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Abstract

Fractal Geometry (GF) comprises problems characterized by having particular geometric and analytical properties based on the concept of self-similarity. Fractals constitute a breaking point in relation to classical Geometry. In addition, many natural phenomena exhibit fractal features, leading to several useful practical applications. In spite of this, fractals are rarely included in curriculum design and, let alone, seen in actual classrooms. This also leads to a lack of resources for teaching fractals at different educational levels. In this context, generative AI (GenAI) can bring an opportunity to produce resources for teaching fractals. In this work we present the results of a study oriented to analyze and compare the productions of in-service Mathematics teachers and GenAI models (namely ChatGPT, Copilot and Gemini) related to a particular fractal: the Sierpinski Triangle. The solutions to the problem as well as the class proposals built by both teachers and chatbots were compared in terms of a number of proposed categories (such as correctness of the results, level of mathematical justification, specificity of class proposal in several aspects). Our findings have multiple practical implications in terms of the design of methodological proposals including GenAI in fractals teaching.

Introduction

Benoît Mandelbrot is considered the precursor of Fractal Geometry as a disciplinary field in Mathematics (Mandelbrot, 1972). Although some examples of fractals were already known, Mandelbrot managed to break with the idea of considering them as "mathematical monsters". Fractals emerge from the need of studying certain curves for which classical Geometry was insufficient. A well-known example is that of measuring the British coast (Ramos Canos, 2022): this is a very irregular curve that can be understood from a small portion of itself. This is the characteristic that defines fractals and that differentiates them from any other figure: self-similarity.

The study of fractals started to become very popular in the mathematical community, but not so much in the scholarly community. On the one hand, it is interesting to study fractals due to their particularities, and because they can be identified in different phenomena in nature, for example, in snowflakes (Koch fractal); in artworks, such as Escher's Metamorphosis (which would later become part of Escher's fractals); in medicine applications, specifically in the area of oncology where they can be used to characterize cancerous structures (Rodríguez et al., 2016); among others. On the other hand, it is interesting to study fractals because Fractal Geometry differs from the classical one. For example, fractals are curves with infinite perimeter, but enclosing finite areas (Koch flake) and even null areas (Sierpinski Triangle fractal).

These characteristics are interesting to study not only from the mathematics point of view, but also from the application one due to the large number of real uses that fractals have in everyday life. However, this fact seems not to correlate with the importance given to fractals in official curricula designs at the secondary school level. In this regard, Ramos Canos (2022) points out that fractals are mathematical objects that have been widely used for mathematical dissemination but that do not appear in the mathematics curriculum of the Valencian community. In this respect, Chavil et al. (2020) indicates that the concept of fractal is not present in Peruvian secondary school despite its potential at a mathematical level and its applicability. Something similar occurs in the curriculum design of secondary schools in Argentina. The importance of fractals study is recognized and some of the 24 provincial curriculum designs even include Fractal Geometry, but this subject is rarely studied within classrooms. In a survey on research in Geometry education, Sinclair et al. (2016) noted that there appears to have been little or no research on fractal teaching and learning during the previous period of 10 years.

On the one hand, there are those who consider that one of the possible reasons for this absence is the lack of Mathematics teachers training in the area of Fractal Geometry (Chavil et al., 2020; Martin et al., 2019). On the other hand, there are those who refer to the scarcity of specific resources to teach this type of Geometry in schools. In this regard, some researchers suggest some alternatives. For example, Márquez (2017) proposes a classroom sequence to introduce fractals in secondary school based on GeoGebra, and other dynamic geometry resources. Authors in (Redondo et al., 2004) make a compilation and synthesis of some key concepts of Fractal Geometry and try to adapt them, based on activities, to a proposal for secondary level students. These attempts at class proposals are vital if we want to address the problem of an insufficient presence of Fractal Geometry content in secondary school, but it is only a starting point.

Given that Fractal Geometry is a breaking point in relation to classical Geometry and, in consequence, it is often absent in Mathematics education at different levels, the resources for its teaching are also scarce. The emergence of new conversational generative AI technologies and their massive adoption in the form of chatbots (such as ChatGPT, Copilot and Gemini), may represent an opportunity for teachers to take advantage of these powerful technologies in the creation of resources for the classroom. These innovative resources can guide students in the processes of acquiring complex mathematical concepts related to fractals, such as self-similarity, and other mathematical notions associated with them (infinity, series, recursion, iterations, among others).

In this context, this work aims to investigate how to combine the following three issues: Fractal Geometry, the design of class proposals on the topic and the resources from conversational AI. To do this, we carried out a study with a particular fractal, namely the Sierpinski Triangle, analyzing the responses provided by three in-service Mathematics teachers and by five chatbots (ChatGPT, Copilot in its three conversation styles, Creative, Balanced and Precise, and Gemini) from two points of view. On the one hand, we analyzed the solutions provided by teachers and chatbots when calculating the perimeter and area of the fractal triangle. On the other hand, we considered the teaching proposals that were designed by the teachers and by the chatbots to bring the Sierpinski Triangle to the classroom.

The study presented in this work was guided by the following research questions:

- RQ1: Do the solutions provided by Mathematics teachers to the problem differ from those offered by GenAI? If so, how?
- RQ2: Is GenAI capable of providing mathematical proofs of the solutions at the same level as Math teachers?
- RQ3: Do the class proposals provided by Math teachers differ from those offered by GenAI? If so, how?
- RQ4: Is GenAI capable of providing class proposals at the same level as Math teachers?

The article is organized as follows. In the following section, we analyze some background information and related works in Fractal Geometry and Mathematics education and their potential relation with generative AI. Then, we present the mathematical construction of the Sierpinski triangle, and we describe how to calculate its perimeter and its area. In the next section we detail the materials and methods employed to carry out this study, describing the process of data collection and the definition of the categories for the analysis of both solutions and class proposals. Then, we report the results obtained when analyzing the different categories. Finally, we summarize our conclusions.

Background and Related Research

Fractals in Mathematics Education

In the context of secondary education in Argentina, the notion of fractal is included to be taught in some of the official programs in the last school year (students of 17-18 years old). It is placed in the last year because it is a good opportunity to study, recover, discuss and expand some mathematical notions studied in previous years, such as: limit, sequences, series, recursion, similarity transformations, exponential and logarithmic equations. Fractals also model objects that exhibit structure at various levels, and they are used in computer graphics that, in certain cases, describe shapes in nature: Helge Koch showed a curve with infinite perimeter, which encloses a region of the plane of finite area, represented by a snowflake-shaped figure (Dirección General de Escuelas de la Provincia de Buenos Aires, 2011). In relation to the Koch snowflake fractal, Corica et al (2024) presents the results of exploring, analyzing and comparing the responses that an in-service Mathematics teacher and generative artificial intelligence (GenAI) models (ChatGPT 3.5, Bard and Bing Chat), concluding that there are significant differences between the procedures, graphic representations and validations of the responses obtained from the teacher and the chatbots.

With respect to fractal teaching at the secondary school level, some researchers have demonstrated its feasibility. For example, Cañibano et al. (2011) propose introducing the concept of fractal dimension through the box counting method applied to a physiographic accident, a lagoon. Martin (2015) introduces fractals from the question: How to measure the coast of Mar del Plata (a city in Argentina)? (Chavil et al., 2020) propose to approach the study of fractals using the immersive virtual reality software NeoTrie VR. The work reported in (Ramos Canos, 2022) presents an activity developed during seven class sessions, with the objective that students can search for information about fractals and build their own conception of what they are. It proposes to study with a specific fractal and obtain some of its properties. Márquez (2017) designed and implemented a sequence of tasks, based on the theory of didactic situations (Brousseau, 1998) with the objective of measuring and analyzing fractal dimension learning using the principle of self-similarity with GeoGebra and other technological resources.

In the context of higher education, Hershkowitz et al. (2023) investigate how master's students construct (fully or partially) the concept of self-similarity while recursively constructing the Sierpiński triangle, while working in small groups and whole-class settings in a Mathematics education course as part of a research-based master's degree. The results show that the knowledge construction processes of different students varied: some of them think recursively in finite cases while others think more directly in the infinite case.

Generative AI and Fractals

GenAI bases the generation of new content on models learned from large volumes of data with deep learning techniques. In the case of conversational AI whose main goal is to maintain a dialogue, being chatbots its main and more visible product, the task of text generation falls on their underlying Large Language Models (LLMs). Thus, ChatGPT uses the GPT series of models, including GPT 3.5 (Brown et al., 2020) and GPT 4 (OpenAI, 2023), which is also used by Copilot, while Google Bard uses PaLM-2 which has its origin in PaLM (Chowdhery et al., 2022) and the newly launched Gemini uses the model of the same name.

Fractal Geometry is assumed to exist in the basic knowledge of the world. Based on this observation, Kataoka et al. (2022) investigated the use of fractal images for training an image recognition system instead of using photos of real objects. For this purpose, the authors created a dataset (FractalDB) with a variety of fractals (similar to leaves, snowflakes or snails), and pre-trained a convolutional neural network. The results were comparable to those of models trained entirely on state-of-the-art datasets (such as ImageNet or Places).

In the same way that self-similarity emerges in the diverse phenomena in nature, in (Alabdulmohsin et al., 2024) it is argued that the presence of this property in the structure of language may be interconnected with the intelligence exhibited by LLMs. The authors argue that short-term patterns/dependencies in language, such as those that occur within paragraphs, mirror the patterns/dependencies over larger scopes, like entire documents. This work found a concrete estimate of the fractal parameters of the language, including the fractal dimension, in experiments with PaLM-2 and other LLMs. It also showed that incorporating fractal parameters can remarkably improve the ability to predict the subsequent performance in LLM over known metrics. This work provides an interesting new perspective in the study of how next-word prediction tasks in LLMs can lead to an understanding of text structure at multiple levels of granularity, from words and clauses to broader contexts and intentions.

Studies oriented to assess the performance of LLMs in mathematical reasoning have been mainly concerned with the construction of benchmark datasets and the quantitative analysis of the results of a given model with respect to them (Shakarian et al., 2023; Frieder et al., 2023). Although their findings may provide an overall view of the performance of LLMs in the Mathematics domain, there is still a lack of understanding of their strengths and weaknesses in specific Mathematics fields, such as Geometry, and restricted to particular languages such as Spanish.

Finding solutions to complex Geometry problems like fractals is a particularly challenging task for generative AI based on multimodal LLMs, as it not only involves knowledge of fundamental concepts (theorems) and their correct application, but also the use of spatial reasoning skills. In fact, even the most advanced multimodal models present difficulties in accurately understanding geometric figures and the relationships between fundamental elements such as points and lines, and accurately interpreting basic concepts such as the degrees of an angle (Gao et al., 2023). Solutions to this problem include training with specific datasets to enrich the models, such as the G-LLaVa model (Gao et al., 2023). However, these models are not widely accessible and are limited to the language of their training data, usually English.

The Sierpiński Triangle Fractal

Among classical fractals, the Sierpiński triangle (sometimes spelled Sierpinski), also called the Sierpiński gasket or Sierpiński sieve has a prominent place in Fractal Geometry since, among other characteristics, it has an infinite perimeter and an area equal to zero. The construction of the triangle is as follows. It starts with an equilateral triangle. The first step consists in dividing it into four congruent equilateral triangles (which is achieved by joining the midpoints of its sides) and removing the central triangle, that is, leaving the three equilateral triangles at the vertices. The second step of construction consists of repeating the first step with each of the three triangles obtained in the previous step. The process is repeated infinitely, obtaining the final result of the Sierpinski triangle (see Figure 1).

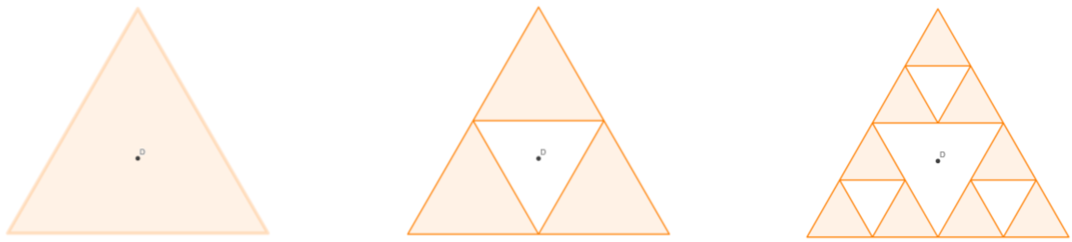


Figure 1. Sierpinski Triangle

This fractal can be found in several mathematical contexts, such as dynamical systems, graph theory, applied mathematics (in the manufacture of high-frequency antennas, particularly in cell phones), and even in objects as old as the Pascal's triangle or the Towers of Hanoi game (see for example (Stewart, 1999; Mesa, 2001; Falconer, 2003, p. 309)).

To calculate the area of the Sierpinski triangle, let's suppose that it has a side length of L. Taking into account that the triangle is an equilateral one, the perimeter of the initial triangle (iteration $n = 0$) is $P_0 = 3L$. Following the construction protocol, the next iteration ($n = 1$) is composed of three equilateral triangles with side length $\frac{L}{2}$, where each triangle has a perimeter equal to $3\frac{L}{2}$. Thus, the total perimeter for the first iteration is $P_1 = 3\frac{3L}{2} = 3\frac{3}{2}L = \frac{3}{2}3L$. Considering that $P_0 = 3L$, P_1 can be expressed in terms of P_0, resulting in $P_1 = \frac{3}{2}P_0$. Iteration 2 ($n = 2$) will have triple the number of triangles from the previous iteration, that is, there will be 9 triangles whose side length is equal to half of the side length of the previous ones, this is $\frac{L}{4}$. The perimeter of each of these triangles with side $\frac{L}{4}$ is equal to $3\frac{L}{4}$. Then, the total perimeter considering the 9 triangles is $P_2 = 9\frac{3L}{4} = 3\frac{9}{4}L = \left(\frac{3}{2}\right)^2 3L$. Taking into account that $P_0 = 3L$, P_2 can be expressed in terms of P_0, as $P_2 = \left(\frac{3}{2}\right)^2 P_0$. Iteration 3 ($n = 3$) will contain 27 triangles of side length $\frac{L}{8}$. Thus, each of the 27 triangles will have a perimeter equal to $3\frac{L}{8}$. The total perimeter of the 27 triangles is therefore $P_3 = 27\frac{3L}{8} = 3\frac{27}{8}L = \left(\frac{3}{2}\right)^3 3L = \left(\frac{3}{2}\right)^3 P_0$. Following this process, the sequence S_p that allows us to obtain the total perimeter of the Sierpinski Triangle at any iteration is shown in equations 1 and 2:

$$S_p = \{P_0, P_1, P_2, P_3, \ldots, P_n\} \tag{1}$$

$$S_p = \left\{P_0, \frac{3}{2}P_0, \left(\frac{3}{2}\right)^2 P_0, \left(\frac{3}{2}\right)^3 P_0, \ldots, \left(\frac{3}{2}\right)^n P_0\right\} \tag{2}$$

Then, the total perimeter P results from calculating the limit as n tends to infinite in the general term of the sequence S_p, as shown in equation 3.

$$P = \lim_{n\to\infty} \left(\frac{3}{2}\right)^n P_0 = \infty \tag{3}$$

To calculate the area, the procedure is analogous. It starts by considering the area of the initial triangle, denoted as A_0, and the areas of the triangles in the following iterations are determined from it. For $n = 1$, the areas of three of the four congruent triangles are considered. This leads to the area of the first iteration, which is $A_1 = \frac{3}{4}A_0$. In the next iteration ($n = 2$), each of the three triangles are divided into four congruent triangles and the central one is removed. In this way, the total number of triangles is 9. Each of these 9 triangles has an area equal to ¼ of $\frac{1}{4}A_0$, that is to say $\frac{1}{16}A_0$. Since there are 9 congruent triangles of area $\frac{1}{16}A_0$, the total area in iteration 2 will be $A_2 = \frac{9}{16}A_0 = \left(\frac{3}{4}\right)^2 A_0$. Following this process, the total area for iteration 3 will be $A_3 = \left(\frac{3}{4}\right)^3 A_0$. Finally, we construct the sequence S_A that enables us to obtain the total area of the Sierpinski triangle in any iteration as shown in equations 4 and 5:

$$S_A = \{A_0, A_1, A_2, A_3, \ldots, A_n\} \tag{4}$$

$$S_A = \left\{A_0, \frac{3}{4}A_0, \left(\frac{3}{4}\right)^2 A_0, \left(\frac{3}{4}\right)^3 A_0, \ldots, \left(\frac{3}{4}\right)^n A_0\right\} \tag{5}$$

Then, the total area A results from calculating the limit as n tends to infinity of the general term of the sequence S_A, which is as follows:

$$A = \lim_{n\to\infty} \left(\frac{3}{4}\right)^n A_0 = 0.A_0 = 0 \tag{6}$$

The initial area A_0 can be easily calculated using the Pythagorean Theorem and turns out to be $A_0 = \frac{L\frac{1}{2}\sqrt{3L}}{2} = \frac{\sqrt{3}}{4}L^2$.

In the next section we describe the methodology used in our study.

Methods

This work reports the findings of an exploratory research focused on assessing the performance of generative, conversational AI in providing class proposals and solutions to the problem of calculating the perimeter and area of the Sierpinski Triangle fractal. The responses to the problem of three in-service Math teachers and the following conversational AI resources were analyzed: ChatGPT 3.5 (hereinafter, ChatGPT), Gemini and Copilot (in its three conversation styles: creative, balanced and precise). Also, the classroom proposals designed by teachers and chatbots for the same problem were compared.

The methodology for collecting the responses of a group of in-service math teachers and the AI generated ones is described next, whereas the categories established to analyze and compare these responses are detailed in the next subsection.

Data Collection

The three Mathematics teachers are in service teachers at the secondary school level in Argentina. The solutions provided to the problem as well as the class proposals were developed in the context of a final project of a continuing training course. This course focuses on solving complex mathematical problems leveraging on technological resources, such as dynamic geometry software, spreadsheets, block programming and the use of conversational AI (in particular, ChatGPT). The final work consisted of the following sequence of tasks, to be solved individually:

T1: Solve the following problem "using pencil and paper", that is, solve it in the role of mathematicians (this is the translation of the task provided to teachers in Spanish): "The Sierpinski triangle is a fractal, with Waclaw Sierpinski in 1915 as his mentor. For its construction, we start from an equilateral triangle. The first step consists of dividing it into four congruent equilateral triangles (which is achieved by joining the midpoints of the sides) and eliminating the central triangle, that is, leaving the three equilateral triangles at the vertices. The second step of construction consists of repeating the first step on each of the three triangles obtained in the previous step. The process is then repeated infinite times, obtaining the Sierpinski triangle as the final result. Calculate the perimeter and area of the Sierpinski triangle. Justify the results obtained using mathematical concepts."

T2: Solve the problem using GeoGebra, block programming environments such as Scratch, spreadsheets like Excel or generative AI, such as ChatGPT or another chatbot that you consider relevant, freely designing the prompts. That is, teachers were free to choose the instruction sentences to provide as input to the selected chatbot and also how to interact with it.

T3: Explain the advantages (and/or disadvantages) of using one environment over the others in terms of providing a solution to the same problem.

T4: Adapt the problem with the aim of taking it to a "real" Mathematics class incorporating at least one of the technological environments: detail the adaptations and their reasons, indicate what mathematical knowledge students could be studying in this class, what the context would be and how you, as teachers, would manage that class.

The teachers delivered the four tasks in writing via the course platform. In this work, we focus on the analysis of tasks T1 and T4 specifically. For the chatbots, we designed three different prompts that were provided as input in the following order (translated from the prompts given to teachers in Spanish):

Prompt 1: The problem includes the problem described in T1, followed by the question "Can you calculate the perimeter and area of the Sierpinski triangle?"

Prompt 2: "Could you mathematically prove the results?

Prompt 3: Adapt the problem to take it to a "real" Math class: detail the adaptations and their reasons, indicate what mathematical knowledge can be studied, what the context would be and how you as a teacher would manage that class.

Given that prompt 1 and prompt 2 correspond to task T1 while prompt 3 corresponds to task T4 assigned to the teachers, it is possible to analyze all the responses (those of the teachers and the chatbots) using Table 1. Teachers who solved the problem of calculating the perimeter and the area of the Sierpinski triangle are identified as Pi, where i indicates the order number of each teacher.

Categories of Analysis

The comparison of teachers and chatbots responses regarding both the mathematical solutions to the problem and the class proposals was carried out throughout a number of categories or aspects enabling a qualitative analysis. The categories defined for evaluating the solutions given to calculate the perimeter and area of the Sierpinski triangle are the following:

1. Number of iterations before presenting the general term, either for the perimeter or area.
2. In the calculation of the perimeter, determine: a) if the corresponding sequence is constructed, b) if the general term is presented, c) if the correct value of the perimeter is reached (which must be infinite) and d) the type of justification for the result achieved.
3. In the calculation of the area, determine: a) if the corresponding sequence is constructed, b) if the general term is presented, c) if the correct value of the area is reached (which must be zero) and d) the type of justification for the result achieved.

Regarding the proposals for the classroom, we defined the following categories:

1. Level and school year: the proposal identifies the school year (and/or age of the students) for which the proposal is addressed. For example, last year of the secondary level.

2. Mathematical notions to be taught: the mathematical content which is intended to teach or study is indicated in the proposal. For example, teach the limit of functions.
3. Mathematical notions involved: all the mathematical concepts necessary to solve the problem (and their corresponding adaptation) are considered. For example, equilateral triangle, iteration, sequence, limit, etc. In this category, word clouds were used to illustrate the notions covered by the different proposals.
4. Adaptations made to the problem: the modifications made to the problem, from which the class proposals were designed. For example, the class proposed requires the problem to be segmented into three activities.
5. Time assigned for the proposal: the time (in number of hours) proposed to develop the proposal in the classroom is indicated. For example, three and a half hours.
6. Classroom equipment: it is considered whether the classroom is equipped with specific equipment such as blackboard, projector, computers, etc.
7. Technological resources to be used: the software, applications, web pages, among others, that students can use when solving the problem are detailed. For example, GeoGebra, Excel, ChatGPT, among others.
8. Dynamics/organization of the class: the stages proposed by the teachers and the chatbots that must be followed to complete the class are explained. For example, if the proposal begins with an explanation from the teacher, or a task to solve and how it develops starting from it.
9. Evaluation/assessment: if an evaluation instance is proposed and, if so, of what type. For example, evaluating through a questionnaire.
10. Student role in the class: it refers to the task that has to be done in parallel to the class or remains to be done in the afterwards of the class. It is observed if the proposal comments on whether the student has to solve problems that the teacher proposes, listen to what the teacher presents or copy the explanations from the blackboard.

Table 1 summarizes the analysis dimensions and their intersection. The identified categories are hereafter denoted S1 to S3 for those related to the solutions provided and C1 to C10 for those concerning the class proposals.

Table 1. Table for the Analysis of Teachers and Chatbot Responses

	Teachers			**Chatbots**				
					Copilot			
	P1	P2	P3	ChatGPT	Creative	Balanced	Precise	Gemini
Solution provided to the problem								
S1. Number of iterations before presenting the general term								
S2. Perimeter calculation								
S3. Area calculation								
Classroom proposal								
C1. Level and school year								
C2. Mathematical notions to be taught								
C3. Mathematical notions involved								
C4. Adaptations made to the problem								

C5. Time assigned to the proposal
C6. Classroom equipment
C7. Technological resources ro be used
C8. Dynamics/organization of the class
C9. Evaluation/assessment
C10. Student role in the class

Results and Discussion

Analysis of Problem Solutions

As mentioned in the introduction, our study was guided by four research questions, the two first relate to the solutions provided to the problem:

RQ1: Do the solutions provided by Math teachers to the problem differ from those offered by GenAI? If so, how?

RQ2: Is GenAI able to provide mathematical proofs of solutions at the same level as Math teachers?

Regarding the solutions provided by the mathematics teachers, the three perform 4 iterations (category S1) before formulating the general expression of both the perimeter and the area; while three of the chatbots (ChatGPT and Copilot in its three versions) directly present the general term without any prior iteration. Only one of the chatbots, Gemini, goes through four iterations before introducing the general term. This result is important to illustrate how chatbots work and its implications in the teaching process. The text for answering the problem is built by LLMs based on the next-word prediction mechanism, this is, estimating the probability of a word given the previous ones according to the occurrences observed in a vast number of texts. Thus, the most likely text for answering the problem contains the general term but not the steps for reaching it, because in the observed data the reasoning process that leads to the solution is not described either. In turn, this prevents chatbots from providing the deduction steps that would help students in the learning process, as Mathematics teachers do.

Regarding the sequence construction (category S2a) for the perimeter, two out of the three teachers (P2 and P3) propose an adequate solution, whereas the third (P1) builds a series (see Figure 2). The latter is not correct because the perimeter of the next iteration does not result from adding the previous one. This is also an interesting result since it shows how chatbots build the answer; the next-word prediction mechanism is inherently forward; it does not have a backtracking process to check or correct intermediate results. On the other hand, none of the chatbots explicitly allude to the notion of sequence. ChatGPT refers to the limit of a geometric series.

$$p_t = p_1 \cdot \left\{ \left(\frac{3}{2}\right)^0 + \left(\frac{3}{2}\right)^1 + \left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^3 + \dots + \left(\frac{3}{2}\right)^{n-1} \right\}$$

La razón $r = \frac{3}{2} > 1$, entonces cuando $n \to \infty$ el perímetro tiende a infinito.

Figure 2. Fragment of Solution Proposed by Teacher P1

The text in Figure 2 is translated as follows: “The ratio r=3/2 >1, then when n → ∞ the perimeter tends to infinity”

Only two of the teachers provide a correct solution (P2 and P3) regarding the general term (category S2b) for the perimeter. Instead, ChatGPT and Gemini present the correct general term by recursion and based on the initial perimeter, respectively. Copilot in its three versions offers incorrect general terms for calculating the perimeter (see Figure 3).

$P_n = 3 \cdot \left(\frac{1}{2}\right)^n \cdot P_0$ $\qquad$ $P_n = P_0\,(½)^n$ $\qquad$ $P_n = 3^n \cdot \frac{1}{2^n} = 3^{n-1}$

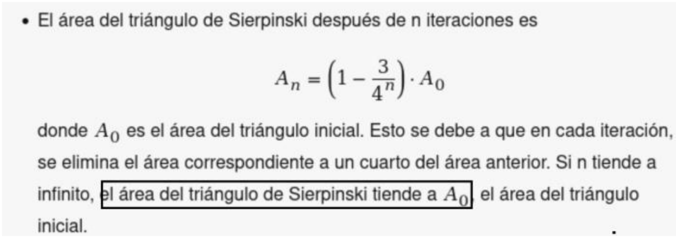
Figure 3. The Expression in the Middle Corresponds to the More Balanced Copilot Answer, the One in the Right to the More Precise One and the One on the Left is the More Creative One

In relation to the value of the perimeter (category S2c), the three teachers concluded that it tends to infinity. All of the chatbots give the correct value except for the answer of the more creative Copilot style, which states that the perimeter is zero. It seems contradictory to arrive at a correct result having offered incorrect algebraic expressions (as it is the case of Copilot), but this is possible because it is intuitive for teachers to determine that the limit of the perimeter tends to an infinite value, whereas it is the more likely answer (the most probable text) for chatbots.

Regarding the type of justification of each teacher (category S2d), the analysis shows that only one of them proposes a mathematical justification made through parallel projections. In the case of the chatbots, the creative Copilot and Gemini justify by induction over n (the number of iterations) that the general term is the proposed one. The difference between these two is that Gemini starts from a correct expression, while creative Copilot does not.

Regarding the area calculation (category S3), P1 formulates a geometric series, while P2 and P3 construct the sequence necessary for it (category S3a). The difference between the latter two is that P2 calculates the base and height of the initial triangle to determine the value of the area, while P3 considers A0 as its initial area, without determining the base and height. None of the chatbots explicitly refer to the expression of sequence. Concerning the general term for the area (category S3b), only two of the teachers provided a correct solution (P2 and P3). On the chatbots side, ChatGPT is the only one that presents a correct general term and it does so by recursion and specifying the initial area of the triangle. The more creative answer of Copilot (Fig. 3) and the more precise one return incorrect general terms; while balanced Copilot and Gemini answers do not present the general term.

Considering the area value (category S3c), the three teachers correctly infer that it is zero. Out of the three chatbots that introduce the general term, two obtain the correct area value (ChatGPT and more precise Copilot), while the third one (more creative Copilot) claims that the area value tends to the initial area (Fig. 4, the highlighted segment can be translated as "the area of the Sierpinski triangle tends to A0"). Those chatbots that do not provide the general term (balanced Copilot and Gemini) reached the correct result anyway. Intuitively, teachers can infer that by removing areas in each iteration, the value will tend to zero. However, chatbots reached this solution for being the more likely one in the data observed, in spite of the construction with the intermediate reasoning steps.


- El área del triángulo de Sierpinski después de n iteraciones es

$$A_n = \left(1 - \frac{3}{4^n}\right) \cdot A_0$$

donde A_0 es el área del triángulo inicial. Esto se debe a que en cada iteración, se elimina el área correspondiente a un cuarto del área anterior. Si n tiende a infinito, el área del triángulo de Sierpinski tiende a A_0, el área del triángulo inicial.

Figure 4. General Terms and Value for the Area according to the More Creative Answer of Copilot

Finally, none of the three teachers offer any type of mathematical justification (category S3d) for the solution. Instead, three of the chatbots provide a colloquial justification (ChatGPT, balanced Copilot and precise Copilot). For example, the balanced Copilot answer states the following (translated from Spanish): “Each time we create these similar triangles within a previous triangle, not only the newly created triangles are similar to the original one, but each triangle created has an area equal to 1/4 of the area of the previous triangle. Since we repeat this process infinitely, the total area of the Sierpinski triangle tends to zero”. Thus, ChatGPT proposes a colloquial form of its justification, but it does this incorrectly. It alludes to the fact that the area is reduced to the half in each iteration because we are eliminating a central triangle. The more creative answer of Copilot suggests looking for demos on external Web sites and offers some links and, finally, Gemini justifies starting from infinite series, which is incorrect.

Analysis of Class Proposals

The next research questions, related to the class proposals, are the following:
RQ3: Do the class proposals provided by Math teachers differ from those offered by GenAI? If so, how?
RQ4: Is GenAI capable of providing class proposals at the same level as Math teachers?

Regarding the class proposals, the three designed by the Math teachers, consider a specific target student (category C1): teacher P1 proposes a student of the first year of a technical school, while P2 and P3 opt for students of the last two years of high school (5th or 6th). In the case of the chatbots, two do not specify the target students (ChatGPT, creative Copilot) and the other three (balanced Copilot, precise Copilot and Gemini) propose the secondary level in general (more balanced Copilot) and undergraduate or higher education students (more precise Copilot and Gemini). In this category, the teachers' proposals turn out to be more specific than those of the chatbots. Because of their training, it is clear for teachers that it is not the same to design a classroom proposal for secondary school than for higher education.

In relation to the mathematical notions that are expected to be taught based on the class proposal (category C2), two teachers explain this content: P2 claims to teach the notion of limit and P3 aims to characterize the concept of geometric sequence. With respect to the chatbots, the only one that makes this aspect explicit is Gemini, which provides a too general statement saying that it will be an introductory activity to the topic of fractals. In this category, once again, it is worth noticing the level of specificity of teachers over the one of chatbots. This can be attributed to the background and experience of teachers in general, and maybe of these three individuals in particular (who work at the secondary school level), which allows them to determine that their students' learning requires to clearly identify specific goals that enable them to propose especially tailored tasks.

In terms of the mathematical notions involved in solving the problem (category C3), the lists given by the chatbots are much more exhaustive than the ones offered by the teachers. This is evident in the word clouds generated starting from the lists provided by teachers (see Figure 5 left) and the ones from chatbots (see Figure 5 right). In the case of teachers, the most frequent words are: concept (5 times), formula (3), definition (3), triangle (2) and equilateral triangle (2), limit (2), perimeter (2) and area (2). In the case of chatbots: area (9), perimeter (8), fractal (5), limit (5), properties (4), equilateral triangle (4) and iteration (4).

Figure 5. Word Clouds of Mathematical Notions Mentioned in the Proposals of Teachers (Left) and the Proposals of Chatbots (Right)

The difference in absolute word frequencies may be due to two reasons. On the one hand, the number of chatbots (there are 5) compared to the number of teachers (there are 3 and one of them did not provide the list). On the other hand, if we analyze the areas of Math and the most relevant concepts linked to the problems, we can observe that the notions listed by teachers seem to be more related to teaching concepts, while chatbots focus on the solution to the problem.

Regarding adaptations to the problem (category C4), teacher P1 reduces the problem to a triangle with side length of 10 and she proposes to explain the construction of the Sierpinski triangle up to iteration 4. Then, the proposal asks to obtain the algebraic expression that allows to calculate the number of triangles for each iteration, the perimeter and the corresponding area. Finally, the teacher proposes to use GeoGebra to represent the problem. Teacher P1's response does not offer any other details with respect to the rest of the categories. Thus, this proposal can be considered as the least detailed and complete. Teacher P2 proposes a segmentation of the problem in two parts: in the first, it proposes to explain, with the help of visual support, the triangle construction in GeoGebra so that the students can calculate the perimeter and area for the first iterations. In a second part, he proposes to introduce the definition of limit based on the fractal, this is with the intention of using it as an example, so that the students end up solving a practical exercise on this notion. Teacher P3 segments the problem into three activities. In the first, it is requested to calculate the perimeter for the first four iterations and construct the perimeter sequence. The second activity requires calculating the area for the first four iterations and constructing the area sequence. In these two first activities, P3 offers a dynamic resource in GeoGebra (created by itself) as a visual "aid". In the third activity, the teacher proposes an equilateral triangle with side length of 10 and requires students to use spreadsheets to create tables that allow them to deduce the tendency of the perimeter and area. Therefore, the most elaborated adaptation is the one proposed by P3.

Regarding chatbots, ChatGPT does not refer to adaptations of the problem. Creative Copilot proposes to use the problem as a mathematical exploration, beginning with the presentation of the Sierpinski triangle accompanied by images and the use of some software that allows the triangle to be generated interactively, so that students can ask questions. Balanced Copilot refers, in addition, to the need of simplifying the language in which the problem is written, to showing an image or animation of the triangle and giving concrete examples. Precise Copilot version suggests that students draw or construct the fractal with specific materials, such as paper and scissors. Finally, Gemini proposes to start with an equilateral triangle with a side length of 1cm and follows with the calculation the perimeter and area of the first three or four iterations. Similar to Precise Copilot, it suggests constructing the triangle with specific materials, but it also includes the possibility of using software, such as GeoGebra or Fractal Explorer, to visualize the first iterations and calculate its perimeter and area. Among all chatbots responses, Gemini could be considered the one that proposes the most concrete adaptation, despite the fact that none of them do so in detail.

In relation to the expected class duration (category C5), only two of the teachers (P2 and P3) specify it, while P1 does not, and the five chatbots do not mention it. Teacher P2, and ChatGPT, balanced Copilot and creative Copilot propose the blackboard and the projector as the necessary equipment for the classroom (category C6). The rest of the teachers and chatbots do not explain further about this aspect.

Regarding the use of technological resources to be used during the class (category C7), teacher P2 suggests GeoGebra and teacher P3 proposes GeoGebra and Excel. In this category, chatbots offer a greater number of options of resources than teachers. ChatGPT refers to spreadsheets, GeoGebra or Cabri Geometry, Scratch or Blockly and itself (ChatGPT). Creative Copilot lists the following resources: Wolfram Alpha, GeoGebra, and Exploring fractals (an educational project that proposes activities to introduce students to the world of fractals). Balanced Copilot suggests GeoGebra, Desmos, Matplotlib (in Python), Python, Java, C++, Mathematica, SymPy (in Python), Maple, the creation of some interactive web pages, SVG or Canva graphics. Precise Copilot suggests Python and Jupyter Notebook. Finally, Gemini refers to GeoGebra, Cinderella, Python, Mathematica and online simulators, such as Interactive Sierpinski Triangle and Interactive Fractals. However, it is important to highlight that in terms of the effective use of these resources teachers only mention GeoGebra and Excel and they actually incorporate them into their proposals. On the other hand, chatbots provide a large list of resources but do not include them in their class proposals.

In relation to the dynamics and organization of the class (category C8), teacher P2 begins the class by explaining through a dynamic construction of the fractal (of its own creation) and manifests to be willing to answer students' questions during the explanation. P2 then moves forward to explain how to calculate the perimeter and area of the fractal so that students can compute some values. Lastly, P2 returns to the explanation instance to define the limit, and then gives practical work on this notion. This characteristic of the class allows P2 to be characterized as an explanatory teacher. Teacher P3 starts the class by organizing the students into groups, whose task is to solve the three previously designed activities. This teacher expects students to formulate explanations, justifications, answers and also questions to these activities. P3 offers discussion moments and validation of results, with interventions from the teacher. This type of dynamic falls more into the category of a constructivist class. P1 does not say anything about the planned dynamics.

In the case of chatbots, ChatGPT and balanced Copilot distinguish the following stages of class organization: presenting the problem, guiding students to build the fractal, exploring practical applications and evaluating (formative evaluation). Precise Copilot version proposes to begin with a brief introduction to the Sierpinski triangle, its history and images. Then it proposes that students build the fractal in groups and "compete" to determine who does it with more iterations. Subsequently, it offers discussion spaces on constructions, patterns, area and perimeter, so that students have prior knowledge and an intuitive idea of fractals, in order to then be able to offer the "formal lesson". The teacher explains the concepts of fractals, infinite series, limits, and shows the applications of the Sierpinski triangle in other areas. Along the same line, Gemini's proposal begins the class with a presentation of the Sierpinski triangle showing images and videos. It continues organizing the students into groups and giving them concrete materials and some software with the goal of building the fractal. Later, it expects to discuss the iterations, perimeter, area, and applications of the Sierpinski triangle. In summary, these four chatbots propose a class focused on an initial explanation accompanied by images and videos of the fractal, as a fundamental part of teaching. The nature of class proposals provided by the four chatbots allow us to characterize them as explainers. Finally, the Creative Copilot shows a subtle difference with the others by first proposing to give the problem to the students so that they can explore, investigate and ask questions. That is, it does not propose to start with an explanation but let them explore by themselves first. Secondly, it provides the space and materials (such as paper and scissors) for students to build the Sierpinski triangle, also incorporating interactive software. It finally requests to present the results for a joint synthesis. These characteristics allow the proposal to be framed in a constructivist class.

Regarding the evaluation instance (category C9), none of the three teachers make reference to this aspect in their proposals. On the chatbots side, creative and balanced Copilot answers do not make this aspect explicit either. ChatGPT refers to a formative evaluation, without giving further details. Precise Copilot proposes evaluating with a questionnaire or by asking students to calculate the perimeter and area after a certain number of steps. Gemini indicates to evaluate students based on their participation in the practical activity, their ability to formulate conjectures and explain them, and the quality of their work on the research project. Recognizing the role that evaluation occupies in educational systems, a possible reason why teachers have not referred to this instance may be that it was an explicit part of task T4.

From the proposals formulated and considering category C8, class dynamics/organization, it is also possible to analyze the student's role (category C10), in the sense of the task that they can carry out during class. Teachers P1 and P2 do not explain the expected role of students; P2 reduces the student's activity to the task of listening, copying her explanations and solving previously exemplified tasks; what we can call an “spectator student”. P3 expands the role of students to a more leading role, as he provides activities that require resolution in groups, the formulation of explanations, justifications, answers and also new questions, which can be labeled as an “active student.” With respect to the five chatbots, the only one that expects something similar to an active student is the Creative Copilot version, since the remaining four chatbots are more aligned with the characteristics of the spectator student.

Conclusion

In this section, we summarize the conclusions we draw in relation to the four posed research questions that guided our study, focusing on the most salient aspects that we found during the analysis of teachers and chatbots' answers.

Regarding the first research question (RQ1): Do the solutions provided by Mathematics teachers to the problem differ from those offered by GenAI? If so, how? One of the first most significant differences is the number of iterations that teachers perform prior to determining the general term of the sequence corresponding to the area and the perimeter. Only one of the chatbots (Gemini) agrees with the teachers on this point. The remaining chatbots straightforwardly present the general term, which can be seen as an expected consequence of the text generation mechanism as they found this part of the text as the most likely one in the observed texts (Web pages, etc.). Another significant difference is that the general terms of sequences are incorrect for three of the chatbots (the three versions of Copilot) while the three teachers provide a correct solution for the general term, despite the fact that one of them (P1) confuses sequences with series. Another difference that stands out lies in the result of the value of the perimeter and area. The three teachers arrived at a correct value for both, while creative Copilot gives an incorrect result for both perimeter and area. These results can be attributed to the complexity of fractal Geometry and the performance of GenAI at solving Geometry problems. As discussed by Parra et al. (2024), numerous mistakes of different categories are made by chatbots in even more simple Geometry problems.

Regarding the second research question (RQ2): Is GenAI capable of providing mathematical proofs of solutions at the same level as mathematics teachers? If we consider the level of justifications that the community of mathematicians would expect, regarding mathematical rigor and formality, neither the teachers nor the chatbots managed to provide adequate responses. Leaving aside this very specific requirement of a mathematician’s work, in the case of calculating the perimeter, we identified a slight difference between one of the teachers and two of the chatbots, which are the only ones that offered some acceptable justification. The teacher uses parallel projections and an associated theorem (with its corresponding proof) to justify the congruence of the triangles by dividing the respective sides in half. Two of the chatbots (C creative Copilot and Gemini) propose and use the principle of mathematical induction on n (the number of iterations) to justify that the general expression is well defined, even though this Copilot response is incorrect. In the case of the area, the difference is in favor of the chatbots since none of the teachers offer any type of justification, while the chatbots present at least a colloquial explanation (ChatGPT, balanced Copilot and precise Copilot).

Regarding the third research question (RQ3): Do the class proposals provided by Mathematics teachers differ from those offered by GenAI? If so, how? Teachers differ significantly from chatbots in terms of the specificity of the class proposal. This difference is observable in most of its components except in the evaluation instance. We believe this exception may be due to the fact that we did not explain to the teachers that they should propose an evaluation. Teachers propose a topic to teach in a given school level/year, mentioning only the closest mathematical concepts, and propose some type of path that allows students to learn that knowledge. Chatbots, instead, formulate some general ideas about the topics that could be taught, also proposing different educational levels for the same proposal (secondary or higher education). This level of generality of chatbots could be reduced and transformed into something more specific by continuing with the conversation (chain of prompts), which requires the background of a Mathematics teacher for designing the appropriate prompts. In this context, it will be necessary to further study the aspects of class proposal designs using chatbots. As a future research line, we are left to delve deeper into these interactions between chatbots and teachers, and investigate them in depth.

Finally, regarding research question 4 (RQ4): Is GenAI able to provide class proposals at the same level as mathematics teachers? Taking into account the aspects mentioned above, chatbots could generate more specific proposals, and more in line with those of teachers, as long as we interact conveniently with them. This leads us to think that chatbots could work as "good assistants" for teachers for classroom proposals design. Particularly they could be used to teach fractals, since the resources available to study this topic in secondary school classrooms are scarce.

The insights reached in this study might have several practical implications. First, they can be a source not only for school teachers to evaluate how to properly use conversational AI-powered tools as educational resources and take advantage of the technology. Second, they can also be valuable for higher education professors that are training future school teachers to introduce them into this matter with proper assessments of their capabilities and limitations. Furthermore, it can be useful for the design of new automatic assistance tools leveraging on GenAI techniques to guide teachers and students in their training/teaching activities.

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