"I'll convince you this is the best move for me."	Justifying, Evaluating	Promotes argumentation and critical evaluation
	"Suggest a different move I could have made."	Prompting/Helping, Reflecting	Encourages critique and alternative strategies
	"Explain exactly what you just did."	Clarifying	Builds precision and accountability
2v2 (Paired Game)	*"What are we going to do?"*	*Generating, Predicting*	Fosters joint decision-making
	"How can we figure this out together?	Generating, Clarifying	Encourages shared problem-solving
	"Here's one idea, what do you think?"	Generating, Evaluating	Invites negotiation and co-construction
	"Let's each explain our thinking before deciding."	Justifying, Clarifying	Invites negotiation and co-construction
	"Convince me this is the best move."	Justifying, Evaluating	Encourages persuasion and reasoning
	"Suggest a better option."	Prompting/Helping, Generating	Promotes idea refinement
	"What could we have done differently?"	Reflecting	Supports collective reflection

Conclusion and Limitations

This study offers new insights into how gameplay structures shape student interaction in mathematics classrooms. However, a key limitation is inferring internal thinking and participation from observable behaviour. The analytical framework used in this study assumes that reasoning behaviours can be inferred through students' dialogue and overt actions. However, these also have internal aspects that may not be externally visible or verbalised. Students may think deeply, regulate emotions, or participate nonverbally, yet these internal processes remain uncaptured by a coding scheme focused solely on spoken interaction. This particularly impacts the third research aim concerning student-to-student interaction. While triangulation from teacher interviews, student reflections, and transcripts boosted credibility, it could not fully bridge the gap between observable behaviour and internal experience. Future studies could use think-aloud protocols or video-based self-reflection methods to more fully assess internal processes, particularly metacognitive and affective dimensions.

Future studies could also include pre- and post-reflection data to examine whether students' experiences with 1v1 and 2v2 gameplay influence their attitudes, engagement, and perceived learning with games. Comparing students' expectations with their lived experiences would provide a clearer understanding of how exposure to different formats shapes students' preferences and participation.

In summary, our findings indicate that while 1v1 games support individual strategic thinking and autonomy, the 2v2 format offers a uniquely rich environment for collaborative mathematical reasoning and peer support, fostering shared understanding and distributed problem-solving. However, the advantages of 2v2 gameplay are not inherent; they depend on teacher practice**.** Through intentional facilitation, scaffolding of collaboration, and purposeful questioning, teachers can transform paired gameplay into rich opportunities for mathematical reasoning. Ultimately, collaboration flourishes not through format alone but through the teacher's deliberate orchestration of learning.

Statements and Declarations

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Appendices

Appendix 1. Framework Alignment Table (Construct Mapping)

Our code	Nearest SSMT category (Trakulphadetkrai, 2022)	Nearest Loong et al. (2013) descriptor	Note on fit/mismatch
Generating ideas Proposing a mathematical strategy, move, or action to be taken.	Idea Formulation (Exploration)	Generalizing/ Conjecturing	SSMT: Trakulphadetkrai's framework explicitly has "Idea Formulation (Exploration)" which includes asking peers to generate ideas/solutions and providing them autonomously. Loong et al. (2022): "Generalizing" is a key reasoning action heading in their simplified rubric. They identify conjecturing as a level of "Generalizing," referring to ideas that are often testable claims or hypotheses within a problem-solving context.
Evaluating Expressing support, disagreement, or assessment of an action, strategy, or outcome (one's own or a peer's).	Idea Formulation (Evaluation)	Analyzing	SSMT: Trakulphadetkrai's framework explicitly has "Idea Formulation (Evaluation)" which includes asking peers to evaluate and providing positive or negative evaluations. Loong et al. (2013): "Analysing" is one of the three key reasoning action headings in their simplified rubric.
Justifying Explaining the reasoning behind a decision, move, or strategy, providing a mathematical or strategic rationale to another person.	Justification	Justifying	SSMT: Trakulphadetkrai's framework has a direct talk type named "Justification," which includes asking for and providing justification/reasoning. Loong et al. (2013): "Justifying" is one of the three key reasoning actions headings in their simplified rubric.
Clarifying Making the situation or idea clearer, often for the benefit of others, to	Clarifying	Generalizing/ Explaining	SSMT: Trakulphadetkrai's framework has a direct talk type named "Clarification," which includes asking for and providing clarification. Loong et al. (2013): While not a direct

Our code	Nearest SSMT category (Trakulphadetkrai, 2022)	Nearest Loong et al. (2013) descriptor	Note on fit/mismatch
ensure shared understanding of ideas, game state, rules, or opponent's actions. The act of clarifying predominantly involves ensuring procedural and numerical accuracy.			category, they discuss actions like explaining under their "Generalizing" category. A student clarifying an idea or a move would be engaged in explaining their thinking more clearly.
Predicting Speculating what might happen next in the game, including hopes, fears, guesses about future events, or estimating probabilities.	Speculation / Prediction	Analyzing/ Predicting	SSMT: Trakulphadetkrai's framework includes "Speculation / Prediction" as a talk type. Loong et al. (2013): While not a direct category, they discuss actions like predicting and hypothesizing under their "Analyzing" category.
Reflecting and Self-Talk Looking back on previous actions or decisions, considering what could have been done differently, commenting on past mistakes, or planning future improvements based on prior experience. Constructing or working through a mathematical solution aloud, computational reasoning, or purely internal cognitive processing.	Metacognition/ Self-Talk	Justifying/ Reviewing	SSMT: Trakulphadetkrai's framework explicitly includes "Metacognition," which covers "Self-Talk," which is described as verbalizing one's thinking process, self-reflection and correction. Loong et al. (2013): While not a direct category, they discuss actions like reflecting under their "Justifying" category.
Helping/Prompting	Thinking Facilitation	—	SSMT: Trakulphadetkrai's framework includes "Thinking Facilitation," which

Our code	Nearest SSMT category (Trakulphadetkrai, 2022)	Nearest Loong et al. (2013) descriptor	Note on fit/mismatch
Soliciting a response, input, decision, or action from a peer that constructively advances the shared task or helps/guides a peer to execute a task correctly.			involves asking peer questions to check that they understand something or to help guide others' thinking.
Connecting Making links between mathematical ideas, strategies, or representations.	Making Connections	Generalizing/ Analyzing	SSMT: Trakulphadetkrai's framework has a direct talk type named "Making Connections," which includes asking for and providing clarification. While "Connecting" is not a standalone action in the Loong et al. (2013) rubric, the underlying principle of making connections is integral to several of their reasoning actions. For instance, "Generalizing" involves connecting instances or concepts and applying them.
Off-Topic Talk Making a comment not directly related to the game.	Non- explicitly mathematical	—	SSMT: Trakulphadetkrai's framework categorizes talk as "explicitly mathematical" or "non-explicitly mathematical". Off Topic Talk is considered in their study to be non-explicitly mathematical.
Game Management Coordinating, negotiating, or assigning roles/actions; managing game flow and player behaviour; reporting factual dice rolls or scores; discussing/enforcing rules related to game mechanics.	Non- explicitly mathematical/ Role allocation	—	SSMT: Trakulphadetkrai's framework categorizes procedural talk that assigns, negotiates, or enforces roles as "Role Allocation", including disputes. They code these as non-explicitly mathematical interactions.

Our code	Nearest SSMT category (Trakulphadetkrai, 2022)	Nearest Loong et al. (2013) descriptor	Note on fit/mismatch
Emotional Tone Expressing feelings or emotions related to gameplay, outcomes, or peer actions (joy, frustration, impatience, disagreement and teasing, hope etc.).	—	—	We added this code for the gameplay context.

APPENDIX 2. Games Used in This Study

Game 1: Skip Counting Bingo (Russo & Russo, 2018)

Game Materials and Mathematical Focus	Game Rules (One-Against-One)	Game Rules (Paired)
Materials: · 120-chart gameboard · Game dice: one 6-sided dice (alternatives: 10-, 12-, or 20-sided) · Five counters of the same color for each player Understanding · Recognize that some numbers belong to multiple skip-counting sequences (many factors), while primes have only two. · Understand factors as dice rolls, and multiples as results of repeated addition/multiplication. Fluency: · Develop fluency with skip counting and identify counting patterns. Problem solving: · Identify numbers with many factors to maximize Bingo chances. · Understand that lower numbers are often reached first (e.g., 60 before 120). · Strategically avoid selecting prime numbers or those not in multiple skip-counting patterns. Reasoning:	· Each player chooses five numbers greater than 20 and marks them on the 120-chart, which serves as the gameboard. · The first player rolls the dice, and everyone starts counting by the number on the dice, using the 120-chart to keep track. · Counting continues until one of the player's chosen Bingo numbers is called. That player removes their counter from the gameboard. · The next player rolls the dice, and counting begins again. · Play continues until one player removes all five counters and calls "Bingo!" to win.	· Each pair chooses five numbers greater than 20 and marks them on the 120-chart, which serves as the gameboard. · The dice are rolled to determine which pair will begin. The pair with the highest roll starts and play proceeds clockwise. · The first pair rolls the dice, and everyone counts by the number shown, using the 120-chart to keep track. · Continue counting until one of the pairs' chosen Bingo numbers is called out. That pair removes their counter from the gameboard. · The next pair rolls the dice and counting resumes. · Play continues until one pair removes all five counters and calls "Bingo!" to win.

Game Materials and Mathematical Focus	Game Rules (One-Against-One)	Game Rules (Paired)
· Analyze trade-offs, like 60 being a "better" number than 24 due to more factors, but 24 being landed on earlier.		

Game 2: Doubles Bingo (Russo, 2016)

Game Materials and Mathematical Focus	Game Rules (One-Against-One)	Game Rules (Paired)
Materials: 120-chart gameboard Game dice: one 6-sided dice (alternatives: 10-, 12-, or 20-sided) Five counters of the same colour for each player Understanding: · Understand doubling as adding a number to itself or multiplying by two. · Recognize each number in a doubling sequence is double the previous number. · Distinguish doubling (adding increasing amounts) from skip counting (adding constant amounts). · Identify that doubles are even numbers because two is a factor of all doubles. Fluency: · Develop fluency in doubling and partitioning with place value. Problem solving: · Analyze which doubling sequences can reach selected numbers based on the rolled start	· Each player chooses five numbers greater than 20 and marks them on the 120-chart, which serves as the gameboard. · The first player rolls the dice, and everyone starts doubling from the number rolled, using the 120-chart to keep track. · Doubling continues until one of the player's chosen Bingo numbers is called. That player removes their counter from the gameboard. · The next player rolls the dice, and doubling begins again. · Play continues until one player removes all five counters and calls "Bingo!" to win.	· Each pair chooses five numbers greater than 20 and marks them on the 120-chart, which serves as the gameboard. · The first pair rolls the dice, and everyone starts doubling from the number rolled. · Doubling continues until a pair's chosen Bingo number is called. That pair removes their counter from the gameboard. · The next pair rolls the dice and doubling resumes. · Play continuesuntil one pair removes all five counters and calls "Bingo!" to win.

Game Materials and Mathematical Focus	Game Rules (One-Against-One)	Game Rules (Paired)
value. Reasoning: · Recognize that odd numbers will not appear in doubling sequences. · Understand the largest reachable number depends on the dice used (e.g., maximum 112 with 10-sided dice, 120 with 20-sided dice rolling 15). · Note that even numbers where half is odd (e.g., 66) are excluded from doubling sequences.		

Game 3: Choc-Chip Cookies Game (Russo et al., 2022)

Game Materials and Mathematical Focus	Game Rules (One-Against-One)	Game Rules (Paired)
Materials: Choc-Chip Cookie gameboard. Game dice: one 6-sided dice (alternatives: 10-, 12-, or 20-sided) Understanding: · Understand that the commutative property (e.g., 4x5 vs. 5x4) represent different structures despite producing equal quantities. · Recognize that easily multiplied or skip-counted numbers support efficient calculation. Fluency: · Apply partitioning and known facts to simplify multiplication. · Represent multiplicative	· Each player rolls the game dice to determine how many choc-chips to place on each cookie in a chosen row. · The choc-chips are arranged so the total number can be easily identified without counting individually. The total number of choc-chips used becomes the player's score for that round. The game continues for five rounds, with players filling all rows on the choc-chip cookie gameboard.	· Each pair rolls the game dice to determine how many choc-chips to place on each cookie in a chosen row. · The partners arrange the choc-chips together so the total can be easily identified without counting individually. The total number of choc-chips used becomes the pair's score for that round. · The game continues for five rounds, with all rows completed on the choc-chip cookie gameboard. · The pair with the highest total score after five rounds wins.

Game Materials and Mathematical Focus	Game Rules (One-Against-One)	Game Rules (Paired)
situations using models, moving from concrete (smaller dice) to abstract (larger dice) applications that draw on the distributive property. Problem solving: · Select optimal rows: pair larger dice rolls with rows containing more cookies to maximize score. · Plan ahead and anticipate opponents' moves. · Use landmark numbers (e.g., 10, 15) to simplify calculations. Reasoning: · Understand that all dice outcomes are equally probable. · Strategically assign expected roll ranges to rows to reserve high/low-value rows appropriately.	· The player with the highest total score after five rounds wins.	

Game 4: Reverse Landgrab (Russo & Russo, 2021)

Game Materials and Mathematical Focus	Game Rules (One Against-One)	Game Rules (Paired)
Materials: Grid gameboard Game dice: one 20-sided OR two 10-sided dice (to create a 2-digit number) Understanding: · Distinguish composite numbers (many factors/arrays) from prime numbers (only 1 and itself as factors, forming "skinny" 1xN arrays).	· Each player rolls the dice to generate a number representing an area to claim. · The player draws a rectangle on the grid that corresponds to one of the multiplication facts for that number (e.g., a roll of 12 allows 12×1, 6×2, or 4×3).	· Each pair rolls the dice to generate a number representing an area to claim. · The partners collaborate to draw a rectangle that matches one of the multiplication facts for that number (e.g., a roll of 12 allows 12×1, 6×2, or 4×3). · The rectangle is colored and labelled with the multiplication fact used.

Game Materials and Mathematical Focus	Game Rules (One Against-One)	Game Rules (Paired)
· Understand factors as length and width of rectangles. Fluency: · Develop fluency and flexibility with number facts. · Use arrays to represent multiplicative situations. Problem solving: · Identify all possible rectangles for a given product. Reasoning: · Understand the impact of grid size (e.g., a 12x15 board means a roll of 17 results in a missed turn). · Analyze how the grid size affects possible moves and outcomes.	· The rectangle is colored and labelled with the multiplication fact used.	

Game 5: Three-in-a-row Lucky Numbers (Russo, 2018)

Game Materials and Mathematical Focus	Game Rules (One-Against-One)	Game Rules (Paired)
Materials: 120-chart gameboard. Game dice: one 6-sided and 10-sided (alternatives: two 10-sided, or one 6-sided and one 20-sided die). Understanding: · Recognize that some numbers have many factors (belonging to several counting patterns) while others have very few. · Understand that a factor divides another number with no remainder. · Define a prime number as having	· Each player rolls both dice and uses the numbers rolled to form a multiplication fact. The fact is represented using a model chosen by the teacher (e.g., groups, arrays, or skip counting). · The product is calculated and marked on the chart. · If the number is already taken, it becomes a "lucky number," allowing the	· Each pair rolls both dice and uses the numbers rolled to form a multiplication fact. · The partners represent the fact together using the teacher's chosen model (e.g., groups, arrays, or skip counting). The product is calculated and marked on the chart. · If the opposing pair has already marked that number, it becomes a "lucky number," allowing the team to choose

Game Materials and Mathematical Focus	Game Rules (One-Against-One)	Game Rules (Paired)
only 1 and itself as factors. Fluency: · Develop fluency and flexibility with number facts. Problem solving: · Identify "lucky" prime number candidates (e.g., greater than 7 or ending in 3). · Understand that some numbers (e.g., 16) are rolled more often because they have many factors, while others (e.g., 13) occur less frequently. Reasoning: · Analyze the probability of rolling certain numbers based on their number of factors.	player to choose another number on the chart. · The first player to achieve three rows in any direction wins.	another number on the chart. · The first pair to achieve three sets of three in a row (horizontally, vertically, or diagonally) wins.

APPENDIX 3. Code Book

This codebook outlines the categories used to analyze student dialogue during gameplay. Each code captures a distinct form of reasoning or interaction observed across all games. Illustrative excerpts are included to clarify intent.

Code		Purpose/Definition	Sub-Themes	Illustrative Examples
1	Generating	Proposing or adapting a mathematical move, strategy, or procedural action.	Offering strategies; Stating intentions; Rule innovation	"Should we do 5 × 2 or 1 × 10?" "Let's save that spot." "Instead of 9 we can do 90."
2	Evaluating	Expressing agreement, disagreement, or judgment about a move, strategy, or outcome.	Agreement / disagreement; Corrections / blame; Assessment of performance	"That's not a good idea." "You took so long." "I'm going to win."
3	Justifying	Explaining or defending a decision using mathematical or strategic reasoning.	Explaining decisions; Mathematical justification; Defending behavior	"It doesn't work in any timetables." "We could have gotten higher if we swapped 9 and 10." "I swear that's what I rolled."
4	Clarifying	Making information explicit for shared understanding, ensuring procedural and numerical accuracy.	Stating results; Reiterating numbers; Explaining rules / procedures	"17 and 1 is 18." "Twenty-eight." "No, you're trying to get more land."
5	Predicting	Anticipating future events, outcomes, or opponent actions.	-	"If you roll a 10, my risk is worthless." "I think you're going to win." "We need 27 to win."
6	Reflecting	Commenting on previous actions or lessons learned from play.	Reviewing past play; Learning from experience; Meta-commentary on learning	"We should have put the 9 there." "Next time I'll play it safer." "This game really makes me think."

Code		Purpose/Definition	Sub-Themes	Illustrative Examples
7	Prompting/Helping	Providing or asking for help	-	“Come on, start counting, I’ll help.” “Where should we put it?”
8	Connecting	Linking mathematical ideas or representations or drawing on prior knowledge.	-	“Thirteen is a prime number.” “Double 10 is 20 because 5×2=10.”
GM- Game Management		Managing the flow of play, rules, or reporting game state.	Turn-taking; Rules / conduct; Reporting dice rolls; Game-state setup	“Okay, you roll first.” “No silly stuff.” “Thirteen.”
ET- Emotional Tone		Expressing affective responses to gameplay or peer actions.		“Yay!” “He’s annoying.” “OMG, we got it!”
ST- Self Talk / Metacognition		Thinking aloud or verbalizing internal reasoning.	Calculation; Internal planning; Reflecting on cognition	“1×18 = 18.” “Oh, maybe I’ll do …” “I’m thinking.”
OTT- Off Topic Talk		Utterances unrelated to gameplay.	General commentary; Personal talk; Side remarks	“You know the camera’s on.” “I like Fridays because my mum goes shopping.”

Coding Clarifications and Overlaps

Distinction	**Guideline**
Intent guides coding	Utterances may be multi-coded when they serve more than one clear function.
ST vs Justifying	Self-directed reasoning vs explaining to others.
ET vs Evaluating	Emotion vs analytical judgment.
GM vs Generating	Managing game flow vs proposing strategy.
Clarifying vs Justifying	Making information clear vs explaining *why* something holds.
Short utterances	Context determines function (e.g., “Okay” = GM if procedural; ET if emotive).