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Abstract

Productive struggle (PS) has emerged as a critical construct in mathematics education, emphasizing the importance of persistence, cognitive engagement, and conceptual understanding in problem-solving. However, inconsistencies in its conceptual and operational definitions have hindered its effective implementation and assessment. This study conducted a content analysis of mathematics education literature to examine how PS is defined and operationalized. Findings reveal that while cognitive demand, deeper understanding, and effortful engagement are consistently emphasized, other critical characteristics—such as ingenuity, mathematical relationships, and student-centered learning—are less frequently addressed. Based on these findings, we propose a comprehensive conceptual definition of PS: an effortful, student-centered process involving cognitively demanding tasks that require persistence, ingenuity, and deeper mathematical understanding. Additionally, we identify observable and measurable classroom indicators to support the structured implementation and assessment of PS. By aligning research, teacher training, and instructional practices with this refined conceptualization, this study provides a foundation for enhancing mathematical problem-solving, resilience, and critical thinking in students. Implications for mathematics educators, researchers, and curriculum developers are discussed.

Introduction

For many years, mathematics educators have lacked an agreed-upon conceptual framework for explaining how persistence, creativity, and conceptual understanding interact. The interplay between these elements is increasingly conceptualized as “productive struggle” (PS). Mathematics educators have utilized the PS as a construct across numerous situations within mathematics education (Gray, 2019; Townsend et al., 2018; Warshauer, 2015a). Yet, to our knowledge, the effects of the PS on mathematics teaching and learning remain under-examined. One reason for this lack of measurable and observable effects is the absence of a consensus on the conceptual definition of the PS, which naturally inhibits the ability of researchers to develop instruments to measure the PS. In mathematics education, the term “productive struggle” has been defined in various ways. According to the MIND Research Institute (2021), the “productive struggle is the process of effortful learning that develops grit and creative problem solving". While NCTM's Principles to Actions (2014) defines PS as students delving "more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions" (p. 48). Another definition of the PS is "developing strong habits of mind, such as perseverance and thinking flexibly, instead of simply seeking the correct solution" (Renaissance, 2020). Together these definitions encompass the many conceptualizations as well as actualizations of the PS. However, what remains under examined is how to transform these conceptualizations into measurable attributes of the PS requisite to the operationalization of the construct.

A major obstacle to the advancement of student success in mathematics is often the theory-to-practice conundrum. We argue that the PS is anomalous in that teachers are committed to realizing it in their classrooms despite a lack of theoretical and empirical support. Thus, if researchers could develop tools to support the actualization of the PS they would possibly face a less than normal degree of teacher resistance. It is evident there is a lack of consensus on the most relevant considerations and expectations guiding the implementation of the PS. The operationalization and conceptualization of the PS within mathematics education can bridge this gap. To this end, we conducted a content analysis of the literature examining the PS in mathematics to provide a guiding understanding for operationalization.

Review of Literature

The theoretical underpinnings of the PS can be traced back to earlier research within mathematics problem-solving. Research from the early 90's suggests that mathematical understanding is developed, enriched, and extended by persistence in problem-solving (Hiebert & Werne, 1993). These earlier conceptualizations were most often characterized by considerations of student perseverance, persistence, and ultimately effort. "Effective mathematics teaching uses students' struggles as valuable opportunities to deepen their understanding of mathematics. Students come to realize that they are capable of doing well in mathematics with effort and perseverance in reasoning, sense-making, and problem-solving" (NCTM, 2014, p. 10). This form of persistence supports achievement in mathematics (Roble, 2017). Subsequently, the policy documents of numerous national and international mathematics organizations emphasize the importance of persistence in mathematics problem-solving. For example, the 2019 Trends in International Mathematics and Science Study Framework states "learning mathematics improves problem-solving skills and working through problems can teach persistence and perseverance" (Mullis & Martin, 2017, p. 13). Despite a rich history and dedication to fostering effort, persistence, and perseverance in mathematics, little has changed regarding student success.

Furthermore, we argue that mathematics success requires conceptual understanding, creative thinking, and persistence, all of which can be developed through PS. For instance, there are many mathematics prodigies, most mathematics students, as well as mathematics educators, who believe that experiencing and overcoming challenges in mathematics is a critical part of how we develop a love for mathematics. In our opinion, the following statement provides the best notion for the sense of fulfillment that comes from persevering to resolve a problem:

> A great discovery solves a great problem, but there is a grain of discovery in the solution of any problem.

> Your problem may be modest, but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your means, you may experience the tension and enjoy the triumph of discovery (Polya, 2014, p.17).

Here we posit that a lack of a shared conceptualization and operationalization of the PS is a major hindrance to the teaching and learning of mathematics for and sustained understanding as described in the quotation above.

What is Productive Struggle?

Productive struggle (PS) was first introduced in 2011 by Hiroko K. Warshauer in her dissertation titled "The role of PS in teaching and learning middle school mathematics". The idea of PS has been a hot topic in mathematics education so much there have been conference themes and dissertations dedicated to the idea (e.g., Edgington, 2021; Karatas, 2022; Jarry-Shore, 2021; Roth, 2019). Overall, the idea of PS allows for learning opportunities to arise (Baker et al., 2020), leads to an equitable classroom (Lynch et al., 2018), and is a strategy that gives students more opportunity to grow in their mathematics knowledge (Daily, 2021). Teaching mathematics involves more than just knowing the content. Mathematics teachers are responsible for also developing critical thinkers and problem solvers. One method that is discussed more frequently in recent years is the idea of PS. Some definitions of PS include when learning opportunities arise (Baker et al., 2020), it is a strategy that gives students opportunity to grow in mathematics knowledge (Daily, 2021) and leads to more equitable classroom (Lynch et al., 2018). *Principles to Actions* (NCTM 2014, 2014) defines PS as "understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions" (p. 48). As we can tell with these examples, one could come up with a general description of what PS means, but the various definitions could leave room for misconceptions.

Teachers and the Productive Struggle

Mathematics teachers' classroom practices significantly affect the breadth and depth of student learning (Heibert & Grouws, 2007) and students' disposition towards mathematics (Franke et al., 2007). Mathematics teachers can support students' PS through questioning to help students organize their thoughts through moments of struggle in problem solving (Warshauer, 2015), through a patient disposition, and by acknowledging their persistence (Baker et al., 2020). Monitoring students' PS is a metacognitive strategy for teachers to observe the ways in which students progress toward solving a complex task (Wilson & Conyers, 2016). Through teachers utilizing questioning and placing emphasis on the problem solving process as opposed to the correct solution, Amidon et al. (2020) found that students demonstrated less shame and avoidance towards mathematics. Further, designing curricula requires teachers to engage with PS themselves as they grapple with developing their own greater understanding of the subject matter and plan how to transfer the knowledge to students through challenging tasks (Trinter & Hughes, 2021). A task that elicits PS often requires students to engage in non-routine problem solving and apply conceptual understanding to real-world situations (Stein et al., 2009). Effectively utilizing questioning strategies, observing instances of students' PS, and planning engaging tasks that elicit students to grapple with applying content knowledge are objectives that require teacher training.

Exposure to product struggle through pre-service teacher (PST) training and professional development can impact mathematics teachers' instruction (DuCloux et al., 2018; Jansen et al., 2020). More specifically, recent research on exposing teachers early in their teacher preparation programs as PSTs has shown to have a positive impact on cultivating their expertise and willingness to incorporate PS tasks into their teaching practices (El-ahwal & Shahin, 2020; Sun, 2018; Warshauer et al., 2019; Warshauer et al., 2021). For instance, Warshauer et al. (2021) found that pre-service teachers with little to no prior knowledge of the PS were able to develop their own understanding of ways to utilize PS tasks to support students' understanding of mathematics. PSTs in Warshauer et al.'s (2021) participated in video observations of mathematics lessons from NCTM (2014) vignettes paired with writing assignments that prompted students to reflect. The video observations paired with the written reflections provided evidence of pre-service teachers' conceptualization as PS as an opportunity for deep student learning (Warshauer et al., 2021). Further, analysis of the written responses revealed an area of growth for pre-service teachers and a challenge for teacher preparation programs, the mathematical interpretation of students' struggles. Although pre-service teachers demonstrate an eagerness to implement PS (e.g., Warshauer et al., 2019), supporting PSTs' ability to anticipate and identify students' mathematical struggles in the problem solving process is essential for effective classroom implementation.

Students and the Productive Struggle

Research has shown that students who construct their own methods to solve mathematical problems compared to students who employ procedural methods taught directly in class demonstrate higher levels of content mastery on assessments (Jonsson et al., 2014; Kapur, 2011, 2014). Moreover, allowing students the space to try innovative methods and learn through failure fosters student effort to persist in PS (Granberg, 2016; Granberg & Olsson, 2015). Students' deep mathematics learning is supported by teachers utilizing cognitively demanding tasks that prompt students to connect prior knowledge to new learning and novel scenarios (Warshauer et al., 2015a). Elementary school students in Russo and Hopkins' (2017) study expressed enjoyment and the value of cognitively demanding tasks with scaffolding from the teacher. Despite the burgeoning body of literature on preparing teachers to engage students in PS, there is a dearth of empirical study of its impact on students (Young et al., 2024). Our present study provides an operational and conceptual definition of the PS to assist in guiding future empirical study of PS in mathematics classrooms.

Method

A content analysis was conducted to examine the conceptual and operational definitions of PS in mathematics education literature. Content analysis is a systematic approach to analyzing qualitative data that involves identifying core themes and patterns within a body of text (Krippendorff, 2018; Patton, 2014). This method was selected because it allows for the systematic classification of text into categories and helps to uncover trends in the literature regarding the definition and operationalization of PS in mathematics education. Here the volume of qualitative material is the conceptual and operational definitions present in the mathematics education literature. The present study is guided by two research questions:

1. *How is the productive struggle conceptually defined in mathematics educational research?*

2. *How do the operational definitions of productive struggle inform observable and measurable activities for classroom implementation?*

Data Sources

Over the last decade, literature on the PS has expanded beyond the research spaces to popular media outlets used by teachers to locate instructional resources. Thus, the literature used in the content analysis was both empirical and non-empirical, published between 2011 and 2020, to reflect the full scope of available resources. Literature for this study was identified from a scoping review, please see Young et al. (2024) for systematic search procedures.

After conducting the systematic search (Young et al., 2024), the researchers then created conditional formatting rules to highlight the title or abstract a certain color if it contained a reference or mention of the PS. Next, each researcher was assigned a subsample of documents to read and review for initial eligibility. This initial eligibility included or excluded articles based on a review of the title and abstract. Then, the purpose, aims, and objectives of the remaining documents were examined to ensure that the PS was directly related to the document. The documents were then separated into three tabs within Microsoft Excel: one for articles focused on the PS, another for articles that only made mention of the PS, and a final tab for articles examining the PS in non-mathematics education contexts. Subsequently, 29 articles were selected for inclusion.

Data Analysis

A coding protocol was developed to extract and classify conceptual and operational definitions of PS from the selected articles (Hsieh & Shannon, 2005). To ensure consistency, the research team conducted a training phase in which two articles were coded collectively over a series of meetings. After achieving inter-rater reliability, two researchers independently coded the remaining articles. Discrepancies were resolved through discussion until consensus was reached.

The following information was gathered on the coding form to answer *research question 1*: APA citation, year, abstract, purpose, location (international or United States), participants (i.e., K-12 students, teachers, college students, and/or other), conceptual definition, and operational definition based on the information provided by the authors in the body of the text. Articles were further categorized based on whether they provided only a conceptual definition, only an operational definition, or both. Summary statistics were calculated to assess frequency and percentage trends across the sample (Miles et al., 2014).

To triangulate the findings (Patton, 1999), ten mathematics education researchers were surveyed about their definitions and interpretations for the PS, as well as any references or documentation that guided their definitions. We chose three of the most well documented definitions from each of the researchers surveyed and then used an ordinal scaled survey to gather the perceptions of ten mathematics classroom teachers. This process was conducted to identify the three definitions that would be used as our conceptual definition baseline. These three conceptual definitions can be found in Table 1 below.

Table 1. Baseline Conceptual Definitions for the PS in Mathematics

Authors	Commonly Referenced Definitions
NCTM	"In Principles to Actions: Ensuring Mathematical Success for All, NCTM (2014) defines productive struggle as students delving "more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions" (p. 48)."
MIND Research Institute	Productive struggle is the process of effortful learning that develops grit and creative problem solving. When students face problems they don't immediately know how to solve (like on new assessments), we don't want them to give up. We want them to engage in making connections to things they already know, think creatively and try different avenues towards solutions (MIND Research Institute, 2019).
Hiebert and Grouws	Students' productive struggle refers to students' "effort to make sense of mathematics, to figure something out that is not immediately apparent" (Hiebert & Grouws, 2007, p. 287)

Similarities across the three definitions are bolded and from these similarities eight potential characteristics of the PS in mathematics (i.e., success, deeper understanding, cognitive demand, effort, process, student-centered, mathematics relationships, and ingenuity) were derived and used to guide our exploratory analysis of the conceptual definitions presented within the mathematics education literature (see Table 2 and Table 3). Appropriately, the units of analysis for this content analysis were conceptual and operational definitions which were located by reading each document and extracting the pertinent sections and related citations for PS. Explanation of the coding rationale used to distinguish between conceptual and operational definitions is provided in Table 2, along with an example of each definition category.

Table 2. Code & Characteristic Descriptions and Examples

Code	Description	Example
Conceptual Definition	Authors defined the productive struggle using prior research or theoretical underpinnings	"By students' productive struggles, I refer to a student's effort to make sense of mathematics, to figure something out that is not immediately apparent'" (Hiebert & Grouws, 2007, as cited in Warshauer, 2015a, p. 376).
Operational Definition	Authors provided the operational definition of the productive struggle within their study and defined its measurable attributes	"Productive Struggles were identified by examining whether they were able to recall, reconstruct, or adjust their prior knowledge to become more useful to solve at least parts of the given problem. Productive Struggles were identified by examining whether students were able to engage in activities that addressed their errors such that they gained any more useful insight bringing them closer to constructing the rule or parts of the rule correctly." (Granberg, 2016, p. 39).

Table 3. Characteristics and Descriptions

Characteristic	Description
cognitive demand	A *cognitively demanding* mathematical task requires students to engage in higher-order thinking, reasoning, and problem-solving rather than executing routine procedures. Tasks that maintain a high level of cognitive demand require students to justify their thinking, make connections between concepts, and explore multiple solution pathways (Stein et al., 2009).

Characteristic	Description
deeper understanding	A *deeper understanding* of mathematics goes beyond procedural fluency to include conceptual comprehension, strategic competence, and adaptive reasoning (Kilpatrick et al., 2001). Students achieve deeper understanding when they can explain why mathematical procedures work, make connections between concepts, and transfer their knowledge to new contexts (Hiebert & Carpenter, 1992).
effortful	*Effortful* learning refers to the sustained cognitive and metacognitive engagement that students exhibit when solving mathematical problems. Productive struggle, which requires effortful learning, enhances mathematical reasoning and deepens understanding by encouraging persistence through challenges (Hiebert & Grouws, 2007; Kapur, 2014).
ingenuity	*Ingenuity* in mathematics education refers to students' ability to apply creative and flexible thinking when solving mathematical problems. It involves the capacity to generate novel solution strategies, adapt existing knowledge, and make connections between mathematical ideas (Leikin & Levav-Waynberg, 2007). Creativity and ingenuity are essential components of mathematical problem-solving, fostering deeper engagement and innovation in mathematical thinking (Silver, 1997).
mathematics relationships	Understanding *mathematical relationships* involves recognizing and reasoning about connections between mathematical concepts, representations, and structures. Developing relational understanding, as opposed to instrumental understanding, allows students to see how mathematical ideas are interrelated and applicable across different contexts (Skemp, 1976). The ability to recognize mathematical relationships enhances problem-solving skills and conceptual fluency (Carpenter & Lehrer, 1999).
process	The *process* of learning mathematics involves engaging in reasoning, problem-solving, and sense-making rather than focusing solely on correct answers. The *Standards for Mathematical Practice* emphasize that learning should involve a process where students make conjectures, construct arguments, and persevere in solving problems (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA & CCSSO], 2010).
student-centered	A *student-centered* approach in mathematics education emphasizes active engagement, inquiry, and personal construction of knowledge rather than passive reception of information. This pedagogy aligns with constructivist theories, where students develop mathematical understanding through exploration, discussion, and reflection (Boaler, 2016; Schoenfeld, 1992).
success	*Success* in problem-solving is often tied to perseverance, the ability to apply multiple strategies, and the capacity to generalize mathematical ideas (Kilpatrick et al., 2001).

Finally, to answer *research question 2* on observable and measurable attributes of PS, the research team identified and synthesized key recommendations from operational definitions found in the literature. These recommendations, outlined in Table 5, offer practical guidance for implementing PS in mathematics classrooms.

Findings

The final sample of studies included journal articles, conference proceedings, and dissertations/theses. Based on the data extracted from the $n = 28$ included studies, each study included conceptual definitions (i.e., definitions based on prior research and theory) for the PS (see Table 4). Conceptually, PS was most often described as students' engagement in effortful problem-solving that fosters deeper understanding, persistence, and resilience (e.g., Hiebert & Grouws, 2007; NCTM, 2014). These definitions emphasized the importance of grappling with mathematical concepts, making sense of new information, and persisting through challenges. However, our analysis indicated considerable variation in the wording and emphasis of conceptual definitions, leading to inconsistencies in how the construct is understood and applied. Furthermore, only nine studies (32%) included operational definitions of PS. In the following sections we describe how the PS is conceptually defined, identify the most agreed upon components of the PS, and outline measurable and observable activities for implementing PS in mathematics classrooms.

Table 4. Frequency of Trends Across Study Characteristics

Category	Options	Frequency
Location	United States	26(93%)
	International	2(7%)
Participants	Inservice Teachers	7(25%)
	Preservice Teachers	8(29%)
	K-12 Students	6(21%)
	K-12 Students/Teachers	7(25%)
Operationalization	Yes	9(32%)
	No	19(68%)
Conceptualization	Yes	28(100%)
	No	0(0%)

RQ1: How is the productive struggle conceptually defined in mathematics educational research?

From the conceptual and definitions of PS provided in the literature we then identified keywords and phrases related to each of the eight characteristics (see Table 3) derived from our initial three conceptual definitions (see Table 1). These characteristics highlight the essential elements of PS, illustrating its complexity as both a cognitive and affective experience for students (see Appendix A for full coding). While some definitions emphasize struggle as a means to develop problem-solving skills, others frame it as a vehicle for conceptual understanding and mathematical reasoning. In this section we distill the characteristics of PS described in conceptual definitions (see Table 5).

Table 5 . Sample Characteristics Coding of Conceptual Definitions

Characteristic	Frequency	Sample
cognitive demand	14	● expose a lack of knowledge (Amidon et al., 2020, p. 65) ● not immediately apparent (Heibert & Grouws et al. [2007, p. 387] as cited in DeJarnette, [2017, p. 1359])
deeper understanding	11	● forming and making sense of mathematical ideas (Schoenfeld, 1988; 1992; as cited in Bolyard and Valentine [2019, p. 9]) ● students learn mathematics with deeper meaning ([Dixon et al. 2015; Hiebert & Grouws, 2007; Vygotsky, 1978] as cited in Ewing et al. [2019, p. 1])
effortful	11	● grappling with key mathematical ideas (Hiebert & Grouws [2007, pp. 387–388] as cited in Edwards and Beattie [2016, p. 31]) ● where students expend effort (Hiebert & Grouws [2007] as cited in Franke et al. [2015, p.127])

Characteristic	Frequency	Sample
ingenuity	2	● students who fail are more likely to evaluate their methods (Granberg & Olsson [2015] as cited in Granberg [2016, p.176]) ● and the perception that the path to solving the problem is as important as the solution itself (Heibert and Grouws [2007], MIND Research Institute [2019] , NCTM [2014], and Stiles [2017] as cited in Gray (2019, p. 4)
mathematical relationships	6	● dynamic and connected (Bolyard & Valentine, 2017, p. 9) ● draw on mathematics concepts to determine how to solve tasks (Carpenter & Lehrer, [2014] as cited in Polly [2017, p. 254])
process	4	● necessary components of learning mathematics (Ewing, 2016, p. 936) ● temporary failure is expected and accepted as normal, meaning there is a belief that perseverance will lead to eventual success (Heibert and Grouws [2007], MIND Research Institute [2019] , NCTM [2014], and Stiles [2017] as cited in Gray (2019, p. 4)
student-centered	7	● speculate about possible solutions and experiment (Kapur [2016] as cited in Fisher and Frey [2017, p. 85]) ● judicious telling which involves teachers initiating ideas with students in a way that does not take over students' thinking (Freeburn & Arbaugh, 2017, p. 178)
success	9	● figure something out (Heibert & Grouws et al. [2007, p. 387] as cited in DeJarnette [2017, p. 1359]) ● attempting to perform a task and initially failing can improve learning (Kapur [2016] as cited in Fisher and Frey [2017, p. 85])

The first characteristic of *cognitive demand* describes a challenging task that requires students to engage in higher order thinking (Stein et al., 2009). In PS literature, there is a conceptual characterization of PS tasks that expose students' lack of knowledge as they engage in solving a problem with a solution that is not immediately apparent (Heibert & Grouws et al. [2007, p. 387] as cited in DeJarnette, [2017, p. 1359]). Of note, this characteristic had the highest frequency across conceptual definitions of PS. The second characteristic of *deeper understanding* refers to developing students' understanding of mathematics beyond procedural fluency (Kilpatrick et al., 2001). Conceptual definitions of PS describe learning mathematics with a deeper meaning through opportunities for students to develop and make sense of mathematical ideas (Schoenfeld, 1988; 1992; as cited in Bolyard and Valentine [2019, p. 9]). The next characteristic of *effortful* refers to sustained cognitive and metacognitive engagement in problem solving (Hiebert & Grouws [2007, pp. 387–388] as cited in Edwards and Beattie [2016, p. 31]). In the PS studies in mathematics education in our sample, conceptual definitions include an element of students expending effort as they grapple with mathematical ideas. The characteristic of *ingenuity* was the least frequently occurring (see Table 4). Interestingly there was not a robust connection to flexible, creative thinking in the conceptual definitions. The characteristic of *mathematical relationships* describes making connections between mathematical concepts, representations and structures. Conceptual definitions include that PS struggle tasks should encourage students to interrogate mathematical connections and utilize reasoning in problem-solving (Carpenter & Lehrer, [2014] as cited in Polly [2017, p. 254]). The next characteristic of PS, *process,* describes an emphasis on the solution process over the mathematical solution. In conceptual definitions of PS, failure is described as a necessary component of learning mathematics of which persevering through will lead to mathematical understanding (Ewing, 2016, p. 936; (Heibert & Grouws 2007; MIND Research Institute [2019]; NCTM [2014]; Stiles [2017] as cited in Gray [2019, p. 4]). Conceptual definitions of PS described *student-centered* as providing opportunities for students to explore and investigate potential mathematical solutions (Kapur [2016] as cited in Fisher and Frey [2017, p. 85]) with teacher guidance that does not hijack students' thinking (Freeburn & Arbaugh, 2017, p. 178). Lastly, *success* is characterized by perseverance through failing in solving a problem to eventually figuring something out (see Table 4).

The analysis of conceptual definitions of productive struggle in mathematics education highlights the multifaceted nature of this construct, encompassing cognitive, affective, and pedagogical dimensions. While some characteristics, such as cognitive demand and deeper understanding, were consistently emphasized across definitions, others—such as ingenuity—appeared less frequently, suggesting potential gaps in how productive struggle is framed within the literature. The emphasis on effortful engagement, mathematical relationships, and process-oriented learning reinforces the idea that productive struggle is not merely about reaching a correct answer but about fostering a mindset that values persistence and critical thinking. Additionally, the recognition of student-centered approaches underscores the role of instructional design in facilitating productive struggle in ways that support, rather than impede, student learning. Together, these characteristics provide a more structured framework for understanding how productive struggle is conceptualized, offering valuable insights for researchers and educators seeking to implement and assess this construct in mathematics classrooms. Future research should further explore the role of ingenuity and creative problem-solving within productive struggle, ensuring that conceptual definitions fully capture the depth and complexity of student learning experiences.

RQ2: How do the operational definitions of productive struggle inform observable and measurable activities for classroom implementation?

Based on the observed nine operational definitions (see Appendix B) we provide recommended observable and measurable tasks for mathematics classrooms. Observable indicators included students' willingness to restart problems after initial failure, engage in self-reflection, articulate reasoning during mathematical discussions, and apply new strategies based on feedback (Warshauer, 2015a; Granberg, 2016). These operational definitions emphasized productive struggle as an iterative process in which students encounter challenges, refine their thinking, and develop mathematical resilience.

From these studies, we identified several measurable and observable activities that teachers can use to facilitate productive struggle (see Table 6). These include setting appropriate checkpoints within problem-solving tasks (Townsend et al., 2018), providing probing guidance rather than direct solutions (Warshauer, 2011), and creating a classroom culture where struggle is normalized and supported (Wilburne et al., 2018). Additionally, our findings suggest that productive struggle is best facilitated when teachers anticipate student difficulties and provide timely scaffolding to maintain cognitive demand without reducing the challenge of the task. The absence of consistent operational definitions in the literature underscores the need for future research to develop standardized assessment tools that capture the nuances of productive struggle. By formalizing observable behaviors associated with productive struggle, researchers and educators can better evaluate its effectiveness in promoting mathematical understanding and persistence.

Table 6. Measurable and Observable Activities to Better Operationalize the PS

Measurable & Observable Activities	Citations
Teacher provides explicit feedback that identifies examples of student knowledge and recognizes the challenges overcome	Wilburne et al. (2018) – Praise students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems. Warshauer (2015a) – Teachers integrate struggle as part of doing mathematics by acknowledging students' consternation and encouraging perseverance.
Teacher facilitates the learning process by setting appropriate checkpoints throughout activities	Warshauer (2011) – Teachers provide probing guidance to prompt deeper thinking rather than direct solutions. Townsend et al. (2018) – Students working within their zone of productive struggle completed tasks with guidance from teachers.
Teacher makes accommodations rather than modifications to the activities to maintain the cognitive demand	Wilburne et al. (2018) – Teachers anticipate student struggles and provide productive support. Warshauer (2015b) – Teachers encourage students to reflect on their work rather than focus solely on correct answers
Student restarts a problem and works it in whole or in part based on new information provided or obtained	Warshauer (2011) – Students restart problems after teacher guidance through directed or probing questions. Sengupta-Irving & Argwal (2017) – Students engage in problem-solving with an iterative approach.
When faced with an incorrect solution the student reflects on the solution and then proceeds to apply a more appropriate problem-solving approach or mathematics concept with some degree of accuracy.	Warshauer (2015b) – Students struggle with mathematical concepts and refine problem-solving strategies. Granberg (2016) – Students who fail are more likely to evaluate their methods than those who succeed.
Students participate in mathematics discourse surrounding their personal thought process and others that includes the acknowledgement of challenges and the reasoning surrounding their understanding of mathematics concepts and connections.	Townsend et al. (2018) – Students engage in discourse while working through tasks, refining understanding. Warshauer (2015a) – Teachers use questioning strategies to encourage students to articulate their thinking.
When a student receives negative feedback (i.e., poor grade, incorrect response) the student responds by reflecting on the error and providing a new approach that is correct based on the feedback provided.	Warshauer (2011) – Students adjust strategies after receiving feedback from teachers. Wilburne et al. (2018) – Teachers facilitate discussions on mistakes and misconceptions to normalize errors.

Discussion

Our analysis of the reviewed literature on productive struggle (PS) in mathematics education reveals a variety of conceptual definitions and characteristics, indicating both the complexity and variability of the construct in mathematics education literature. The findings indicate that while there is unanimous agreement on the conceptualization of PS—every study included in our review provided a definition—there exists significant variability in how these definitions articulate the nuances of PS. Thus, using observed frequencies for the common characteristics of prior conceptual definitions, we proffer the following comprehensive conceptual definition for the mathematics PS: *Productive Struggle is an effortful student-centered process characterized by the completion of a cognitively demanding mathematics task that requires persistence, ingenuity, and a deeper understanding of the structural relationships within mathematics problems.*

One of the strongest themes emerging from the data is the emphasis on cognitive demand. This characteristic, which surfaced in 14 of the 28 studies, highlights the importance of challenging mathematical tasks in which students are required to utilize their higher-order thinking, reasoning, and problem solving. This finding emphasizes the use of Stein et al. (2009)'s conceptual framework on the relationship between variables that are related to tasks and students' learning outcomes when learning mathematics. Hence, teachers play a significant role in ensuring that students are engaged in cognitively demanding tasks. Warshauer et al (2015a) argued that teachers must support deep mathematics learning with challenging tasks that help students connect what they know to new ideas. However, the variability in how these tasks are constructed and presented in the classroom remains a point of concern here. Teachers may need further guidance on designing tasks that balance cognitive demand with appropriate levels of support.

Both deeper understanding and effortful emerged to be essential elements of PS as well. Both characteristics appeared in 11 studies. Deeper understanding in mathematics values clear comprehension over memorization. It helps students see how ideas connect. This method helps them solve hard problems. Effortful engagement means students take part actively when engaging in mathematical tasks which builds critical thinking and confidence. Combining deeper understanding with hard work leads to students' success in learning mathematics (Boaler, 2015). Therefore, teachers should be encouraged to create classrooms that encourage productive struggle which helps students see challenges as their opportunity to grow.

The remaining five other characteristics (i.e., ingenuity, mathematical relationships, process, student-centered, success) were mentioned less frequently in the reviewed studies. However, their importance in fostering productive struggle in mathematics education should not be overlooked. These characteristics play a vital role in connecting students to the content on a deeper level, helping them engage with challenging problems and foster critical thinking. By adopting a student-centered approach that emphasizes ingenuity and mathematical relationships, for instance, mathematics teachers can create an environment where students feel empowered to tackle challenging problems in their classrooms. Again, we emphasize the importance of the teacher's role in facilitating their lessons to ensure they provide high-quality instruction to their students. A report by the National Mathematics Advisory Panel (2008) emphasized that high-quality instructional practices can significantly influence students' ability to understand and apply mathematical concepts.

The findings of this study underscore the multifaceted nature of PS from conceptual definitions and the necessity of a consensus on what are the characteristics of PS to guide both research and classroom implementation. While prior conceptual definitions consistently highlight cognitive demand, deeper understanding, and effortful engagement, our analysis reveals gaps in how other critical characteristics—such as ingenuity, mathematical relationships, and student-centered learning—are framed within the literature. These elements, though less frequently emphasized, are essential for fostering a holistic PS experience in mathematics education. Our proposed definition of PS—*an effortful, student-centered process characterized by engagement in cognitively demanding mathematical tasks that require persistence, ingenuity, and a deeper understanding of mathematical structures and relationships*—synthesizes these key characteristics into a cohesive, research-informed framework. By integrating these elements, this definition provides a foundation for developing instructional strategies and assessment tools that support and sustain PS as a powerful mechanism for enhancing students' mathematical reasoning, problem-solving abilities, and overall resilience in learning. Moving forward, aligning research, teacher training, and classroom practices with this refined conceptualization will be critical to ensuring that PS is effectively leveraged to maximize student success in mathematics.

Implications for Teachers and Research

Our findings from Research Question 2 suggest that in order to promote consistency in the mathematics literature regarding the concept of productive struggle, it is essential to formalize the operational definitions that capture its complexities. Teachers can create a common language that enhances the understanding and implementation of this instructional approach by establishing standardized assessment tools and clear observable behaviors associated with productive struggle. This consistency will allow researchers to assess the effectiveness of various strategies and practices across different educational contexts. Ultimately, our goal in this field is that this consistent practice can lead to improved teaching methods that support students' mathematical resilience and understanding. For example, the measurable and observable activities presented in Table 5, such as giving feedback, promoting reflection, and encouraging repeated problem-solving, can be a solid framework for mathematics teachers to assess their teaching practices. This way, teachers can track student engagement and growth by measuring these indicators in class. Through these concerted efforts, the mathematics education community can effectively measure the impact of productive struggle on student achievement.

Conclusion

The findings and recommendations of the present study have important implications for research and praxis related to the productive struggle. The aim of the present content analysis was to identify the current conceptual and operational definitions of the productive struggle present across the mathematics education literature, and then map the key characteristics of the productive struggle cited across studies to develop a comprehensive conceptual definition that reflects the current implementation. The goal of this project was to inform the operationalization and measurement of the productive struggle with mathematics education contexts. Our content analysis of the literature suggests that the productive struggle is often conceptualized by induction or rather through citing the descriptions of others and rarely operationalized. Thus, the direct measurement of the productive struggle as a construct was virtually absent across studies. Unfortunately, the current dependence on conceptual rather than operational definitions limits the ability of researchers to measure the efficacy of the productive struggle and its impact on mathematics teaching and learning. As a result, we hope the results of this study move the field forward through the derivation of a formal conceptual definition of the productive struggle as well as the identification of eight essential elements (see Table 3) of the productive struggle and corresponding measurable attributes associated with these characteristics (see Table 6).

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Appendices

Appendix A. Conceptual Definitions and Characteristics

Citation	Definition	Codes
Amidon et al. (2020)	"For a math problem to truly be a problem, it must expose a lack of knowledge and promote productive struggle. Thus, every time a student engages in solving a mathematics problem, the possibility of struggle and, consequently, the possibility of shame exist (Muris & Meesters, 2014; Nathanson, 1992; Tangney et al., 2013; as cited in Amidon et al., [2020, p. 65])."	cognitively demanding
Bolyard and Valentine (2017)	"Productive struggle reframes the discipline as dynamic and connected, positioning the learner as an active participant in forming and making sense of mathematical ideas (Schoenfeld, 1988; 1992; as cited in Bolyard and Valentine [2019, p. 9])."	mathematical relationships, effortful, deeper understanding
DeJarnette (2017)	"Productive struggle in mathematics education most typically refers to extending "effort to make sense of mathematics, to figure something out that is not immediately apparent" (Hiebert & Grouws, 2007, p. 387; as cited in DeJarnette, [2017, p.])."	effort, success, cognitively demanding
Edwards and Beattie (2016)	"We use the word struggle to mean that students expend effort to make sense of mathematics, to figure something out that is not immediately apparent. We do not use struggle to mean needless frustration or extreme levels of challenge created by nonsensical or overly difficult problems...The struggle we have in mind comes from solving problems that are within reach and grappling with key mathematical ideas that are comprehendible but not yet well formed (Hiebert & Grouws, 2007, pp. 387–388; as cited in Edwards and Beattie [2016, p. 31])."	success, cognitively demanding, success, effortful, cognitively demanding
Ewing (2016)	"Productive struggle is a necessary component of learning mathematics with meaning and refers to students grappling to make sense of problems (Hiebert & Grouws, 2007; as cited in Ewing [2016, p. 936])."	process, cognitively demanding
Ewing et al. (2019)	"Students learn mathematics with deeper meaning when they engage in productive struggle—grappling to make sense of problems within their zone of proximal development (Dixon et al. 2015; Hiebert & Grouws, 2007; Vygotsky, 1978). Warshauer (2014) added teachers can engage students in productive struggle by asking them questions instead of telling them the answers; questioning helps students organize their thoughts as they struggle to make sense of problems (as cited in Ewing et al. [2019, p. 1])."	deeper understanding, effortful, mathematical relationships, success
Fisher and Frey (2017)	"Researcher Manu Kapur (2016) has developed the theory of productive struggle—the idea that attempting to perform a task and initially failing can improve learning. Productive struggle, as Kapur envisions it, occurs in two phases. First, students are given a problem or task that they probably can't solve, and they're encouraged to speculate about possible solutions and experiment. Next, after their initial failed attempts, they receive instruction that will assist them in successfully completing the task, and they are encouraged to try again (as cited in Fisher and Frey [2017, p. 85])."	success, cognitively demanding, student-centered, success
Franke et al. (2015)	"productive struggle," where students expend effort to make sense of mathematics and figure out something that is not immediately apparent (Hiebert & Grouws, 2007; as cited in Franke et al. [2015; p.127])."	effort, deeper understanding
Freeburn and Arbaugh (2017)	"Teachers can support students' productive struggle is by using judicious telling, which involves teachers initiating ideas with students in a way that does not take over the students' thinking (Lobato et al., 2005). More specifically, judicious telling can involve revoicing students' contributions to highlight an important mathematical idea, redirecting students to a solution pathway they are unable to find on their own, clarifying directions or contexts in mathematical tasks, and conveying terminology for students' mathematical ideas (Chazan & Ball, 1999; Freeburn, (2015); as cited in Freeburn and Arbaugh [2017, p. 178]).	student-centered
Granberg (2016)	"students who fail are more likely to evaluate their methods than students who succeed (Granberg & Olsson, 2015). It appears that making, discovering and correcting errors may generate effort that can engage students in productive struggle (as cited in Granberg [2016, p. 176])."	ingenuity, effort

Citation	Definition	Codes
Gray (2019)	"There is agreement that productive struggle occurs when: • temporary failure is expected and accepted as normal, meaning there is a belief that perseverance will lead to eventual success, • when the path for solving the problem comes from the student and not authority, • and the perception that the path to solving the problem is as important as the solution itself. " (Heibert and Grouws [2007], MIND Research Institute [2019] , NCTM [2014], and Stiles [2017] as cited in Gray (2019, p. 4)	process, student-centered, ingenuity
Herrera et al. (2019)	"By productive struggle, we mean when "students expend effort in order to make sense of mathematics, to figure out something that is not immediately apparent" (Hiebert & Grouws, 2007, p. 387; as cited in Herrera et al. [2019, p. 1157])	student-centered
Kalinec-Craig (2017)	"When students engage in problem-solving that lacks an obvious answer or strategy, students have more opportunities to develop connections between old and new knowledge (Hiebert et al., 1996; Proulx & Heine, 2009; Schoenfeld, 1992; as cited in Kalinec-Craig [2017, p. 4]).	process, deeper understanding
Leitze and Soots (2015)	"Recent recommendations from national leaders in mathematics education call for teachers to supply more opportunities for students to engage in productive struggle by solving word problems and challenging mathematical tasks (NCTM 2014) for which students draw on mathematics concepts to determine how to solve tasks (Carpenter & Lehrer, 2014; as cited in Letze and Soots [2015, p. 254])."	student-centered, cognitively-demanding, mathematical relationships
Lemly et al. (2019)	Productive struggle occurs when students are given "opportunities for delving more deeply to understand the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions." (NCTM 2014, p. 48; as cited in Lemly et al. [2019, p. 15]).	deeper understanding, mathematical relationships
Pasquale and EDC (2016)	"make sense of problems and persevere in solving them" (NGA Center for Best Practices, 2010; as cited in Pasquale and EDC [2016, p. 2])	Efffortful
Polly (2017)	"students draw on mathematics concepts to determine how to solve tasks" (Carpenter & Lehrer, 2014; as cited in Polly [2017, p. 254])	Mathematical relationships
Roble (2017)	"The National Council of Teachers of Mathematics (NCTM) Principles to Actions define productive struggle in mathematics as opportunities for delving more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions (NCTM, 2014; as cited in Roble, 2017, p. 225)"	deeper understanding, mathematical relationships
Townsend et al. (2018)	"Productive struggle is inherent in understanding mathematics and solving real-world problems. It can also lead students toward mathematical resilience, encourage retention, and build growth mindsets (Boaler, 2016; Dweck, 2006; Hiebert & Grouws, 2007; as cited in Townsend et al. [2018, p. 217-218]." "In Principles to Actions: Ensuring Mathematical Success for All, NCTM (2014) defines productive struggle as students delving 'more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions (as cited in Townsend et al. [2018, p. 218])." "Hiebert and Grouws (2007, p. 387) state that struggle does not mean 'needless frustration' or 'overly difficult' problems but problems within a student's zone of proximal development, as defined by Vygotsky (1978; as cited in Townsend et al., [2018, p. 218])."	deeper understanding, success, deeper understanding, cognitively demanding
Valentine and Bolyard (2018)	"Hiebert and Grouws (2007) concluded that providing opportunities for students to 'struggle with important mathematics' (p. 387) plays a key role in learning that results in conceptual understanding" (Valentine & Bolyard, 2018, p. 4). "Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships" (NCTM, 2014, as cited in Valentine & Bolyard, 2018, p. 5). "By struggling with important mathematics, we mean the opposite of simply being presented with information to be memorized or being asked only to practice what has been demonstrated" (Hiebert &	cognitively demanding, deeper understanding, student-centered, cognitively demanding

Citation	Definition	Codes
	Grouws, 2007, as cited in Valentine & Bolyard, 2018, p. 4).	
Warshauer (2015a)	"The kind of struggle referred to here is described by Hiebert and Grouws (2007) in this way: 'Students expend effort in order to make sense of mathematics, to figure something out that is not immediately apparent'" (as cited in Warshauer, 2015a, p. 390). "Studies suggest that struggling to make sense of mathematics is a necessary component of learning mathematics with understanding" (Warshauer, 2015a, p. 391).	effortful, effortful, cognitively demanding, cognitively demanding, process, deeper understanding
Warshauer (2011)	"Students' productive struggle refers to students' 'effort to make sense of mathematics, to figure something out that is not immediately apparent'" (Hiebert & Grouws, 2007; as cited in Warshauer, 2011, p. ix). "This struggle occurs in the context of students 'solving problems that are within reach and grappling with key mathematical ideas that are comprehensible but not yet well-formed'" (Hiebert & Grouws, 2007; as cited in Warshauer, 2011, p. 10).	effortful, cognitively demanding, success, effortful, cognitively demanding
Warshauer (2015b)	"By students' productive struggles, I refer to a student's 'effort to make sense of mathematics, to figure something out that is not immediately apparent'" (Hiebert & Grouws, 2007; as cited in Warshauer, 2015, p. 376).	effortful, cognitively demanding
Warshauer et al. (2016)	By productive struggle, we mean what occurs when, "students' productive struggle refers to students' 'effort to make sense of mathematics, to figure something out that is not immediately apparent" (Hiebert & Grouws, 2007, p. 387; as cited in Warshauer et al., 2016, p. 917).	effortful, cognitively demanding
Warshauer et al. (2017)	By productive struggle, we mean what occurs when, "students expend effort in order to make sense of mathematics, to figure out something that is not immediately apparent" (Hiebert & Grouws, 2007, p. 387; as cited in Warshauer et al., 2017, p. 893).	student-centered
Wilburne et al. (2018)	"Support productive struggle in learning mathematics; Posing purposeful questions; and Implementing tasks that promote reasoning and problem-solving were most characteristic of the teachers' classroom practices" (Wilburne et al., 2018, p. 240). "Productive struggle involves providing students with tasks that require effort, perseverance, and exploration to make sense of mathematics" (Wilburne et al., 2018, p. 240).	mathematical reasoning, effortful
Zeybeck (2016)	"Hiebert and Grouws (2007) defined struggle as an intellectual effort students expend to make sense of mathematical concepts that are challenging but fall within the students' reasonable capabilities" (as cited in Zeybek, 2016, p. 396). "Teaching that provides students opportunities to struggle with important mathematical ideas has been identified in mathematics education research as one of the key components of teaching that supports the development of students' conceptual understanding of mathematics" (as cited in Zeybek, 2016, p. 396).	effortful, cognitively demanding, success, deeper understanding

Appendix B. Operational Definitions of Productive Struggle

Citation	Definition
Amidon et al. (2020)	"productive struggle in mathematics can become the class norm by situating mathematics in relevant contexts in which students can meaningfully engage with peers, employing knowledge of learning trajectories, and redefining homework." (p. 69)
Edwards and Beattie (2016)	"In Pathways instruction, productive struggle most often occurs in collaborative learning settings in which students explore rich mathematical tasks as they develop strategies to investigate the problem situation or question. Students who are productively struggling are engaged and inquiring, repeatedly making guesses and judgments about how to use mathematics to approach the given situation. Promoting productive struggle involves posing tasks that require substantive mathematical thinking and giving students both the time and encouragement within the classroom culture to engage with the problem." (p. 31)
Ewing (2016)	Trajectory for PSTs to engage with productive struggle: connect content with students, provide access for students, acquire content knowledge which leads to acquiring productive struggle and high expectations for students who are ELLs and learning how to teach productive struggle (from figure 1, p. 936)
Townsend et al. (2018)	"By applying Vygotsky's construct of the zone of proximal development to productive struggle, we developed indicators to analyze students' zones of productive struggle. This allowed us to determine whether their struggle was productive while they worked on the two tasks" (Townsend et al., 2018, p. 218). Observable Indicators: "Students who were working within their zone of productive struggle were successful in completing and understanding the tasks with guidance from their teacher or peers and without continual frustration" (Townsend et al., 2018, p. 218-219). "Students who were working outside of their zone were overwhelmed, expressed negativity toward mathematics, and at times stopped working on the task" (Townsend et al., 2018, p. 219). Measurable Activities: "During the Virus task, there were more indicators of students working outside their zone of productive struggle. For example, more student talk was apathetic or negative in nature, such as 'I don't really care' and 'So am I done?'" (Townsend et al., 2018, p. 220). "Increases in overall student scores on the Car task indicated that students developed a more proficient understanding of the mathematics of algebraic functions as compared to the Virus task" (Townsend et al., 2018, p. 222).
Warshauer (2015a)	Definition: The article provides explicit strategies for teachers to support productive struggle: "Teachers can incorporate into their practice explicit reminders to students that struggling to make sense of mathematics is an important and natural part of learning. Rather than avoiding this phenomenon, teachers can integrate struggle as part of doing mathematics by acknowledging students' consternation, encouraging perseverance, asking questions, and offering time to work through problems" (Warshauer, 2015, p. 393). Measurable and Observable Activities: Strategy 1: "Teachers ask questions that help students focus on their thinking and identify the source of their struggle, then encourage students to build on their thinking or look at other ways to approach the problem without solving the problem for them" (Warshauer, 2015, p. 391). Strategy 2: "Teachers encourage their students to reflect on their work and support student struggle in their effort to explain their thinking and not just in getting correct answers" (Warshauer, 2015, p. 392). Strategy 3: "Teachers give time and help students manage their struggles through adversity and failure by not stepping in too soon or helping too much and thus taking the intellectual work away from the students" (Warshauer, 2015, p. 392). Strategy 4: "Teachers acknowledge that struggle is an important and natural part of learning and doing mathematics" (Warshauer, 2015, p. 392).

Citation	Definition
Warshauer (2011)	Definition: "The study developed a framework to analyze the kinds and patterns of students' struggles, how teachers respond, and the outcomes of these interactions in terms of student understanding" (Warshauer, 2011, p. x). "I identified all the episodes during instruction where students made mistakes, expressed misconceptions, or claimed to be lost or confused, and to which teachers responded" (Warshauer, 2011, p. 7). Measurable and Observable Activities: Types of Struggles: Getting Started: "Students voiced confusion about the problem requirements or failed to initiate a solution" (Warshauer, 2011, p. 82). Carrying Out a Process: "Students encountered difficulties recalling formulas or executing calculations" (Warshauer, 2011, p. 83). Giving Explanations: "Students struggled to articulate or justify their reasoning" (Warshauer, 2011, p. 84). Expressing Misconceptions and Errors: "Students demonstrated incorrect understanding or persisted in using faulty logic" (Warshauer, 2011, p. 85). Teacher Responses: Telling: Directly providing solutions or answers to students (Warshauer, 2011, p. 100). Directed Guidance: Offering step-by-step hints or partial solutions (Warshauer, 2011, p. 105). Probing Guidance: Asking open-ended questions to prompt deeper thinking (Warshauer, 2011, p. 115). Affordance: Allowing students to struggle independently with minimal intervention (Warshauer, 2011, p. 123).
Warshauer (2015b)	Definition: "The study developed a classification structure for student struggles and teacher responses with descriptions of the kinds of student struggle and kinds of teacher responses that occurred" (Warshauer, 2015, p. 375). "A Productive Struggle Framework was developed to capture the episodes of struggle episodes from initiation, to interaction, and to resolution" (Warshauer, 2015, p. 375). Measurable and Observable Activities: Types of Struggles Observed (pp. 384–385): Getting Started: "Students voiced confusion about what the task asked them to do… or showed no work on their paper." Carrying Out a Process: "Students encountered an impasse, such as difficulty recalling a geometry formula or executing an algorithm." Uncertainty in Explanation and Sense-Making: "Students struggled to verbalize their thinking and give reasons for their strategies." Expressing Misconceptions and Errors: "Deep-seated mistaken ideas were used as a basis for solving problems." Teacher Responses (p. 386): Telling: Providing explicit instructions or solutions. Directed Guidance: Offering step-by-step hints or breaking tasks into smaller parts. Probing Guidance: Encouraging students to articulate their thinking through open-ended questions. Affordance: Allowing students time to explore their struggles independently while providing minimal intervention.
Wilburne et al. (2018)	Definition: "Give students time to struggle with tasks and ask questions that scaffold students' thinking without stepping in to do the work for them" (Wilburne et al., 2018, p. 238). "Help students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles" (Wilburne et al., 2018, p. 238). Measurable and Observable Activities: "Praise students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems" (Wilburne et al., 2018, p. 238). "Anticipate what students might struggle with during a lesson and be prepared to support them productively through the struggle" (Wilburne et al., 2018, p. 238).