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To cite this article:

Saadati, F., & Celis, S. (2023). Student motivation in learning mathematics in technical and vocational higher education: Development of an instrument. *International Journal of Education in Mathematics, Science, and Technology (IJEMST), 11*(1), 156-178. https://doi.org/10.46328/ijemst.2194

International Journal of Education in Mathematics, Science and Technology

2023, Vol. 11, No. 1, 156-178 https://doi.org/10.46328/ijemst.2194

Student Motivation in Learning Mathematics in Technical and Vocational Higher Education: Development of an Instrument

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Article Info

Article History

Received:
10 November 2021
Accepted:
03 August 2022

Keywords

Higher education
Mathematics learning
Motivational beliefs
Professional institutes
Validity evidence

Abstract

Mathematics is a challenging subject for most students in technical and vocational institutions. The institutions apply great effort towards developing mathematics knowledge and skills, mainly by influencing students' motivations. This study tries: (1) to present evidence for the validity of an instrument for measuring students' motivational beliefs as their beliefs, self-efficacy, and intrinsic-extrinsic goal orientations regarding mathematics; (2) to explore the relationship between institutional context and the instrument's scores as students' motivational beliefs. The data collected from 1,239 students from two different Chilean institutions were analyzed. The psychometric analysis provided evidence of the reliability and validity of the instrument. The instrument suggests differences in the level of motivational beliefs associated with students' age and the type of institutions. The instrument allows educators to study the relationships between motivational beliefs and the institutional context of technical and vocational schools. It can also be used for the prediction of student success in their mathematics course by the understanding of their motivational beliefs towards mathematics.

Introduction

The teaching of mathematics is a cornerstone of higher and postsecondary education. Most undergraduate programs include some content and level of mathematics in their curricula. Mathematics is crucial, regardless of how selective an institution is, mainly because the demand for workers with mathematics and science-related capabilities is growing globally (Shin et al., 2018). In particular, mathematics represents a significant challenge in open-access institutions—those with no admission requirement, which admit more than 80% of applicants (Doyle, 2010). In Chile, technical/vocational centers and professional institutes (TPI hereafter) are the most common type of open-access institutions. We estimate that about 70% of TPI programs include at least one mathematics course as a mandatory requirement. Due to TPI's open-access character, they receive students with severe weaknesses in mathematical knowledge and skills (Farías & Sevilla, 2015) and fear of the subject (Cox, 2011). Open access institutions, such as TPI, put effort into developing students' mathematics knowledge besides changing their motivations, beliefs, and self-concepts related to mathematics. Most of these efforts are concentrated on first-year students. However, in general, the failure rate in mathematics courses is high. In Chile, these rates are greater than 50% in some cases, which makes mathematics one of the main academic reasons for dropping out, about 30% in TPI's first year (Servicio de Información de Educación Superior [SIES], 2021). Unfortunately, there is little research on this phenomenon in Chile and in many other countries, especially compared to the study of mathematics education at colleges and universities. TPI enrolls about 40% of the entire Chilean higher education system, concentrates on students coming from the most impoverished families, and is an effective instrument for social mobility for those who complete their programs (Espejo, 2016; SIES, 2020a, 2020b). Moreover, previous studies, particularly in community colleges in the US, suggest that open access to mathematics influences retention, grade completion, continuing education, and job expectations (Attewell et al., 2006; Bahr, 2010; Rose & Betts, 2001).

TPI students' perspectives on mathematics are critical but understudied. Measuring their perspective in learning mathematics will allow institutions and educators to improve their efforts to support students. This study seeks to explore TPI students' motivational beliefs regarding mathematics learning by considering students: beliefs about mathematics (Op 't Eynde et al., 2006), self-efficacy beliefs (Bandura, 1994), and intrinsic and extrinsic goal orientations (Mesa, 2012; Ryan & Deci, 2000; Ryan & Deci, 2017). Thus, we developed an instrument to measure the motivational beliefs consisting of four scales: beliefs, self-efficacy, intrinsic goal orientation, and extrinsic goal orientations for TPI students. To test the instrument's validity, we applied it in two TPI institutions, which receive similar types of students, but under very different teaching and learning environments. The first institution is a large private TPI that enrolls more than 40,000 students across the country (*Professional Institute* hereafter). The second is a Navy academic unit, called *Escuela de Grumetes* (Armada Institute hereafter), which trains about 1,100 low-ranked marine infantries at one location. Students at both institutions share similar socio-economic backgrounds and weak mathematics academic preparation. The level of first-year mathematics is also similar and includes the topics such as trigonometry, pre-calculus, and pre-algebra. However, both contexts are significantly different. Students at Professional Institute go to their branches only for class-time, commuting from home or workplace. Students at Armada Institute have a residential regime on an island and live under strict discipline.

In this article, we aim to develop an instrument to measure the motivational beliefs towards mathematics learning (MBtML), discuss the reliability and the validity evidence, and support its applicability by comparing the cases of the Professional and Armada institutes. Thus, we contribute to the field of students' motivational beliefs towards mathematics learning at open-access institutions in higher and postsecondary education. We also offer an exploration of the relationship between institutional context and students' motivational beliefs towards mathematics. In fact, the age of students in this field indicates the gap between completing high school and entering university, which can impact their motivational beliefs. Finally, we discuss our findings and suggest some paths for future research.

Background

The Chilean higher education system consists of 150 institutions divided into three main types. *Centros de formación técnica* (vocational-technical centers) (N=52) offer two-year programs, and professional institutes (N=39) offer four-year programs. These two types of institutions are often considered one sector for students, teachers, practitioners, and public policy. In this study, we call them TPI, which are considered open-access institutions and mainly concentrate on students from low-income families in the higher education system (Espejo, 2016). Finally, universities (N=59) are the only type of institution allowed to grant bachelor's, master's, and doctoral degrees (SIES, 2020a). In 2020, 1,151,727 students are enrolled across these higher education institutions, where 42% are enrolled at TPIs and 58% at universities (SIES, 2020b). Across the Chilean higher education system, teaching follows a traditional approach, focusing on content memorization and evaluation based on content repetition and a general lack of enthusiasm for innovation (Yin et al., 2018; Marchant et al., 2018). Even in an institutional context where mathematics has a student-centered approach, teachers keep a vertical role, directing, interrupting, and evaluating their students' work (Celis et al., 2019).

Overall, most Chilean young adults (about 85%) are expected to enter higher and postsecondary education, even above other OECD countries (OECD, 2019a). However, Chile ranks at the bottom in mathematics achievement in international standardized tests, such as PISA (OECD, 2019b). Mathematics teaching in Chile, as in many countries, is fundamentally traditional, and students struggle with a lack of problem-solving skills during school, causing them to underperform in national and international exams (Saadati et al., 2021). This mix of a lower achievement in mathematics learning during school years of (K-12), compared to other countries, and massive access to higher education, creates a teaching challenge for institutions, especially for TPIs. Other countries face similar scenarios, such as the US, where mathematics teaching has been an issue of increasing research interest (e.g., Bahr, 2010; Chen, 2016; Hodara & Jaggars, 2014; Melguizo et al., 2014).

Public and institutional policies address teaching mathematics at open access by implementing placement tests, remedial or developmental courses, or support or pedagogical strategies for regular courses (e.g., Bahr, 2013; Hodara & Jaggars, 2014; Melguizo et al., 2014). The debate on what strategies are more effective is still undergoing (Logue et al., 2016; Xu & Dadgar, 2018). No matter what strategy, the consensus suggests students need to be at the center of the teaching and learning process (Freeman, 2014). In mathematics education, it is often referred to as "reformed teaching," and more generally in STEM (Science, Technology, Engineering, and Mathematics) "active learning." The challenges of enacting this form of teaching have triggered inquiries about teachers' (Mesa et al., 2014) and students' motivational beliefs (Chen & Lin, 2020; Mesa, 2012; Stage & Kloosterman, 1995; Wang & Liou, 2017).

Beliefs are usually classified between two poles (e.g., Geisler & Rolka, 2020; Giaconi et al., 2018; Saadati et al., 2019). The traditional pole conceives mathematics as a collection of facts and rules that must be learned step-by-step through memorization and practice (Kizilgunes et al., 2009; Stage & Kloosterman, 1995). In this view, the teacher carries high authority – defining what is deemed to be the correct way of addressing mathematics. For traditionally minded students, a good teacher is then someone who explains mathematics to students in an orderly and precise manner. This type of student would be resistant to changes. Students on the opposite pole can perceive mathematics as a subject to explore, where to being confused or puzzled is a valuable part of the process. For this type of student, there is space for original thinking and creativity in the mathematical classroom. In open-access institutions, previous studies have found that most students hold more traditional beliefs about mathematics in less demanding programs (Drobnic Vidic, 2015). Geisler and Rolka (2020) also found that viewing mathematics as a static body of knowledge tends to remain stable when students are transitioning to college. If students' beliefs are aligned with the institutional teaching approach, students would be positively motivated towards the learning of mathematics (Mesa, 2012).

Another element that is distinctive among students in open-access institutions is fear of mathematics (Cox, 2011). Part of this fear is related to a low self-efficacy towards doing mathematics. Self-efficacy or the belief "in one's capabilities to organize and execute the courses of action required to produce given attainments" (Bandura, 1997, p. 3) is a well-studied construct, in higher education. Previous research has found that self-efficacy positively influences academic achievement and the desire of continuing studying mathematics (e.g., Peters, 2013). Self-efficacy also influences motivation to learn mathematics. Students with high mathematical self-efficacy are more willing to work on problem solving tasks than those with low levels of self-efficacy (Chen & Lin, 2020; Chen & Starobin, 2017).

Conceptual Framework

Motivation can be conceived as a goal-oriented behavior that represent engagement, dedication, and enjoyment of a particular task (Mesa, 2012; Middleton & Spanias, 1999), for example, learning of mathematics. There are a plenty of studies that have theoretically and empirically supported the idea of the association between students' motivations and their success in learning achievement (Wigfield & Eccles, 2002). For the initial motivation theorists, the focus was based on students' motivation like drives and needs or the patterns of rewards and punishments that they received in school or other settings (Wentzel & Wigfield, 2009). This idea could predict that a student motivated towards mathematics would dedicate more time, and with a positive disposition, to work on the subject, which would likely influence learning. However, there is some contradictory evidence from the countries with high-performing, but low-motivated students in learning science and mathematics such as Taiwan (Wang & Liou, 2017), which can highlight the importance of conceptualizing students' motivation as a multifaced construct.

The efforts of scholars in this field have established a variety of rich constructs mainly developed based on different motivational theories such as social cognitive theory, self-regulation, self-determination, and expectancy-value theories. Linnenbrink and Pintrich (2002) point out that "social cognitive models stress that students can be motivated in multiple ways and the important issue is understanding how and why they are motivated for school achievement" (p. 313). There is also an emphasis on the role of motivation in students' self-regulation; motivation enables students' academic success, mainly towards promoting students' self-regulated learning (Linnenbrink & Pintrich, 2002; Pintrich, 1999). Self-determination theory also focuses more on the orientation of motivation rather than the level of motivation by distinguishing between different reasons or goals that cause an action to happen (Ryan & Deci, 2000; Ryan & Deci, 2017).

To have a multifaced understanding of students' motivation, Pintrich et al. (1991) developed an instrument called Motivated Strategies for Learning Questionnaire (MSLQ) in two sections (motivation and the learning strategy). The motivation section was developed based on expectancy-value theories and designed in three main components; value component (goal orientation and task value), expectancy component (self-efficacy and control of learning beliefs) and affect component (test anxiety). Eccles and Wigfield (2002) believe that the focus on belief, values, and goal constructs can explain the reason why students prefer to engage or disengage in some activities and lead to important advances in the field of motivation. By considering the common key propositions of each theory and the contexts of TPI and open-access institutions in higher education, our hypothesis is that students' motivations towards learning mathematics are supposed to be influenced by their beliefs about the subject, perceived expectations of self, and goal-orientations regarding mathematics tasks. Accordingly, we are interested in three components: (i) beliefs about learning mathematics, (ii) self-efficacy, and (iii) intrinsic and extrinsic goal-orientations.

Beliefs about Learning Mathematics

Students' beliefs can be categorized in multiple ways, from an epistemological belief system about the structure and source of knowledge to justifications for learning and knowing (Op 't Eynde et al., 2006). Here, we focus on how students conceive the learning of mathematics in a bipolar fashion, from a static, rote, or surface approach to learning to a more meaningful or deep approach (Chan, 2003; Kizilgunes et al., 2009). Students from the first pole expect content delivery by the teacher as authority. In the second pole, students view the learning of the content as a process of exploration and discovery (Chan, 2003; Geisler & Rolka; 2020; Kizilgunes et al., 2009). Previous evidence suggest that students' beliefs are domain-specific, in which mathematics stand-out (Buehl et al., 2002; Op 't Eynde et al., 2006).

Nevertheless, there is some debate as to whether students' beliefs about mathematics change during their progress of education from primary schools to colleges (Geisler & Rolka; 2020). In postsecondary-level mathematics, students normally encounter difficulties with working on more sophisticated mathematics problems, which cognitively demands more efforts, shifts of conceptual interpretations, and maturity of feelings about their abilities to reinforce understanding (Skilling et al., 2020). On the other hand, Geisler and Rolka (2020) found that students with rote beliefs remain stable during the transition, but meaningful beliefs tend to decrease as students struggle with the subject. Students' beliefs about mathematics are highly influencing their learning strategies and goals, it is then likely that when they feel comfortable with mathematics, they challenge themselves to achieve more (Middleton & Spanias, 1999). There is also some evidence among first-year and second-year college students (under 24 year old) that beliefs influence academic achievement and motivation in females more than male students in open-access institutions (Stage & Kloosterman, 1995).

Self-efficacy

Self-efficacy comes from Bandura's social cognitive theories; however, it is the integrated components of the three mentioned theories of social cognitive, self-regulation, and expectancy-value. Bandura (1994) defines self-efficacy as an individual's beliefs in his/her own capabilities to achieve certain levels of performance. According to Bandura (1994), four sources feed students' self-efficacy: mastery experiences (or previous achievements), vicarious experiences (or seeing the achievements of similar others), social persuasion (others' opinions and beliefs on own capacities), and emotional arousal (emotions triggered by the performance). Self-efficacy in different studies referred to different sources of beliefs about one's own capability. For example, Green (2020) showed that students' self-efficacy regarding their beliefs about the capacity to cope with daily challenges and stress is indirectly related to their vocational identity development. Researchers were also interested in identifying the role of academic self-efficacy as a factor that can influence academic performance among undergraduate students (Honicke & Broadbent, 2016). Academic self-efficacy, in fact, referred to students' confidence in their ability to complete the educational requirements with relatively global measures (Lent et al., 1989). Lent and colleagues (1989) created two specific subscales of self-efficacy as engineering self-efficacy and science self-efficacy to capture the impact of self-efficacy to inventoried vocational interests. As mathematics is an important subject for students in TPIs and is related to their future careers, there is a need for considering self-efficacy at a more specific level. In the case of this study, student self-efficacy is understood as a student's beliefs in his/her own abilities to learn mathematics and reach the academic performance requirement of his/her program.

Intrinsic and Extrinsic Goal-Orientations

Based on the self-regulation and expectancy-value theories, from students' point of view, the importance of a task influences their performance. Task importance or goal orientation refers to personal value as an intrinsic or utility value (Eccles & Wigfield, 2002). Intrinsic value is defined as an individual's enjoyment of performing some activities related to her/his subject of interest. Utility value is determined as the positive value that an individual put on a task based on how the task is related to her/his current and future goals. In certain respects, intrinsic value is similar to the notion of intrinsic motivation and utility value or usefulness is similar to extrinsic motivation (Eccles & Wigfield, 2002; Wigfield et al., 2009). On one hand, intrinsic motivation refers to engaging with and enjoying a task because it is inherently fulfilling for the individual, without the need for external incentives (Ryan & Deci, 2000; 2017). In mathematics, an intrinsically motivated student works on math with the goal of learning more mathematics to improve knowledge and skills for his/her own sake. On the other hand, extrinsic motivations refer to engagement motivated by external forces, such as rewards (e.g., grades), failure-avoidance, and future academic or job performance (Ryan & Deci, 2000; 2017). Extrinsically motivated students engage in mathematics guided by goals other than improving in math. In summary, we consider the goal orientation construct as the degree to which an individual perceives his/herself engaging in a task for intrinsic or extrinsic reasons.

Students' beliefs, self-efficacy, and goal orientations are all constructs influenced by the context surrounding each student. According to Eccles and Wigfield (2002), "it is difficult if not impossible to understand students' motivation without understanding the contexts they are experiencing" (p. 128). In particular, the influence of the institutional context on student motivation has received research attention (e.g., Drobnic Vidic, 2015; Geisler & Rolka, 2020; Mesa, 2012; Peters, 2013). For institutions, this influence is important since motivation is a factor in improving academic achievement, retention, and academic progress. The main venue in which institutions attempt to influence students' motivation is through the teaching of mathematics, setting the norms of what instructors are expected to do (Chazan, Herbst, & Clark, 2016). For instance, a more demanding course promotes more positive beliefs about mathematics than less demanding ones (Drobnic Vidic, 2015). However, these results, with the exception of Mesa's (2012) study, are obtained from a university or college-level math. There is little research on students' motivation in the institutional context of technical and vocational higher education, such as TPI. Thus, the goal of this study is twofold: develop an instrument to measure students' motivational beliefs in learning mathematics in technical and vocational higher education (i.e., beliefs, self-efficacy, and goal-orientation) and explore how student motivation is in turn related to different institutional contexts.

Methodology

According to the foci of this study, the first part aims to develop an instrument that we call "MBtML", and review its validity evidence of components and items in order to measure students' motivation in mathematics learning. The next part focuses on the application of the instrument in two different institutional contexts in technical and vocational higher education to the relationship between these contexts and students' motivation.

Participants

The target group was first-year students from two different higher education institutions, different branches of Professional Institute and a single campus at Armada Institute. After discussing and agreeing with the school officials, the researchers distributed the questionnaire and the consent form among the mathematics teachers of both schools and asked them to first give the consent form and then the questionnaire (for the students who agreed to participate in the study) at the beginning of the class. Students were asked to answer a questionnaire that took almost 20 minutes. The data collection happened during the last month of the school year. In total, we received a sample of 1,239 responses, which were used in this study. The details of the participants' demography are presented in Table 1.

Table 1. Participants: Sample Size and Demography

Demographic Variables		Exploratory Analysis		Confirmatory Analysis		Armada Institute		Professional Institute	
Gender	Male	485	78%	479	77%	685	85%	279	64%
	Female	119	19%	127	21%	115	14%	131	30%
	Did not declare gender	16	3%	13	2%	4	1%	25	6%
Age (years)	Under 20	294	47%	272	44%	471	59%	95	22%
	20-22	181	29%	207	33%	244	30%	144	33%
	22-24	70	11%	72	12%	76	9%	66	15%
	24 & above	61	10%	56	9%	4	.5%	113	26%
	Did not declare age	14	3%	12	2%	9	1.5%	17	4%
Total		620	100%	619	100%	804	65%	435	35%

Among them, 804 students were from the Armada institute, and 435 were from four different branches of the Professional Institute. Overall, there were 246 (20%) female, and 964 (78%) male students, while 29 (2%) persons did not identify their gender. The small number of female students is in relation to the respective population of the population in the TPIs (e.g., Cabero Almenara et al., 2019). Moreover, where 26 (2%) of students did not the range of their age, there were 566 (46%) students under age 20, 388 (31%) between 20 – 22 years old, 142 (12%) between 22 –24 years old, and 117 (9%) students were 24 years old or older.

Procedures and Analysis

First Part - Scale Development

The scale is constructed to measure students' demographic characteristics, their motivational beliefs in mathematics including expectations and beliefs about mathematics, their perceived self-efficacy, and goal orientation in learning mathematics. The demographic characteristics part is also composed of questions about gender, age, semester enrolled, and the name of their mathematics teacher. The preliminary version of the scale was created with 25 items established through the relevant base work. We adapted and translated items from the "Patterns of Adaptative Learning Scales" (PALS) survey used by Mesa (2012) since it was applied to community college students, equivalent to the Chilean TPI context. Mesa's PALS (2012) used seven scales with 41 items; we subsequently translated 28 of them related to students' motivation and self-efficacy and added four items related to perceptions about the use of mathematics. The method of scale development allows us to have the structure of the motivation based on all the main constructs and theories by using a variety of available resources and instruments at a shorter scale suitable for this group of students. This process of item selections and modifications was developed by a team of three researchers, including the second author.

Later, the evidence for validity as well as reliability of the instrument has been checked. In the process of validation, we provided relevant evidence based on the Standards for Educational and Psychological Testing (American Educational Research Association [AERA], American Psychological Association, & National Council on Measurement in Education, 2014) to ensure a scientific basis for the interpretations of the test scores for the proposed use that are evaluated. In fact, "validity refers to the degree to which evidence and theory support the interpretations of test scores for proposed uses of tests" (AERA et al., 2014; p.11). Test content validity evidence was checked by gathering information from a panel of content experts. The set of adapted items was reviewed by a group of three experts in math and science education at the Professional Institute with over five years of experience. They were contacted face-to-face and asked to check the items and respond to what extent the items accorded with the concept of students' motivation. They were also asked to make comments on each item if necessary. The feedback and comments were used to improve the clarity of the statements and their relevance to the three dimensions.

The response processes were checked through a pilot study (Gall et al., 2007). The preliminary version of the scale was constructed in three parts and in a form of a 32-item self-reported questionnaire with a 7- point agreement scale in order to increase the reliability (Symonds, 1924). It was tested in a pilot study with 120 students from one of the first-year mathematics courses in an engineering school at a research university in Santiago, Chile. The participants were asked to reflect on the clarity of the items and their relevance to their mathematics learning experiences. The results of piloting suggested keeping the 25 items and two subscales. The first subscale, students' expectations and beliefs about mathematics, with 10 items specifically address students' beliefs in mathematics from two different perspectives as traditionalism (e.g., Q.2) vs reformism (e.g., Q.1). The second subscale, perceived self-efficacy (e.g., Q11) and goal orientation (e.g., Q17), was composed of 15 items.

The first part of the main study aimed to show the validity evidence from the internal structure of the instrument as well as the reliability (Gall et al., 2007). The psychometric results have been checked by conducting Exploratory Factor Analysis (EFA) and Confirmatory Factor Analysis (CFA). The data collected from the sample of 1,239 students is randomly split up into two sub-samples. The EFA is carried out with one-half of the sample or a total of 620 students to evaluate whether the factors cluster in the expected way (Thompson, 2007) and identify the underlying separable three dimensions, representing the study's theoretical construct.

In addition, a varimax rotation is used as one of the recommended orthogonal rotations (Thompson, 2007) in order to help and improve the interpretation of the results. Later, to confirm the EFA structure, confirmatory factor analysis (CFA) is performed with the other half of the sample of 619 students. The CFA is used as a complementary analysis for the explanatory factor analysis. As Rios and Wells (2014) declared "CFA provides evidence to support the validity of an internal structure of a measurement instrument by verifying the number of underlying dimensions and the pattern of item-to-factor relationships (i.e., factor loadings)" (p. 109). The adequate goodness-of-fit indices should be $\chi^2/df > 5$; CFI>.9; IFI>.9; and RMSEA<.08 (Byrne, 2010; Hooper et al., 2008). Besides, the internal consistency reliability was checked to estimate the "coefficient of precision from a set of real test scores" (Bostic & Sondergeld, 2015; Crocker & Algina, 2006, p. 117) by calculating the α-Cronbach.

Second Part - Application in the Contexts

The second part of the study, indeed, focuses on testing the validity evidence for relationship to other variables. For example, the external variables (e.g., gender and type of institutes) may include measures of some criteria that the test is expected to predict (AERA et al., 2014). The interpretation of the instrument's scores needs to show the groups differences based on the theories, for example in displaying the differences in students' beliefs about self and beliefs about mathematics in dissimilar situations. Two groups of first year students of different age levels from two different types of higher education institutes (as discussed before) have participated in this survey study and their responses were used to provide evidence about the degree to which these relationships can be consistent with the construct underlying the proposed test score interpretations (AERA et al., 2014). In this part of the study, we researched whether the students' motivation in learning mathematics measured by the instrument differed significantly according to their age levels and types of institutes.

The Kolmogorov–Smirnov test was applied to check the normality of the scores in different dependent variables according to the factor analysis and for each independent variable (i.e., age levels and the type of institutes). The normality test concluded that the dependent variables are not normally distributed ($p < .05$). Therefore, to check the criterion validity, the groups were compared using nonparametric tests. The Mann–Whitney *U* test was used to compare differences of students' motivation in two type of institutes, and Kruskal–Wallis tests for comparing the groups according to the age levels. Furthermore, the value of the effect sizes (*r*) were calculated by the formula of $r=Z/\sqrt{N}$ in which N is the total number of cases (Pallant, 2013).

Results

In the first part of this section, we report the results of exploratory and confirmatory factor analyses as the validity evidence of the instrument's internal structure. We then present descriptive statistics of confirmed constructs or dimensions and their reliability. In the second part, we applied the instrument in the contexts and review the instrument's validity evidence based on its relationship with other variables. It is, as we mentioned earlier, a way to check the validity evidence of the instrument to capture the differences in motivational beliefs for students in different age and educational contexts.

Instrument Development

Prior to performing the component analysis, the suitability of data for factor analysis and the strength of the inter-correlation among the items was assessed by two statistical techniques; Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy, and Bartlett's Test of Sphericity (Pallant, 2013). Inspection of the correlation matrix revealed the presence of many coefficients of .3 and above. Moreover, the results support the factorability of the correlation matrix since the KMO value was .94, exceeding the recommended value of .6 (Kaiser, 1970) and Bartlett's Test of Sphericity was statistically significant ($p < .001$).

The initial analysis revealed the presence of five components with an eigenvalue above 1 which explained 38%, 8%, 7%, 5%, and 4% of the variance respectively. However, the scree-plot did not clearly support five factors. In order to discover a better pattern of constructs, the number of factors was set as four and six (Costello & Osborne, 2005). The results of repetitive EFA while deleting items with factor loading below .40 and examining the scree-plots determine four factors with 21 items as the best matrix. This was further supported by the results of the parallel analysis, which showed that four components with eigenvalues exceeding the corresponding criterion values explained a total of 66% of the variance. The final four factors (see Table 2) extracted are entitled *Beliefs about Mathematics in a General View*, *Self-efficacy*, *Intrinsic Goal*, and *Extrinsic Goal*. To aid in the interpretation of these four components, Varimax rotation as an orthogonal approach was performed to minimize the number of variables that have high loadings on each factor.

The first factor named *Beliefs about Mathematics* measures the degree of students' perception of mathematics as a discipline that needs specific ways of learning, which is not based on learners' perspectives. Thus, 6 items are included in this dimension. Q.1, Q.7, and Q.10 are based on a reformed perspective, which presents a set of beliefs about students' creativity and the role of a teacher as a facilitator. Q.2, Q.6, and Q.9 are written through a traditional perspective that focuses on beliefs about students' procedural and superficial thinking and teacher authority in the classroom. However, the students did not find the existing items in two different poles as traditional and reformed beliefs, but they see that the value of all items is in one direction. This factor did not follow what the theory suggested. The target group has a different view, not as a traditionalist or reformist, but rather as a kind of combined belief in learning and teaching mathematics. The second factor, *Self-efficacy*, is the degree to which students believe in their capability in learning mathematics. The next two factors are related to students' goal orientation in learning mathematics. *Intrinsic Goal*, as the third factor, identifies the degree to which students believe that mathematical activities are interesting and meaningful. The last factor, *Extrinsic Goal*, is the degree to which students believe that learning mathematics is positively related to their future work.

Table 2 Pattern Matrix of Students' Motivational Beliefs in Learning Mathematics

Factor	Item No.	Factor Loading 1	2	3	4
Beliefs about Mathematics (B.M.)	Q.1	.40			
	Q.2	.43			
	Q.6	.55			
	Q.7	.70			
	Q.9	.65			
	Q.10	.51			
Self-efficacy (S.E.)	Q.11		.76		
	Q.12		.67		
	Q.13		.78		
	Q.14		.82		
	Q.15		.81		
	Q.16		.77		
Intrinsic Goals (I.G.)	Q.17			.61	
	Q.18			.64	
	Q.19			.75	
	Q.20			.80	
	Q.21			.81	
Extrinsic Goals (E.G.)	Q.22				.60
	Q.23				.76
	Q.24				.90
	Q.25				.88

The CFA model was tested by using the other half of the sample (see Table 3). The model is defined according to what the EFA suggests, with 4 factors or latent variables corresponding to the defined dimensions as *Beliefs* (B.M.), *Self-efficacy* (S.E.), *Intrinsic Goal* (I.G.), and *Extrinsic Goal* (E.G.) in learning mathematics. In the model, the items Q.3, Q.4, Q.5, and Q.8 were not included because of the low factor loading in the initial analysis. The correlations between the factors were defined as free parameters (Table 3). The model provided adequate goodness-of-fit indices for the data: Overall χ^2 (183, N=619)=724.516; CFI=.922; IFI=.923; RMSEA=.069 (90% confidence interval=.039; .060). All the items have also had statistically significant factor loading.

Table 3. Standardized Regression Weights of Model 2

Item	B.M	S.E.	I.G.	E.G.
Q.1	.61			
Q.7	.51			
Q.10	.43			
Q.2	.58			
Q.6	.50			
Q.9	.44			
Q.11		. 75		
Q.12		. 67		
Q.13		.77		
Q.14		.84		
Q.15		.84		
Q.16		.87		
Q.17			.77	
Q.18			.66	
Q.19			.72	
Q.20			.88	
Q.21			.88	
Q.22				.75
Q.23				.86
Q.24				.82
Q.25				.77
Correlation among factors	B.M.	S.E.	I.G.	E.G.
B.M.	1			
S.E.	.69*	1		
I.G.	.76*	.68*	1	
E.G.	.54*	.44*	.63*	1

* $p < .001$

Finally, descriptive statistics of the dimensions of the instruments were checked to ensure their appropriateness as measurement constructs. Descriptive statistics of students' motivational beliefs (see Table 4) show that the means (*M*) of all the dimensions ranged from 5.81 to 6.38 with standard deviations (*SD*) from .73 to 1.20. Moreover, the factor of self-efficacy beliefs has the highest mean of 6.38 (*SD*= .85) while the extrinsic goal has the lowest of 5.81 (*SD*=1.20).

The Alpha Cronbach was also used to check the inter-reliability of the components besides CFA. For the S.E., I.G., and E.G., values of Alpha bigger than .88 likewise high inter-item corrections indicate a strong internal validity for each of these components. The value of Alpha for the B.M. was .67, which is poor, but acceptable reliability by many researchers in the field of the affects in science education (Taber, 2017).

Table 4. Descriptive statistics and reliability of the dimensions (N = 1239)

Dimension	No. of Items	*M*	*SD*	Min.	Max.	Alpha Cronbach	Inter-item correlations ranging
B.M.	3	6.15	.73	1	7	.67	.15 - .38
S.E.	6	6.38	.85	1	7	.92	.54 - .76
I.G.	5	6.26	.93	1	7	.90	.57 - .83
E.G.	4	5.81	1.20	1	7	.88	.53 - .77

The lowest mean with the highest standard deviation occurs for Extrinsic Motivation. It can be related to the participants of the study; they were from two different higher institutes with different future careers as we discussed before. To make sure that the instrument is sensible in the contextual differences, the results of the second part of the study are presented here.

Application in the Contexts

Two different non-parametric tests, the Mann-Whitney U and Kruskal-Wallis tests, were performed to examine the relations between motivational beliefs factors and demographic or independent variables; (1) age and (2) type of institutes.

The first analysis reveals that significant differences occurred in all factors of students' motivational beliefs (B.M., S.E., I.G., and E.G.) based on the two different types of institutes. As Table 5 shows, students in Armada reported a significantly higher level of motivational beliefs in learning mathematics than the students in Professional institute regarding the scores of S.E. with U=107030.5; p-value<.001; r=.33 indicated a medium effect size, I.G. with U=98898; p-value<.001; r=.37 a quite large effect size, E.G. with U=135628.500; p-value<.001; r=.18 a small effect, B.M. with U=129712; p-value<.001; r=.21 almost medium effect. As results reveal, the students from both institutes have reported the highest difference in their level of intrinsic motivation and then their self-efficacy.

Table 5. Mann-Whitney U test results according to the type of institutes

Dimensions	Institutes	n	Mean Rank	Sum of Ranks	*U*	*P*	*r*
B.M.	Professional	435	516.19	224542.00	129712.000	.000**	.21
	Armada	804	676.17	543638.00			
S.E.	Professional	435	464.05	201860.50	107030.500	.000**	.33
	Armada	804	704.38	566319.50			
I.G.	Professional	435	445.35	193728.00	98898.000	.000**	.37
	Armada	804	714.49	574452.00			
E.G.	Professional	435	529.79	230458.50	135628.500	.000**	.18
	Armada	804	665.25	531536.50			

**p < .01.

In the second analysis, we ran Kruskal-Wallis tests to check if there are significant differences in students' level of motivational beliefs in each dimension across four age levels. Table 6 shows there were statistically significant differences in only two dimensions across four different age groups (G1. n=566, G2. n=388, G3. n=142, and G4. n=117), which are S.E. with $\chi^2(3, 1213)=14.982$, $p<.01$, and I.G. with $\chi^2(3, 1213)=19.059$, $p<.01$. An inspection of the post-hoc analysis indicated that students in their 24th and above reported a significantly lower level of S.E. in comparison with those students younger than 22 years old in G1 and G2. Moreover, this group of students reported a significantly lower level of I.G. than the other students in G1, G2, and G3.

Table 6. Kruskal-Wallis Test Results according to Age Level

Dimensions	Age level		n	Mean Rank	*df.*	χ^2	*P*	Differences
B.M.	G1.	<20	566	610.62	3	2.998	.392	
	G2.	Between 20-22	388	622.85				
	G3.	Between 22-24	142	576.32				
	G4.	<24	117	574.14				
S.E.	G1.	>20	566	633.96	3	14.982	.002**	G1- G4**
	G2.	Between 20-22	388	610.07				G2- G4**
	G3.	Between 22-24	142	571.49				
	G4.	<24	117	509.49				
I.G.	G1.	>20	566	630.07	3	19.059	.000**	G1- G4**
	G2.	Between 20-22	388	619.32				G2- G4**
	G3.	Between 22-24	142	582.63				G3-G4*
	G4.	<24	117	484.11				
E.G.	G1.	>20	563	623.17	3	4.303	.231	
	G2.	Between 20-22	387	597.63				
	G3.	Between 22-24	141	587.85				
	G4.	<24	117	557.43				

$*p < .05$, $**p < .01$.

To sum up, the results indicate that the four dimensions as reformed beliefs, traditional beliefs, self-efficacy, intrinsic goal orientation, and extrinsic goal orientation developed with 21 items in this study were reliable enough to measure motivational beliefs in learning mathematics for students in different age levels and from different educational contexts.

Discussion and Conclusion

Open-access institutions are an important part of higher and postsecondary education in most countries. Keeping students motivated towards mathematics learning is one of the central challenges at these institutions. Motivation is also important regarding student success and future careers. Developing an instrument or a scale to capture students' motivational beliefs towards mathematics was the main purpose of this study. To do so, we first determined the dimensions of motivational beliefs towards mathematics learning based on its definitions, the related theories, and the literature. Then the initial instrument included two scales and 25 items, which were chosen in relation to the three dimensions: Beliefs about mathematics, self-efficacy, and goal orientations. Its content was modified slightly after reviewing the experts' feedback and a pilot study with more than 100 students. Then, the revised version of the instrument with 25 items was administrated with 1,239 students from two different open access TPI with branches across Chile. Theoretically, we expected to have five factors, however, the results of the EFA confirmed four dimensions or scales based on the dataset in the study. There were items related to the beliefs that were written in the two different perspectives as static vs. dynamic view of mathematics. The common variances shared between these items confirmed one single factor in students' responses to beliefs-related items and the relationship between these items. It indicates that we cannot dichotomize this group of students into traditionalists vs. reformers based on their beliefs about mathematics teaching and learning. They have a sort of mixed beliefs that still leave room for students' creativity while accepting teachers' authority. The results also suggest an influence of the institute context on the reinforcement of these mixed beliefs. The same pattern was found for Chilean mathematics teachers in a study done by Saadati et al. (2019). Therefore, four items were removed, and the confirmatory analysis confirmed the four-factor model derived from the EFA with adequate goodness-of-fit indices. In the end, the MBtM instrument was acquired with four scales of 21 items. The scales of the instrument were beliefs about mathematics, self-efficacy, intrinsic goal, and extrinsic goal orientation. To sum up, the qualitative and statistical methods to check the validity evidence allowed us to have confidence "that results from mathematics measures are consistently measuring (reliability evidence) what we expect them to measure (validity evidence) is critical in mathematics education research", which is important as Bostic and Sondergeld (2015) suggest, "more instruments need to be developed and empirically evaluated through the use of more modern analytical approaches" (p. 289).

The correlation analysis shows significant correlations among the factors in this study. The highest correlation was between beliefs about mathematics and intrinsic motivation (r=.76), while the lowest correlation was between self-efficacy and extrinsic motivation (r=.44). Moreover, self-efficacy is more correlated with intrinsic activity rather than the extrinsic one since the correlation between self-efficacy and intrinsic motivation was larger (r=.68). Lent et al. (1989) declared that self-efficacy in science and engineering learning is necessary to promote and maintain students' interest in doing an activity (or their intrinsic motivation), but the effect is limited because the additional rises in self-efficacy do not yield greater interest. The results also suggested a significant correlation (r=.63) between two separated dimensions of goal-orientations (intrinsic and extrinsic), which is interesting and relevant to the context of the study and our target group. This distinction supports the Ryan and Deci (2000)'s work where they critique the self-determination theory in order to clarify the critical distinction between these two types of motivation as "those that emanate from one's sense of self and those that are accompanied by the experience of pressure and control and are not representative of one's self" (p. 65). It can also be supported directly by the context of the study, in which students from postsecondary vocational education system participated. Hattie (2009) declared that "as students get older, personality as well as cognitive proficiency can combine to better predict subsequent performance (especially motivation-related personality variables)" (p.45). Most TPI students have experienced low academic achievement, insufficient skills in the learning of mathematics during their secondary schools, and low motivational levels towards this subject. However, the same group, with enough autonomy choose to continue their postsecondary education, showing conscious valuing of mathematics activity, its relatedness to their future careers, and self-endorsement of the goals, which is called extrinsic motivation (Ryan & Deci, 2000).

In the second part of the study, the results in terms of the validity evidence based on the relations to other variables are twofold; the influence of age and the institutions. Regarding age, the group of students 24 and older, reported significantly lower levels of self-efficacy and intrinsic motivation than their younger peers. Among this group of students, there were students who could not finish school sooner. One reason would be the lack of interest in their future careers, or not finding a relationship between their academic courses and their career. In this case, the results can be explained better with the study by Stipanovic et al. (2017), showing a higher level of academic self-efficacy of students as a factor that increases their motivation to complete school sooner, increase their willingness and interest in taking on more challenging courses like mathematics, and making them prepare for work. The difference between this group of students and others can also be supported by the gap or educational discontinuity between their secondary and postsecondary education. This gap can cause a failure in mathematics performance, which results in low self-efficacy since academic self-efficacy is moderately correlated with academic performance (Honicke & Broadbent, 2016). This result is particularly important for technical and vocational higher education, where students enter and exit at different points in life.

When considering the institutional culture from where students participated in our study, apparently the Armada institute firmly established motivational beliefs in the students, which could align well with their professional needs. The Armada was, indeed, more successful in comparison with the Professional Institute at setting a higher level of positive beliefs, self-efficacy, intrinsic, and extrinsic motivation towards mathematics learning among students. Shin et al. (2018) declared that differences in schools' curriculum and educational environment can cause differences in students' career development related to mathematics and science. Therefore, we should focus on the goal of education, the curriculum and pedagogical activities, and the educational environment in this institution, to explain these differences. We speculate that at least three different institutional and contextual factors might explain this difference. First, although the math content in first-year mathematics is similar in both institutions, Armada is able to fully contextualize the content to that of army-related situations, which offers a clearer perspective about the usefulness of the mathematical knowledge. On the contrary, after first-year mathematics, Professional Institute students follow a broad spectrum of programs and disciplines, with different mathematics requirements. For numerous students, mathematics might be seen as a barrier rather than a tool for their future careers. Second, in Armada, within-cohort hierarchy is determined by a point system built upon different types of achievements and grades. Mathematics is one of the subjects with the highest weight in that system. Therefore, students with better math grades will benefit directly with more options than their peers for choosing specializations (e.g., mechanical engineering, pilot, coastguard) and future promotions. This system acts as a direct boost to students' extrinsic motivation. Third, in Armada, all students must reside in the school's camp located on an island. This culture of living in a group as well as the educational model promoted based on teamworking can contribute to increasing students' self-efficacy and intrinsic motivation.

Even though the interpretation of the instrument's scores exhibited sensitivity to age and contexts, the results should be interpreted with caution since our study has also some limitations. First, we should highlight that the results of this instrument provide a glimpse of the students' motivational beliefs scale developed based on the items borrowed from pre-existing scales. As we reported, 5 items did not work well within the EFA and they have been deleted. We agreed with Desimone and Le Floch, (2004), that the cognitive interview techniques could be an effective step in the survey development to revise and improve the validity and reliability of the items. Therefore, conducting an interview with a small number of students is recommended asking them to discuss their responses to each item concerning its' clarity, assessments, relativeness, and be able to receive even unanticipated interpretations. Second, there were only 20% female students in the full sample. Because of the low percentages of female participants, we decided to not include gender influence in motivational beliefs. Future studies can investigate whether this scale is sensitive enough at capturing gender issues on different scales or not. For this, the validity and reliability evidence of the scale can be measured within different groups of females and males from different institutes and/or of different ages. This information gives us evidence for the validity of the instrument and the consequences of its testing (AERA et al., 2014). Finally, we recommend checking the validity evidence based on relations to other organizations and institutes in Chile and other countries and considering other external variables (e.g., student's grades).

Despite the limitations, the development of this scale gives us insight into the factors that can measure students' motivation. These factors are important for predicting students' behavioural tendencies in willingness to do certain activities in learning mathematics. The importance of understanding students' motivational beliefs, their engagement, and self-determination provide an explanation for how students take an active and reflective role in their own learning (Ljubin-Golub et al., 2019) and it also predicts what an individual will do in his/her future career. The understanding of their motivational belief tendencies in learning mathematics can also help teachers, educators, and higher education policymakers to predict students' perseverance, conscientiousness, and finally mathematical achievement. It also can help them to assess and evaluate changes in the curriculum and the instructions.

Acknowledgements

Funding from PIA-CONICYT Basal Funds for Centers of Excellence Project FB0003, CONICYT/Fondecyt Nº11160656, and CONICYT/Fondecyt Nº11220514 is gratefully acknowledged.

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Appendix. Scale for Students' Motivational Beliefs in Learning Mathematics

No.	Original Item	Translated Item
Q.1	Hacer matemáticas deja espacio para el pensamiento original y la creatividad.	Doing math leaves room for original thinking and creativity.
Q.2	Hacer matemáticas suele ser una cuestión de trabajar lógicamente, paso a paso.	Doing math is usually a matter of working logically, step by step.
Q.6	Un buen profesor de matemáticas es una persona que explica de forma clara y completa cómo debe resolverse cada problema.	A good math teacher is a person who explains clearly and completely how each problem should be solved.
Q.7	Cuando los estudiantes resuelven el mismo problema de matemáticas utilizando dos o más estrategias diferentes, el profesor debería hacerles compartir sus soluciones.	When students solve the same math problem using two or more different strategies, the teacher should let them share their solutions.
Q.9	Los estudiantes nunca deberían salir de la clase de matemáticas sintiéndose confusos y perplejos.	Students should never leave the math class feeling confused and perplexed.
Q.10	La cuestión más importante no es si la respuesta a cualquier problema matemático es correcta, sino más bien que los estudiantes pueden explicarla.	The most important issue is not whether a student's answer to a math problem is correct or not, but rather if he/she can explain it.
Q.11	Estoy seguro de que puedo aprender matemáticas.	I'm sure that I can learn math.
Q.12	Creo que podría hacerlo mejor que hasta ahora en matemáticas	I think I could do better in math so far.
Q.13	Estoy seguro de que puedo comprender el contenido más difícil de este año en matemáticas.	I am sure that I could understand the most difficult content of this year in mathematics.
Q.14	Puedo sacar buenas notas en matemáticas.	I can get good grades in math.
Q.15	Sé que puedo tener éxito con las matemáticas.	I know I can succeed with math.
Q.16	Estoy seguro de que puedo dominar las habilidades que se enseñan en matemáticas.	I'm sure I can master the skills that are taught in math.
Q.17	Quiero aprender muchas cosas nuevas este año, en matemáticas.	I want to learn many new things this year, in math.
Q.18	Quiero entender perfectamente todos los ejercicios de las clases de matemáticas.	I want to understand perfectly all the exercises of the math classes.
Q.19	Intento aprender tanto como puedo en cada clase de matemáticas.	I try to learn as much as I can in each math class.
Q.20	Este año quiero desarrollar mis habilidades matemáticas.	This year I want to develop my math skills.
Q.21	Este año intento aprender muchas nuevas habilidades en matemáticas.	This year I tried to learn many new math skills.
Q.22	Aprender más matemáticas me entrega conocimientos y habilidades relevantes para mi futuro laboral.	Learning more math gives me knowledge and skills relevant to my future career.
Q.23	Aprender más matemáticas me abre oportunidades laborales mejor remuneradas.	Learning more mathematics opens up better paid job opportunities.
Q.24	Aprender más matemáticas ahora me permitirá, en el futuro, poder subir de cargo en mi trabajo.	Learning more mathematics will now allow me, in the future, to be able to increase my workload.
Q.25	Saber matemáticas me convertirá en un trabajador valorado en mi empleo.	Knowing mathematics will make me a valued worker in my job.