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Gameplay in Perspective: Applications of a Conceptual Framework to Analyze Features of Mathematics Classroom Games in Consideration of Students' Experiences

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Abstract

Games are often used to foster student engagement and motivation to learn content, such as mathematics. Although digital games dominate game-based learning research, the table games commonly used in classrooms warrant investigation. Especially for mathematics learning, prior research has not taken into account content-specific frameworks. Integer arithmetic (i.e., calculations with negative numbers) is a difficult topic that is crucial for later mathematics. Thus, this study synthesized multiple theoretical perspectives to understand students' experiences playing games during an integer unit. A mixed methods design study was conducted to answer the primary question: How was student experience (motivation to learn and engagement) related to game features? Student perspectives about the three integer games they played and observations of student engagement during gameplay were analyzed using perspectives from game design and the Cognitive Demand framework for mathematical tasks. Overall, students positively perceived playing games, however, individuals' motivation and engagement varied in relation to game features. Students provided valuable insights for research and practice as to which game features were engaging or motivating and why. Speed-based synchronous games that exclusively used skill, rather than a chance-skill balance induced stress and decreased motivation for many students. Thus, a critical implication was to first do no harm by selecting or designing classroom games that have features of turn-taking and chance. We suggest recommendations for mathematics learning and provide the STEM Classroom Games Features Framework to benefit research and practice of any STEM content area.

Introduction

An important role of mathematics teachers is to facilitate students' motivation to learn and meaningfully engage with challenging mathematical activities. Games are often used to increase motivation and engagement to learn any subject and particularly mathematics (Bayeck, 2020; Ernest, 1986; White & McCoy, 2019). Hence, teachers actively seek and share games to foster this motivation and engagement. Consider, for example, that a Google search for "math game" and “middle school,” the grade range relevant to this study, yielded 23 million results. Researchers have also claimed that games for learning are uniquely situated to bridge the traditional classroom learning environment and entertainment settings to foster motivation to learn (Plass et al., 2015; Garris, Ahlers, & Driskell, 2002). However, theorizing and empirical studies about student experiences with gameplay for mathematics learning, particularly in authentic classrooms, are missing. Just as early claims about manipulative use or children's books were overgeneralized as universally valuable for learning mathematics (see Ball, 1992; Nurnberger-Haag, 2017; Nurnberger-Haag, 2018c; Nurnberger-Haag et al., 2021), it appears this has also been true for games. To theorize effectively about game-based learning in classrooms, we should draw on broad perspectives from game design while simultaneously using focused mathematics education frameworks, which Spangler and Williams (2019) argue are most useful for understanding mathematics learning.

Recently, studies have focused on learning with digital games (Baek & Touati, 2020; Bayeck et al., 2020), despite the practical need to better understand student experiences with games played in physical classrooms. Game format affects students' experiences, so learning with analog (non-digital) games must be studied (Barbara, 2017). Although there is little agreement about how to define the construct of game (Plass et al., 2015), there is more agreement about classes of games. The focus of this study are the table games used in mathematics classrooms. Table games are analog games that encompass various classes such as board games that have a physical board on which pieces are moved, card games, and other games people typically play in-person around the same space, such as a table (Bayeck, 2020). Despite the popularity of card games, most analog gameplay research has focused on board games, those with pieces that move around or are captured on a board (Gobet et al., 2004).

Effective mathematics teaching involves planning that includes anticipating and monitoring student thinking, and highly complex in-the moment decision making that is influenced by teachers’ intentions, knowledge, beliefs, and goals (Depaepe et al., 2013; Stahnke et al., 2016; Stein, Smith, Henningsen & Silver, 2009). Thus, teachers and researchers need frameworks and insights prior to use (e.g., how to choose games and plan for their use) as well as during mathematical gameplay (i.e., how to interpret and make decisions about their students’ learning with games). Thus, this study investigated student experiences with three tabletop mathematics games used in authentic classrooms to provide these insights to inform research and practice.

Review of Literature and Conceptual Framework

Our review of literature begins with a brief overview of student understanding of integer operations. Although this study provides valuable insights to teachers and researchers about student experience with specific integer games, a primary purpose of this study was to use this context to inform new and deeper understanding of gameplay for mathematics learning beyond the topic of integers. Thus, we focus more on combining the theoretical perspectives we suspected could be crucial to seeing student experiences with gameplay in mathematics classrooms with more accurate and nuanced understanding. Figure 1 provides a visual overview of this conceptual framework with which we analyze the data in this study. First, to understand student experiences playing games, research should draw on perspectives of authentic gameplay. Second, to understand learning through gameplay in a mathematics classroom, frameworks from mathematics education must be considered. Cognitive Demand of Mathematical Tasks has been a crucial framework to improve the resources and contexts in which students learn (National Council of Teachers of Mathematics, 2014). We see games as a unique type of mathematical task, yet we were unable to find others who have applied this framework to games.

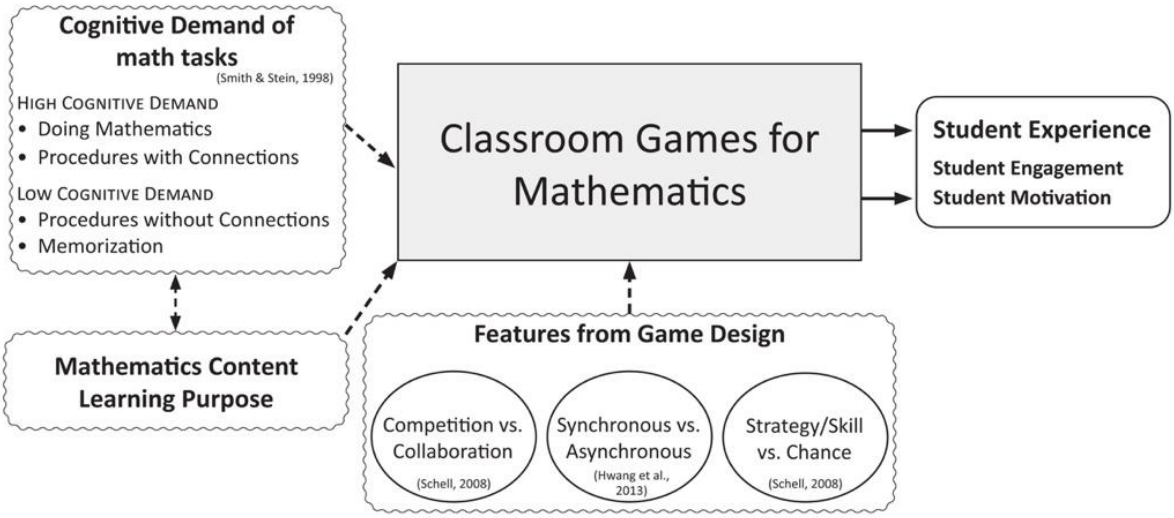

Figure 1. Mathematics Classroom Games Features Framework

Integer Knowledge and Learning

Proficient knowledge of integer arithmetic and number sense requires ordering numbers, procedural proficiency with all four basic operations, several meanings of the negative sign, and the ability to apply this knowledge to varied situations (e.g., Bofferding, 2014; Chiu, 2001; National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010; Thompson & Dreyfus, 1988; Vlassis, 2008). The Common Core State Standards for Mathematics (NGA, 2010) used in the United States suggests grade levels for beginning to understand operations with negative numbers, which means students are approximately ages 11 to 12 when they are introduced to and subsequently expected to master integer arithmetic (NGA & CCSSO, 2010).

Operations on negative numbers, however, are conceptually difficult (e.g., Fischbein, 1987). Many challenges students have with subsequent mathematics are due to difficulties with integer arithmetic (Altiparmak & Özdoğan, 2010; Bishop et al., 2014; Gallardo, 2002; Ryan & Williams, 2007). Studies in multiple countries with students who are 13 to 14 years-old who should have already mastered integer arithmetic demonstrate this difficulty and how this missing knowledge will be problematic when needed in subsequent years to succeed with more advanced mathematics. For instance, student accuracy ranged from about 33% to 67% on integer subtraction items depending on item structure (Nurnberger-Haag, Kratky, & Karpinski, 2022, p.11; Periasamy & Zaman, 2009, p.366; Ryan & Williams, 2007, p.218). A Rasch analysis of the Integer Test of Primary Operations confirmed that integer subtraction overall was the most difficult operation for middle grade students (Nurnberger-Haag et al., 2022). Subtraction problem structures that result in a positive solution are the most difficult (Nurnberger-Haag et al., 2022). Integer subtraction is particularly difficult, because in early elementary school students' extensive experience with subtraction always decreases quantities, which makes it counter-intuitive that subtracting a negative integer increases the value (Periasamy and Zaman, 2009; Ryan & Williams, 2007). Integer multiplication and division are also counterintuitive, because products or quotients of two negative numbers yield a positive number (Nurnberger-Haag, et al., 2022). Although consistent rules can be memorized for multiplication and division of integers, only about half of grade 8 students accurately answered these on a standardized test (Ryan & Williams, 2007, p. 218). Moreover, Nurnberger-Haag and colleagues (2022) found that regardless of whether the dividend was positive or negative, division by -1 was most difficult. This difficulty is problematic given that students must have this prerequisite knowledge to accurately factor while solving algebraic equations (Nurnberger-Haag, et al., 2022). Due to these conceptual difficulties with integer arithmetic, teachers and researchers continuously seek and design new ways to facilitate student learning as well as engage and motivate students to learn this difficult topic. Integer games are one such approach (e.g., Lappan et al., 2014; Nurnberger-Haag & Wernet, 2019; Wessman-Enzinger & Bofferding, 2014).

Motivation and Engagement

Motivation and engagement are theoretical perspectives from psychology commonly used to investigate authentic gameplay in general as well as game-based learning (see, e.g., Filsecker & Hickey, 2014; Garris et al., 2002). Here, we use these perspectives to describe the student experience of playing learning games in the classroom. *Motivation* is a broad construct used to describe the reasons underlying people's behavior (Middleton & Jansen, 2011) and their choice, persistence, and performance when engaging in an activity (Brophy, 2004). Whereas motivation is the generally non-observable mechanism underlying people's behavior, *engagement* is the observable manifestation of motivation (Skinner et al., 2008). Investigations of gameplay in classrooms require consideration of the construct, *motivation to learn,* which is the "tendenc[y] to find academic activities meaningful and worthwhile and to try to get the intended learning benefits from them" (Brophy, 2004, p. 16). By focusing on the learning goals of activities, this definition distinguishes it from extrinsic and intrinsic motivation that depend on rewards and enjoyment, respectively (Brophy, 2004). Expectancy-value theory is a helpful framework for understanding student motivation to learn through and engage with mathematics (Yurt, 2015). Expectancy-value theory holds that achievement motivation can be explained by a person's expectation for success in an activity, the value they place on success as reward, level of enjoyment, and beliefs about their current competence in relation to anticipation of success with a particular activity (Brophy, 2004; Wigfield & Eccles, 2000; Yurt, 2015). Through this lens, a student's willingness to play a math learning game is a product of their belief in their ability to successfully play the game, intrinsic interest in playing the game, and how much they value the learning experience.

Motivation to learn mathematics, expectation of success, math avoidance and math anxiety are interrelated (Ashcraft & Moore, 2009). Math anxiety is the experience of feeling "tension, apprehension, nervousness, and worry" as well as physiological responses consistent with fight or flight responses when engaging in math-related activities (Ashcraft, 2002; Luttenberger et al., 2018, p.312-313; Buckley et al., 2016). Three facets of math anxiety are classroom anxiety (i.e., simply being in a math classroom can invoke anxiety and may include a fear of math teachers), numerical anxiety (felt while attempting to do any mathematical computation or manipulation of numbers) and test anxiety (Luttenberger et al., 2018). Unfortunately, children as young as first grade have been documented with math anxiety (Maloney & Beilock, 2012; Ramirez et al., 2013). Math anxiety may increase and peak during middle school, because adolescents tend to reevaluate their beliefs about their academic abilities by continuously monitoring and processing their perceptions of success and failure in relation to peers (Scarpello, 2007).

Math anxiety can diminish working memory capacity, which in turn, appears to reduce procedural fluency on mathematics tasks (Beilock & Maloney, 2015; Ramirez et al., 2013; Luttenberger et al., 2018). The ubiquity of math anxiety, particularly in the United States (U.S.), is problematic because high levels of math anxiety have been associated with lower math achievement, lowered confidence in mathematical ability, and a tendency to avoid mathematical situations and take fewer elective math classes, which often results in limited career choices (Ashcraft & Moore, 2009; Barroso et al., 2021; Ahmed, 2012; Zakaria, 2012; Lou et al., 2009; Scarpello, 2007; Ashcraft, 2002; Luttenberger et al., 2018). Mathematics anxiety among middle school students have also found to have a negative correlation with achievement (Luo, et al., 2009). It is important to clarify, though, that the relationships between math anxiety and motivation are complex. Learners with high levels of intrinsic motivation may benefit from a moderate degree of math anxiety, because it may cause them to try harder to succeed (Luttenberger et al., 2018; Erturan & Jansen, 2015). This is consistent with mathematics education perspectives that effective tasks must be challenging and may require just enough anxiety to foster interest and persistence (Smith & Stein, 1998).

Perspectives from Game Design

Research is needed on how to foster positive attitudes about learning mathematics (Olivares & Ceglie, 2020). Games offer enjoyable experiences that people want to participate in—that is, they are intrinsically motivating and found in every culture in the world (Bishop, 1988; Plass et al., 2013). Games are a common way to foster student engagement with learning in the classroom and potentially mitigate the math anxiety that is so pervasive across the world, but especially in the U.S. (Ersozlu & Karakus, 2019; Luttenberger et al., 2018). However, the construct of game is not simple, indeed no common definition exists (Plass et al., 2015). Game design is incredibly complex, comprising an entire commercial industry as well as areas of research. We use the term *authentic game* in this study to refer to games played strictly for enjoyment outside of school. To design authentic games that encourage interest, motivation, and engagement, it is crucial to consider how a potential player would experience the game. Schell (2008), for example, clarified 72 lenses to potentially consider when designing a game. These lenses encompass digital and analog (tabletop) games, so not all lenses are relevant to every game type. For instance, a game designer or someone selecting a game for others should look for elements of surprise that are essential to people's perceptions of entertainment (Schell, 2008, p.26). Schell (2008) offers some specific focusing questions such as considering whether the rules allow players to surprise themselves or each other. For this study we incorporate this idea and focus on opposing features that our results revealed were important to recommend as a starting point for more fully theorizing about game-based learning with tabletop games in classrooms: competition versus collaboration, synchronous versus asynchronous play, and skill versus chance (Hwang et al., 2013; Schell, 2008; see Figure 1).

Although competitive games are most common, collaborative games exist (Schell, 2008). A collaborative game is one in which people work together to succeed at a shared task or goal, rather than succeed against or in comparison to others (Baek & Touati, 2020). Schell's (2008) lenses on game design are independent from any other choices, thus, regardless of whether a game is competitive or collaborative, it can have any other combination of features. Gameplay can be synchronous in that all players are simultaneously playing, or a game may be asynchronous, which has also been described as sequential, turn-based or having a turn-taking feature (Barbara, 2017; Hwang et al., 2013). For literacy vocabulary learning, some learners have perceived the synchronous game feature as a more negative experience than asynchronous play when combined with the features of competition and timed play (Hwang et al., 2013).

In authentic games people prefer varying amounts of balance between the level of strategy or skill required to play and features of chance (Schell, 2008). This chance versus skill balance will "determine the character of your game" (Schell, 2008, p. 183). Schell noted that "one very common method of balancing these is to alternate the use of chance and skill" (2008, p.184). In other words, a player uses a randomizing feature such as dice, cards, or spinners and then uses their game strategy knowledge or skill to deal with or make decisions based on what luck dealt them. The chance-skill balance is crucial, because "too much chance negates the effects of player skill and vice versa" (Schell, 2008, p. 183). Rolling dice creates a situation due to chance but then deciding what to do with that result involves skill that "can create an alternating pattern of tension and relaxation" that many people enjoy (Schell, 2008, p. 183). Overall, authentic games that are primarily based on chance foster more relaxed feelings and games with less chance offer players a sense of control (Schell, 2008, p. 183-4).

This balance of skill versus chance is from authentic game design theory, whereas our study is about learning content. Hence, we use the term strategy or game-strategy to reflect the intended meaning from game design theory. To differentiate the "math skill" elements that are crucial to consider when investigating gameplay for mathematics learning. We see game-strategy and math skill as being on the same end of the continuum of Strategy/Skill to Chance, yet each is a distinctly important construct to consider when selecting and designing games for mathematics learning.

Bayeck et al.'s (2020) systematic literature review on learning with board games identified a theme that board games are used to facilitate mathematics learning. Yet, we noticed these were all early childhood studies. Studies of board game play of upper elementary or middle school students learning mathematics were not found in their analysis. Our own searches for relevant research about learning with tabletop math games also revealed that middle school students' experiences with this common school phenomenon need to be studied. Given how middle school is a unique and crucial period of academic as well as personal change with high levels of math anxiety (Luo, et al., 2009; Scarpello, 2007), middle school students' experiences with games warrants considered study.

Cognitive Demand of Mathematical Tasks

Board game research has primarily been studied with perspectives from psychology (Gobet, Retschitzki & de Voogt 2004). Those studies that have investigated learning with games have employed broad constructs, such as affect, motivation, and other factors and that if games are used for learning, then a cognitive aspect must be considered (e.g., Garris et al., 2002; Plass et al., 2015). However, given the specificity of theorizing expected within the field of mathematics education (Cai et al., 2019; Leatham, 2019; Spangler & Williams, 2019), extant frameworks about cognition with games from other disciplines (e.g., Garris, et al., 2002) have been too broad to be useful to advance scholarly or practical understanding of game-based mathematics learning. Thus, an innovation of this study was to ensure that theoretical perspectives draw from research specific to mathematics learning. To this end we used cognitive demand of mathematical tasks (See Figure 1).

The Cognitive Demand of Mathematical Tasks framework provided clarity to the field and a shared language for characterizations of mathematical tasks in relation to how these relate to student learning (Henningsen & Stein, 1997). To our knowledge, however, this crucial framework developed to provide teachers and researchers insights for choosing and designing effective mathematical tasks for student learning has not yet been applied to instructional games. We considered games to be a specialized task type, so we applied this framework to math games. Linking cognitive demand with game-based learning offers one way to ensure that theories of mathematics learning are used to evaluate classroom games. More importantly, this framework is firmly established within mathematics education such that teachers as well as researchers are now well-acquainted with this framework. Therefore, our use of the framework to notice features of games as features of mathematical tasks should offer teachers and researchers a coherent and productive extension.

Low cognitive demand tasks are either of the type Memorization or Procedures without Connections (Smith & Stein, 1998). High cognitive demand tasks are either Procedures with Connections or Doing Mathematics (Smith & Stein, 1998). Memorization is, as it sounds, such that speedy recall of facts are characteristics of this level (Smith & Stein, 1998). Procedures with and without Connections have in common that there are procedures involved, yet the conceptual thinking differs (Smith & Stein, 1998). Procedures without connections are what is typically thought of in traditional mathematics in which procedures are used to calculate, but the user likely does not know the reasons these procedures work (Smith & Stein, 1998). Procedures with Connections in contrast is a high-level task because a student either can explain and make connections between procedures or can explain the conceptual bases for procedures (Smith & Stein, 1998). Doing Mathematics is the highest-level task in which greater persistence is needed and possibly some productive anxiety to determine generalizations, relationships, identify patterns, make conjectures or other types of activities that truly reflect engagement with the disciplinary ideas of mathematics (Smith & Stein, 1998).

Purpose of Study

The purpose of this study was to apply the same level of scrutiny to games for mathematics learning as has been given to other classroom resources such as tasks or textbooks. Gameplay in a classroom is generally perceived and promoted as an effective way to enhance student engagement and motivation (Bayeck, 2020). This perspective, however, is too general to account for the variety of games, content area differences or honor the diverse needs and perspectives of students. As Leatham (2019) emphasized, a theoretical framework in mathematics education must be sufficiently specific to the phenomenon we intend to understand. Thus, this study collected data about students' experiences playing a variety of games about the same mathematics content, integer operations.

To ultimately design better experiences for students, we interpreted those experiences using diverse perspectives to see better the complex phenomenon of gameplay for mathematics learning. That is, in addition to using engagement and motivation theory already common to gameplay research, we applied a framework specific to mathematics learning and perspectives from game-design (see Figure 1), neither of which seem to have previously been applied to classroom games for mathematics learning. With this initial study we sought to answer the questions: *RQ1: How, if at all, was student experience of gameplay (motivation and engagement) related to game features?* Two related subquestions were: *RQ1a In what, if any, ways were the integer tabletop games played in these authentic classrooms perceived as authentic games and why?* and *RQ1b: How did students perceive the value of these integer games as effective learning tasks?* Given that games are often used with the intention to motivate and increase enjoyment for the students who might be lowest achieving, we also asked *RQ2: Was student engagement and motivation by game related to achievement level?* For broader implications, we also address: *RQ3 What does the STEM Classroom Game Features Framework aide scholars and teachers to understand about classroom gameplay?*

Method

To uncover student experience of learning mathematics through gameplay, data were collected about students' perspectives on learning and experience of gameplay and supplemented by an analysis of observed engagement. Using popular ways of characterizing this mixed methods educational design, it would be QUAL+quant (Creswell & Plano Clark, 2007). Leech & Onwuegbuzie (2009), however, offered better specification of mixed methods research methodologies to describe the study in terms of mixing, timing, and emphasis. Thus, we describe this study as a Fully-Mixed Concurrent Dominant Status Design (Leech & Onwuegbuzie, 2009) in which qualitative research was dominant. Both qualitative (interview responses and documentation of gameplay via video) and quantitative data (in the form of student ratings) were collected within the same time period prior to any analysis, making this a concurrent design in which analyses of each data source were conducted and then mixed to draw inferences that synthesize across data types (Leech & Onwuegbuzie, 2009).

Settings, Contexts and Participants

The teacher, who taught all grade 7 students mathematics using student-centered instruction, was selected for a larger study that analyzed an entire unit of learning with multiple integer models. Consents and assents were returned by 32 students who submitted written tests across the two classes with 20 of these students participating in post-interviews.

Gameplay

The teacher had all students play three games: *Go-High/Go-Low* (Nurnberger-Haag & Wernet, 2019), *Integer*

Product Game (Lappan et al., 2014) and *Integers 24*™ (Suntex International, 2007). The first two games were designed for classroom use. From game design perspectives, both would be considered board games of asynchronous (i.e., sequential play). Integers 24 is a proprietary game that uses a single card on each round of gameplay on which all players play synchronously.

Go-High/Go-Low was played early in the unit while learning addition and subtraction of integers for the purpose of sparking the conceptual realization that subtraction does not always result in a smaller number (Nurnberger-Haag & Wernet, 2019, https://library.osu.edu/ojs/index.php/OJSM/article/view/6663/5345). Integers 24 was played after multiplication and division of integers was introduced to provide game-based practice of all four operations. Additional unit instruction continued until review day when all students as trios rotated through ten-minute centers to play each game: Go-High/Go-Low, Integers 24, and the Integer Product Game.

Go-High/Go-Low was designed and used originally by a classroom mathematics teacher prior to dissemination for other teachers. It is composed of a single-page printable game board (Nurnberger-Haag & Wernet, 2019), but there are essentially two related games on the board. The goal of the first game was to have the highest score (Go-High play). The goal of the second game was to have the lowest score (Go-Low play). Go-High/Go-Low involves turn-taking (asynchronous) in which a student rolls a sign die (+ or - on three sides each of a cube die) and a numeral die (6, 10 or 12-sided), then decides whether to add or subtract the rolled integer to obtain the greatest or least integers. Thus, using Schell's (2008) explanation, this game alternates chance and skill. The game sheet requires students to record their gameplay equations so that their peers can verify their calculation accuracy before determining a winner. Winners are determined by listing all player names and final scores in order for each game (Nurnberger-Haag & Wernet, 2019).

Integer Product is a competitive printable board game that is part of a commonly used middle school textbook series Connected Mathematics (Lappan et al., 2014), which was originally designed with support from a National Science Foundation grant. The game is played through turn-taking for a pair of students using a six-by-six board of integer numerals in which the goal is to use factors -6 to 6 to obtain four markers in a row with blocking allowed. Only a single game board is provided.

Integers 24 is a competitive card game with leveled cards from 1 to 3 levels of difficulty from which players can choose prior to each round of gameplay. Whitehill (2008) would define Integers 24 as a proprietary game that is available for public purchase. Each card is printed with four integers from -9 to 9. The goal of the game is to make either 24 or -24 by adding, subtracting, multiplying and/or dividing all the integers on the shared card faster than any other player. Thus, Integers 24 involves synchronous gameplay. The player then must state their calculations to the other players to gain approval that this was correct before the player takes the card. The player with the most cards wins the game.

Data Collection

As part of the larger study, instruction was video recorded. Beginning the next school day after the unit tests, those who consented and assented to be interviewed were asked questions about their experiences during the unit. Because the larger study purpose was to focus on the learning of integer operations with models, and the need to investigate student perceptions of games arose during the last class, no measures of anxiety, math beliefs or attitudes were used.

Interviews

The first author conducted all interviews of twenty students as part of the larger study, which included questions about their experience with gameplay. All three games were put on a table in front of the student to spark their memory of playing each game and enhance communication with the researcher. They were asked the following questions to elicit their perspectives on their experiences with these games beginning with telling the students that *People like different games for different reasons, so we'd really like to know your opinions about the games you played for negative numbers:*

- *Which, if any of the games would you play outside of school if you had it? Why?*
- *Which games do you recommend teachers use to help students learn negative number operations and Why?*
- *How did you feel while playing (Go-High/Go-Low, Integer Product, and Integers 24)?*

We also asked students to rate each game using a Likert scale. Students placed slips of paper with 1 through 5 ratings on each game in response to the interviewer asking them to rate their perception of fun (*How Fun Was Each Game?* 5 Really Fun, 4 Mostly Fun, 3 Ok, 2 Not very fun, 1 not fun at all) and then their perception of learning with the games (*How Well Did Each Game Help Me Learn & Review?* 5 *"*Helped me learn or practice a lot better than a worksheet", 4 *"*better than*"* 3 *"*about the same as*"* 2 *"*learned less than*" or* 1 *"*Didn't learn during the game at all*"*).

Observations of Gameplay

Gameplay was documented with a video camera stationed at each game to capture each student group as they rotated through the game centers on review day. Video recordings were then analyzed as described in the data analysis section.

Data Analysis

Student responses to interview questions were analyzed qualitatively to understand their perspectives on each game. To answer RQ2 whether students who were lowest achieving experienced gameplay differently, the post-test scores of the measure used in Nurnberger-Haag (2015) were used. This study was not designed to make claims about pre-post learning with the games, because the games were only one aspect of the entire unit and the purpose of the current study was to investigate students' experiences and voices. Quartiles of post-test performance were used to define low-performance and higher-performing students to analyze whether those with the least integer knowledge around the time of gameplay experienced gameplay differently than students with greater knowledge.

Interviews

To contribute to answering RQ1a and RQ1b, Likert ratings students provided as to whether they would play the game outside of school and whether they recommend it to teachers were documented with descriptive statistics. The five ratings were collapsed into three due to the small sample size and to provide clearer patterns: Really Fun and Mostly Fun were collapsed to "Fun," Ok remained as is, Not Very Fun and Not Fun At All collapsed to "Not Fun." Students' perception of learning were similarly collapsed from five to three ratings. To contribute to answering RQ2 as to whether there were differences among student perceptions of the lowest performing students compared to the other students, chi-square analyses were conducted on these ratings by achievement level.

To answer RQ1, RQ1a, Rq1b so that we could interpret student perspectives from their own words and obtain reasons for their ratings, three research assistants who were unaware of the study purpose (one doctoral candidate in mathematics education and two undergraduate future elementary teachers) looked for themes in transcribed interview responses. They were instructed to use open coding to look for themes and document students whose response fit the theme (Williams & Moser, 2019). To honor every student voice, a single student's response was sufficient to document a theme. During meetings an iterative process was used for axial coding (Williams & Moser, 2019) in which research assistants revisited the themes and data to consolidate, rename, or eliminate themes. At the third level of coding, selective coding (Williams & Moser, 2019), we organized these into kinds, valence and also interpreted all themes in relation to each game and features of games.

Observations of Gameplay

Six focal students were selected using purposeful stratified selection of posttest scores: two with similar scores from the lowest quartile, Median, and highest quartile. A doctoral candidate in mathematics education who had not observed instruction and was unaware of study purpose watched the videos three times to write analytic memos about each focus student. Subsequently, the three research assistants (the doctoral candidate and two undergraduate future elementary teachers) each independently rated the focus student’s engagement using the scale in Table 1 (Fully Engaged, Mostly Engaged, Somewhat Engaged, Not Engaged).

Table 1. Coding Definitions for Focus Student Engagement Level during Integer Gameplay

Engagement Level	**Definition and Clarifications**
Fully Engaged	Student seemed focused on the mathematical game at all times. Social interactions and joking or disagreements were about the mathematics of the game.
Mostly Engaged	Student seemed focused on the mathematics of the game for most of the observation. Social interactions even if not about the mathematics of the game seemed to enhance enjoyment of the gameplay experience even if temporarily appearing off-task from the mathematical game.
Somewhat Engaged	Student engaged with the mathematics of the game at least once during the observation.
Never Engaged	Student did not seem engaged with the mathematical game at any point during the observation.

Ratings for which at least two researchers agreed were used to determine the student's Engagement Level. Of the 18 potential (6 students x 3 games) Engagement Levels from all three raters, they were in complete agreement on 14 ratings. On the other four ratings, two of the three raters agreed and was used as the criteria to report Engagement Level. The dissenting rating in each case differed by just one level, which provides further evidence that researcher perceptions of engagement were trustworthy.

Results

The student data are used to provide insights about the games as the units of analysis. Thus, throughout the results we refer to students using an identification number, because our analysis does not document gender or ethnicity and assigning pseudonyms based on our perception of such identities would create problematic implications of bias. For each qualitative finding we provide 20 to 25% of student voices to communicate trustworthiness of these data and offer sufficient context for reported themes. In this section we report results from each analysis separately. Given our mixed methods study design is such that mixing occurs at the interpretation stage, we mix the results in the Discussion section to interpret these in relation to the research questions.

For each game, we asked interviewed students if they would play the game outside of school and if they would recommend that teachers use that game for in school learning. Table 2 displays these descriptive statistics. Students could choose all or none of these games in response to these interview questions. About three-fourths of students stated that Go-High/Go-Low was a game they would play outside of school and had similar responses about recommending teachers to use (see Table 2). In contrast, fewer than half the students would play Integer Product or Integers 24 outside of school (see Table 2). Students rated these two games better for classroom gameplay, than authentic gameplay, because as shown in Table 2 about half the students recommended these to teachers.

Table 2. Interviewed Students' (*n*=20) Recommendation of Each Game for Authentic Gameplay or Classroom Gameplay

Game	Would Play Outside of School ***n (%)***	Recommend Teachers Use ***n (%)***
Go-High/Go-Low	14 (70.0)	13 (76.5)
Integer Product	6 (35.0)	10 (55.6)
Integers 24	9 (45.0)	10 (58.8)

Note. Missing data due to interviewer omitting the question or uninterpretable response on Recommend Teachers Use: Go-High/Go-Low (*n*=17), Integer Product (*n*=18), Integers 24 (*n*=17).

Table 3 displays the percent of interviewed students who rated how fun each game was and how well each game helped them learn relative to a worksheet. Note that although the distribution for all three games were skewed positive with the majority of participants rating the games "Fun" or "Learn Better" more frequently than for traditional practice review of a worksheet, the distribution differed for Integers 24. The majority of students rated each of the games designed for classroom use (Go-High Go-Low and Product Game) as Fun. All students rated Go-High Go-Low as Fun or OK, whereas the commercially designed Integers 24 game had more students rate it as Not Fun or OK. With regards to perception of learning, a majority of students (at least 70%) rated both Go-High Go-Low and Integers 24 as supporting better learning than completing problems on a worksheet, whereas the efficacy of learning with the Product Game might be in question, in spite of it being designed as part of a textbook to facilitate learning.

Table 3. Students' Judgements of Fun and Learning Overall and By Post-Test Achievement Level

	Fun			**Learning Compared to Worksheet**		
	Fun ***n (%)***	**Ok** ***n (%)***	**Not Fun** ***n (%)***	**Better** ***n (%)***	**Same** ***n (%)***	**Worse** ***n (%)***
				Go-High Go-Low		
Higher Achieving	13 (86.7)	2(13.3)	--	10(66.7)	3(20)	2(13.3)
Lowest Achieving	5(100)	--	--	4(80)	1(20)	--
Totals (*n*=20)	18(90)	2(10)	--	14(70)	4(20)	2(10)
				Integer Product		
Higher Achieving	10(71.4)	3(13.3)	1(7.1)	4(28.6)	7(50)	3(21.4)
Lowest Achieving	4(80)	1(20)	--	1(20)	3(60)	1(20)
Totals (*n*=19)	14(73.7)	4(21.1)	1(5.3)	5(21.1)	10(52.5)	4(26.3)
				Integers 24		
Higher Achieving	8 (53.3)	5(33.3)	2(13.3)	11(73.3)	2(13.3)	2(13.3)
Lowest Achieving	1(20)	1(20)	3(60)	4(80)	--	1(20)
Totals (*n*=20)	9(45)	6(30)	5(25)	15(75)	2(10)	3(15)

Out of 34 total themes, 21 positive, 8 negative, 1 neutral emotion, and 5 game features were found (note that "competitive" was identified as a positive theme as used by students as well as a game feature). Almost three times as many positive themes compared to negative themes were found across the gameplay analysis, which indicates that overall students perceived the games provided a positive classroom learning experience.

Figure 2 displays all themes found in relation to which games were found to have each theme. The themes that represent features of games are shown in bold capital letters (i.e., **COMPETITIVE**, **SPEED**, **MATH SKILL**,

CHANCE, **STRATEGY**). Themes common to all games are positioned within the center of the triangle with 9 positive and two negative themes documented in common. Themes unique to each game are positioned within the loop for that game. Valence of themes in Figure 2 and subsequent figures are notated with positive (+), negative (-), and absence of a symbol indicating neutral valence (i.e., indifferent).

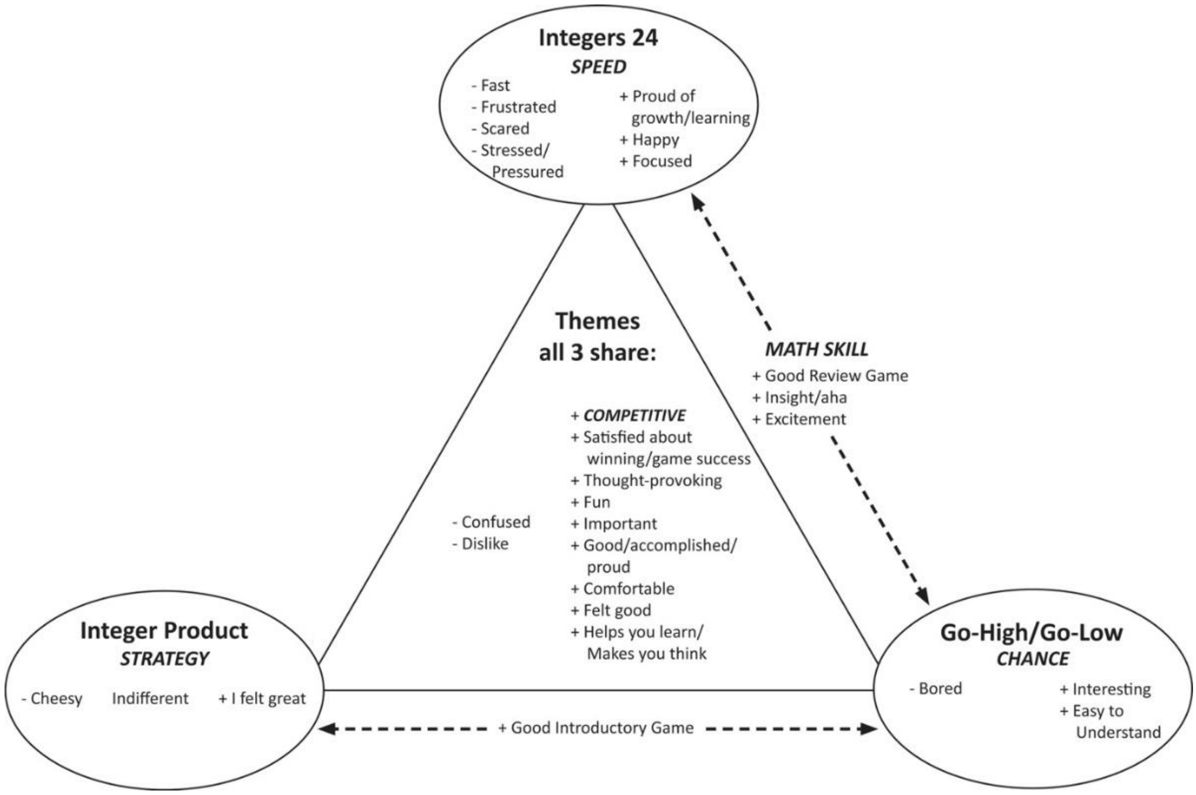

Figure 2. Themes Displayed to Show Which Valence of Themes about Gameplay Experience (+ or -) and Themes That Indicated Game Features (Bolded Capital Letters) Were Common or Unique to Particular Games

The distribution of the valence of themes by game were that Integer Product had 13 positive themes with Go-High/Go-Low and Integers 24 having the most and same number of positive themes (17). However, Integers 24 had twice as many negative themes than Go-High/Go-Low. The within game ratios of positive: negative themes were as follows Go-High/Go-Low (17:3), Integer Product (13:3) and Integers 24 (17:6). To ensure trustworthiness we sought to provide sufficient evidence from student voices throughout the findings. Once we determined which quotations to report for these themes, we assessed the number of different student voices represented by these quotations. We believe the 13 student voices shared through these quotations, which is 65% of interviewees, provides a trustworthy representation of students' perspectives.

Student Experiences in Relation to Game Features

The themes of student experiences that research assistants identified (without being aware of the conceptual framework) that were game design features were: Competitive, Math Skill, Strategy of game, Chance, and Speed (see Figure 2 capitalized words). The following responses illustrate the contexts in which some of the students expressed these themes. In student quotations we bolded the words and phrases indicative of each game feature.

Competitiveness is a Game Feature That Motivates Students

Each of the games were referred to as competitive by some students as a positive game feature. We use two student quotes to provide context for this theme. Although student 4001 explained that none of the games were sufficiently fun to be authentic gameplay (*Yah, um, I probably wouldn't play any of these outside of school),* the student emphatically recommended to teachers for classroom learning one game they perceived as competitive, fun, and provoked learning: *Definitely Go-High/Go-Low because you can like* ***learn while you're still having fun*** *and it's* ***competitive*** *too.* Another student explained why playing two of the games were competitive, which made the games enjoyable and motivating to play:

> *Because I* ***love competitive*** *games and I felt like* ***both of these were really competitive*** [Go-High/Go-Low and Integers 24]*. And I felt like you could win them and it got like* ***really intense****, like* ***really competitive*** *so I feel like* ***that was fun****.*

But in response to Integer Product said: *I felt like there was better games*. (4020)

These students (4001 and 4020) above explicitly used the words "really competitive" and "really intense" as reasons the game was fun.

Students Want Game Feature of Chance in Balance with Game Strategy and Math Skill

The theme of balancing features of chance, game strategy and math skill was found in student reasons as to why they did or did not like each game. To portray the nuance of perspectives from various students we report and interpret four student responses (20% of interviewees).

Student 4010 offered further evidence about competitiveness by implying the value of the competitive theme in an "actual game" and identified specific features of each game that made it feel like an authentic game: strategy or skill of playing the game, chance, and math skill. Note how this student felt like Integer Product felt most game-like due to the strategy of gameplay that they had to block an opponent. Again, we bolded words indicative of the themes for emphasis.

> *Ah because this one* [Integer Product] *is kind of just like an* ***actual game*** *like it's kind of fun trying to like* ***block each other*** *and for this one it's like for this one* [Integers 24] *it's kind of* ***skill-based*** *and this one* [Go-High/Go-Low] *also has a* ***dose of luck*** *in it. So this one it's kind of like* ***you have to be lucky*** *and this one it's kind of just* ***fun blocking*** *each other. (4010)*

This student (4010) appreciated the game-strategy feature of Integer Product, which they emphasized by stating it twice, explaining that Integers 24 entirely requires Math Skill (skill-based), but Go-High/Go-Low was the only game that used chance as a design feature with the phrase "you have to be lucky." Another student (4016) also explained that chance was the reason that the only game they might play outside of school was Go-High/Go-Low: *because it's* ***mostly luck*** *and uh, you know, most people don't want to do math outside of school.*

When explaining why they recommend Go-High/Go-Low, student 4000 did not explicitly compare it to another specific game, yet the student implied using dice in other games was recognized and appreciated as a reason this game felt like authentic gameplay.

> *And I like these two because this one [Go-High/Go-Low]* ***like I really like dice and so like rolling them was fun*** *like getting your number and that was fun and um I like this one [Integer Product] because um kind of* ***like Bingo****.... (4000)*

Notice how student 4000 compared Integer Product to the specific game of Bingo as justification for their enjoyment. The following student (4012) fondly reported that Integers 24 was similar to another math-skill and speed-based game.

> *It's really fun. You were doing math and learning and you were learning* ***doing mental math*** *you were like um and you were having fun all at the same time and you were* ***racing*** *your friends and stuff and It's kind of* ***like Krypto at Math-o-Rama*** *so it was* ***really fun****. It just* ***reminded me so much of Krypto****. (4012)*

Thus, this student explained that Integers 24 reminded them of a highly competitive card game (Krypto) they had played as part of math competitions in which their elementary school team competed against teams from other schools. For those readers who might be unfamiliar with Krypto, each player is dealt six cards with the whole numbers 1 to 25 from a deck of 56 cards. An additional target card is turned over so that players race to add, subtract, multiply, and/or divide all numbers in their hand to make the target number. Thus, the game characteristics similar across the two games of Krypto and Integers 24 are they both use four operations, require calculation of several numbers (Krypto is 6, but 4 for Integers 24) and synchronous play based on speed is a game feature. Whereas Krypto provides each player with a feature of chance (each player is dealt different cards) to balance skill, Integers 24 does not.

Students Experience Speed Due to Synchronous or Asynchronous Feature Differently

The game feature that most impacted students' feelings and perceptions was the feature of speed related to whether the game was synchronous or asynchronous. To fully represent the range of student feelings about these features we report four student responses identified with this theme (20% of interviewees) whose feelings ranged from fear to happiness. The commercial-game Integers 24 was designed based on speed with synchronous gameplay.

The two games designed for classroom use, Integer Product and Go-High/Go-Low, featured turn-taking (i.e., asynchronous play) that allowed for more player control over the speed required for successful game play. In the prior quotation from student 4012 they said, "[Integers 24] was really fun" because they were "racing." Yet, student 4016 below summarizes how some students felt about the relative speed of each game due to synchronous or asynchronous features. Note the student described the synchronous game as "*pressure*" that meant they "*don't really like this game*."

> *The 24 game --this one was a little* ***more pressure****, it was a little harder and uh, yah, I, I don't really like this game that much.* (4016)

In contrast, in the quotation below this same student (4016) referred to the asynchronous games as "calm" and "laid back" or indicated that they had time to "see what the other person is doing to figure out what you're gonna do."

> *Go-High/Go-Low:* [Would play outside of school] *probably because it's mostly luck and uh, you know, most people don't want to do math outside of school. Um, well, on this game, I felt a little* ***more calm*** *… so this game was a little more* ***laid back****.* (4016)
>
> *Integer Product: Uh, this was good because I love* ***strategy*** *and um, I get to* ***see what the other person is doing to figure out what you're gonna do*** *and you have to think a little bit more.* (4016)

As noted in Figure 2, some students felt *happy* and other extremely positive emotions due to the time-pressure of synchronous play because of the speed feature of Integers 24. However, as one student stated when advising which games teachers should offer in their classes: *All those are good I just think [Integers 24] is kind of* ***stressful****, but a lot of my classmates liked it* (4015). Other students expressed more extreme negative emotions they experienced while playing Integers 24. We bolded the emotions and stimuli that caused these emotions in the next student's response.

> ***Stressed****, cause I was* ***with like a group of super smart kids*** *and then once they got it they would slam their hand on it like that so it was kind of* ***scary*** *and I was kind of* ***paranoid the whole time*** *because I don't like the* ***loud noises****, I'm a pacifist (*Interviewer: *So it wasn't just about the speed and them beating you but it was also about the idea of the slapping and the loud noises bothered you too?) Yeah, cause like I get stressed out by small things and I have* ***really bad anxiety*** *and so seeing that made me kind of* ***scared****. (4003)*

A reader might wonder if this student (4003) might find all competitive games stressful due to their generalized anxiety. This student, however, likes competition, which motivates their gameplay. It was the synchronous form of competition that Integers 24 required that was demotivating for this student. The other two competitive games afforded asynchronous play due to turn-taking features. The Integer Product game supported this student without fear as is evident from their response about how the game supported competition: *It was kind of easy and it was kind of* ***fulfilling when you took somebody's spot****.* In spite of the student's generalized anxiety, the features of Go-High/Go-Low made this student feel comfortable:

> *I felt* ***comfortable*** *with it because I knew what to do, like at the beginning of the unit even because I kind of understood the beginning part and so I was able to do that well and then at the end of the unit I was just kind of used to it…*

Thus, each competitive game of Integer Product and Go-High/Go-Low provided a comfortable learning environment for this student in spite of their generalized anxiety. This student (4003) explained that Integer Product felt "fulfilling," because of the competitive game-based strategy of blocking "somebody's spot." The game Go-High/Go-Low was comfortable in terms of the sense of a mathematical success as they reported they were "able to do that well" as an introductory game, with the student's language of "even," this emphasized their interpretation that this level of comfort at the beginning of the unit was surprising to them. Because the student expressed comfort as how they felt during initial gameplay, the interviewer asked if it was also comfortable at the end of the unit, which the student confirmed.

Student Perceptions of Games in Terms of Value for Learning

Some reasons students said they would or would not recommend a game indicate how game features related to whether they perceived learning purposes for playing these games were achieved. To provide the reader a comprehensive sense of student perspectives of the value of these games for learning, we include five student responses (25% of interviewees) in this section.

> *Um, I would say not the 24 game because uh that requires how* ***fast*** *you can do it and some people can do it but they just do it slowly. So it* ***didn't really help most people****.(4016)*
>
> *First time I played [Integers 24] I felt like normal, like ok, and then the second time I played it was just like* ***I can't figure this out quick enough*** *(*Interviewer: *Oh really, at the end you still felt that way?) Yeah (*Interviewer: *Was that more about who you were playing with?) Yeah, cause the people I play with are* ***super quick*** *and then they like I don't know,* ***I almost figured it out****. (4023)*

The next student summarized advice to teachers in relation to the learning purpose. This student described Integers 24 as being useful for those students who have already learned the unit objectives, whereas the other two games facilitate learning.

> *[Integer Product] would be like the second option this [Integers 24] would be the third option because this is* ***like people that already know it*** *so like this [Go-High/Go-Low] is like* ***where you actually teach things to do it****. [*Interviewer*: Did you think the* ***product game was also teaching them****?] Yeah. (4022)*

Another student (4005) had similar opinions as student 4022 while offering a little more rationale including another example of the Integer Product game being compared to an authentic game.

> *[Integer Product] Because it's probably* ***my favorite*** *one to me* ***this is like 4 in Row****.* ***That's technically what the game is. It's 4 in a Row****. But you sort of* ***have to work for it instead of just putting them in****. It's* ***math-related*** *formulas. Like harder/more fun for me. This one too.(Go-High/Go-Low) That one was confusing for some people, not confusing for others, and* ***once they got how to do it*** *[because of the mathematical idea to be learned],* ***they learned a lot****.* (4005)

The next student (4011) explained how a game made them aware of and proud of their learning during the unit, because playing the game a second time on review day provided a context for the student to realize this compared to when they played for the purpose of initial learning.

> *This game I was* ***pretty happy with myself****, the 24 game, because the first time I tried it I couldn't get any of them, like I could not figure it out and then I got 2 of them right (*Interviewer: *On the review day?) Yes on review day, and I thought that was impressive because* ***I saw definitely a growth in what I learned*** *because I could figure that out,* ***I knew I learned something****.* (4011)

Thus, in terms of value for learning, we were able to identify in these qualitative responses that some student valued each game.

Student Perceptions in Terms of Cognitive Demand of Games

When we apply the Cognitive Demand of Mathematical Tasks (Stein & Smith, 1998) to see the games through this theoretical lens, we consider the task level at initial play and subsequent play on review day. We consider the cognitive demand for game play at setup and then as the students experienced it, consistent with Henningsen & Stein's (1997) use for math tasks. Go-High/Go-Low and Integers 24 were set up and experienced by some students as High Cognitive Demand, whereas Integer Product was set up and experienced by students as Low Cognitive Demand.

Go-High/Go-Low is a High Cognitive Demand Game

The first time that students encounter Go-High/Go-Low, the game is intended as a Doing Mathematics level task, because the primary mathematical purpose of the game as designed was to create a context in which students would discover the surprising relationships that subtraction does not always reduce a quantity and addition does not always increase a quantity (Nurnberger-Haag & Wernet, 2019). This focus on discovering and generalizing relationships are features of Doing Mathematics level tasks (Smith & Stein, 1998). Thus, if a student already learned to accurately generalize about subtraction and addition of integers, then in subsequent play they can use these math-skills as game strategies to better win the game while developing fluency (Nurnberger-Haag & Wernet, 2019) at the level of Procedures with Connections.

We also interpret student perspectives through the lens of Cognitive Demand. Student 4020 first talks about enjoying Go-High/Go-Low more due to winning during review day, but having learned more when playing the first time. Thus, engagement with the game as well as motivation to learn are present in this student' response. With regard to learning, notice the theme of insight from Figure 2 and the Doing Math level thinking the student articulated:

> *So I liked it better* [on review day] *so I lost the first time then I won the second time* [review day] *so like I felt like it was easier to do it the second time but I felt like it* ***made me think more the first time*** *so I think I like both of them* [both instances of play] *because I felt like I like how we played it twice so I could like go back and be like "****Wow, I did that I was really like not like I should not have played like adding I should have subtracted that time so I felt like it helped me knowing the second time*** [review day] ***but I also felt like the first time*** [as introductory game] ***it really made me think and really get my learning together****.* (4020)

Revisit this student's explanation (4020) and the emphasis the student made about the value of the game for their conceptual learning during both instances: as an introductory and as a review game. The student said *"how we played it twice so I could like go back and be like Wow,…I should not have played like adding I should have subtracted that time…*" This indicates that the review day play was the context in which they could realize and reflect not only on how to better play at that time, but it also sparked the student to reflect on how their introductory gameplay would have gone differently had they realized the mathematical generalizations during initial play.

The student (4012) who expressed the negative theme of *bored* for Go-High/Go-Low (see Figure 3) was explicit that this negative theme only applied to their experience on review day "*because like hey it's just a little bit of math."* When asked how they felt during their first experience as an introductory game, however, they responded that they felt: "*like hey, this is really fun.*" This provides further evidence that the same game could be experienced by the same students first as a Doing Mathematics level game until the insight and surprise is no longer part of the experience, then changes the perception of their experience to a Procedures with Connections level of demand. Notice, how the next student (4011) reported the intended conjecturing during gameplay that reflects Doing Mathematics level thinking as they explained why they would recommend teachers use this game:

> *If you're **looking to learn** not just for fun I'd **definitely** say the **dice game** (Go-High/Go-Low) because it's fun but **you're actually trying to think if I do subtraction will this work**…**will I get a bigger number.** That one (Go-High/Go-Low) is **definitely interesting** to me because you could land a positive number but it could take you a bit, but you could land a smaller or bigger number with it.* (4011).

Note also that the student 4011 also perceived this high cognitive demand game as "definitely interesting," for the reasons consistent with Doing Mathematics level tasks and stated purpose of the game by the designers to "spark surprise" (Nurnberger-Haag & Wernet, 2019; Smith & Stein, 1998). The following student (4008) reiterates the perspective that the first gameplay of Go-High/Go-Low afforded Doing Mathematics level thinking, explaining that it was "*really helpful with equations and trying to make the largest and smallest,..The first time it really really helped, it just **woah, ok I understand***", whereas for subsequent gameplay the student said, *"I liked it a lot again then too, it wasn't that much of an aha moment."* (4008). These comments indicate the student engaged meaningfully and continued to find the game motivating for additional fluency practice at the Procedures with Connections level.

Integer Product is a Low Cognitive Demand Game

The Integer Product game provided calculation practice of multiplication at the Procedures without Connections level. The stated purpose of this curriculum-based game is to promote fluency of integer multiplication (Lappan et al., 2014). For every potential move, a single multiplication calculation within -36 to 36 needs to be done. This is Procedures without Connections, because even though game-strategies are needed to win, the only math skills needed to accurately place a token is to accurately calculate products of integers.

Some students expressed the *game strategy* feature of the game Integer Product encouraged them to keep persisting with the mathematics. The theme *proud of learning* when accomplishing the mathematics that required the game strategy was also evident:

> *It was sometimes challenging because like definitely putting it where you have to like if you had one on one spot then you have to put it to try to win that was definitely challenging, **to figure out what number you have to do to put it on*** [meaning in order to strategically make the four in a row] *(*Interviewer*: And how did you feel while you were challenged like that?) I felt fine, I definitely **tried to just figure it out**, I did write out a few and I thought that helped and **I felt great that I was able to figure them out**.* (4011).

Although this might suggest evidence of challenging mathematics, the challenge to which the student referred was in relation to the game strategy. In comparison to the ways students talked about Integers 24 and Go-High/Go-Low, the lack of complexity the students' perceived is consistent with our interpretation of the game as a Procedures without Connection level game.

> *Um I felt like it was not quite as fun as 24 but it was more fun than the Go-High/Go-Low game because um you were… **multiplying, applying skills** to the game*. (4012)
>
> *So this one I probably wouldn't* [recommend to teachers] *because **it's just multiplying so you don't really get the full experience***. (4020)

Note both students 4012 and 4020 above referred to the game only being about multiplying even if because of the particular game context student 4012 found it more fun than Go-High/Go-Low for example, but these accolades were not about the mathematical demand of the game. Moreover, procedural skill was emphasized in both student justifications, which is consistent with Procedures without Connections.

Integers 24 is a High Cognitive Demand Game

For Integers 24, Procedures with Connections was the intended level of cognitive demand. Some cards have multiple solutions, which is one characteristic of high cognitive demand tasks. Consider that the intention is that the player would look at all four integers on the card, consider all four operations and the relationships of how each operation would interact with certain kinds of numbers and in relation to the target goal of either 24 or -24 to then determine a sequence of calculations to achieve that goal. The player must remember and justify their sequence of procedures to convince the other players. Thus, at setup the intention of play would be that students must use a well-connected understanding of procedures on integers.

Student voice interpreted in light of the Cognitive Demand framework also corroborates our claims that Integers 24 is a high cognitive demand game. Rather than insight or conceptual ideas, however, student 4008 who previously emphasized the conceptual value of Go-High/Go-Low discussed the procedural operation complexity of Integers 24. This supports the claim that Integers 24 requires Procedures with Connections level thinking. Meaningful engagement with the mathematics of the game is also evident for this high cognitive demand game due to the word "focus."

> Integers 24: *It's **really hard** because you **have to really think** because **there's so many different**, it's really hard you have to subtract and you have to add and you have to divide and all of that**, you can't just do one**, so it really helps with **combining them all in an equation***. (Interviewer: *And how did you feel while playing it?) Umm very, not confused **but very focused and concentrated** because you really have to focus to pick up the, yeah.* (Interviewer: *And that's how you felt every time you played?*) *Yes. (4008)*

Note that the challenge or cognitive demand of the mathematics helped the above student (4008) stay focused and concentrate.

In summary then, our own analysis of the games in terms of cognitive demand and through interpreting student voices with this framework in mind, two of the games were high cognitive demand (Go-High/Go-Low and Integers 24) and one game was low cognitive demand (Integer Product).

Student Experience by Achievement Level

To investigate RQ2 whether students with the lowest achievement perceived or engaged with the games differently, we first report quantitative analyses of student ratings by achievement level and then a qualitative analysis of observed gameplay by achievement level.

Student Ratings of Games Did Not Differ by Achievement Level

To determine if those students who struggled the most with integer arithmetic around the time of gameplay (at the end of the unit) felt differently about each game compared to their higher achieving peers, Chi-square Tests of Association were used. Using the Linear-by-Linear Association Test (appropriate for a 2 X 3 Chi-square test), the relationship between level of achievement (Lowest Achieving or Higher Achieving) and student perception of fun (Fun, Neutral, or Not fun) for the Integers 24 game was not statistically significant but approached significance ($\chi^2[1]=3.455$, $p=.063$).

The association between achievement and perception of fun for the Go-High/Go-Low game was not significant ($p=1.000$). The association between level of posttest performance and perception of fun for the Product game was also not significant ($p=1.000$). Tests of association between students who were lowest achieving compared to those above were not significantly correlated with student perception of learning for either of the three games. For each analysis, a limitation may be that there may have been insufficient sample size to detect a potential effect due to multiple cells in each analysis having fewer than 5 respondents.

Student Engagement during Gameplay is Not Clearly Related to Achievement Level

Qualitative analyses of the themes previously shown demonstrated the same integer game was experienced differently by different students. To investigate if there were patterns of student preferences based on achievement level, a stratified sample of two students from each achievement level were selected to analyze their engagement during gameplay: Lowest achieving (within Q1), Typically achieving (Median) and Highest Achieving (above Q3). Notice in Table 4 that regardless of achievement level, all observed students were at least Somewhat Engaged in each game.

The Go-High-Go-Low game was the game that observers determined to be most engaging, followed by Integer Product and then Integers 24 (see Table 4). Observers' assessments documented five of the six focus students while playing Go-High/Go-Low as Fully Engaged and one as Moderately Engaged. In contrast, Integers 24 observations showed that students were either Fully Engaged or Somewhat Engaged, with no students Moderately Engaged (see Table 4). Student engagement during game play of Integer Product was distributed across each engagement level from Fully to Somewhat Engaged (see Table 4).

Looking at the row for Fully Engaged in Table 4, it is clear that for every game, students from every achievement level were observed being fully engaged. Most importantly, both high and low-achieving students were observed as Somewhat Engaged. Go-High/Go-Low supported students of all achievement levels to engage with the mathematics of the game, with most students being Fully Engaged. Each of the other two games promoted high levels of engagement for only half of the observed students.

A pattern was not evident by particular achievement levels. For example, one student 4003 who was low-achieving was observed as Fully Engaged in Go-High/Go-Low and Integers 24, but only Somewhat Engaged in Integer Product, whereas another student with the same achievement level was observed as Mostly Engaged with Go-High/Go-Low, Fully Engaged in Integer Product, and only Somewhat Engaged with Integers 24. Similarly, students who were high-achieving or at the median level also had differing patterns of engagement.

Table 4. Engagement Level for Each Focus Student by Game and Achievement Level

Engagement Level	**Go-High/Go-Low Game**	**Integer Product Game**	**Integers 24 Game**
Fully Engaged	High-Achieving $_{4008}$ High-Achieving $_{4002}$ Median $_{4000}$ Median $_{4012}$ Low-Achieving $_{4003}$	High-Achieving $_{4008}$ Median $_{4012}$ Low-Achieving $_{4015}$	High-Achieving $_{4008}$ Median $_{4012}$ Low-Achieving $_{4003}$
Mostly Engaged	Low-Achieving $_{4015}$	Median $_{4000}$	
Somewhat Engaged		High-Achieving $_{4002}$ Low-Achieving $_{4003}$	High-Achieving $_{4002}$ Median $_{4000}$ Low-Achieving $_{4015}$
Never Engaged			

Discussion and Conclusions

In the Findings section we reported each analysis separately, so next we synthesize and mix the analyses of this mixed methods study to interpret in relation to our research questions (Leech & Onwuegbuzie, 2009). Figure 3 synthesizes the data within and across games in relation to the Mathematics Classroom Games Features Framework we introduced with Figure 1 (Cognitive Demand of Math Tasks, Content Learning Purpose, and Features of Game Design).

In terms of qualitative findings, only the themes and observed engagement patterns can be succinctly incorporated into such a summary figure, so if the reader wants additional context, we encourage revisiting quotations in the

Findings section. Each integer game was motivating to many students and all observed students engaged at least to some degree with the mathematics while playing each integer game (see Figure 3). These results on majority ratings and experiences are consistent with generalized claims about the value of game-based learning for mathematics (Ernest, 1988).

Interpretations of Student Experiences with the Specific Integer Games

Unlike most studies that rely on majority opinions, our observations during data collection led us to investigate individual student's experiences with these integer games. Although our results are consistent with Plass and colleagues who stated, "different game features elicit different types of engagement in different contents and for different learners" (2015, p.260), our study goes further to specify features by drawing on perspectives of game design (e.g., Schell, 2008) and cognitive demand of mathematics tasks (Henningsen & Stein, 1997; Smith & Stein, 1998). We first address the broad research question RQ1: How was student experience (motivation to learn and engagement) with gameplay related to game features? Based on analyses of student ratings and interview responses in consideration of game design theory (Schell, 2008), we concluded that student preferences need to be understood in relation to whether integer gameplay was synchronous or asynchronous and other important features of authentic games such as strategy and chance. Moreover, when games are used for learning mathematics the cognitive demand levels of games as mathematical tasks should be analyzed as a feature of these integer games. In this study, attention to the cognitive demand game features appeared to reveal a student preference for learning (not strictly enjoyment) with high cognitive demand games. Regarding student engagement (see Figure 3), both high and low cognitive demand integer games were found to be ranked highest in terms of student ratings of fun as well as in researcher ratings of observed engagement. Both games that were designed for classroom learning, were asynchronous, and balanced mathematics skill against features of strategy and/or chance, which yielded primarily positive themes with just three negative themes (noteworthy that only one mildly negative theme was unique to each asynchronous game).

Integer Games for Learning Should Feel Like Authentic Games

During game-based learning students should feel like they are experiencing an authentic game, not "sugar-coating for learning content" (Habgood & Ainsworth, 2011, p.469). RQ1a *In what, if any, ways were the integer tabletop games played in these classrooms perceived as authentic games and why?* To assess how authentically game-like the experiences were from the students' perspectives, we asked if students would play the integer game outside of school. As summarized in Figure 3B/C, less than half the students perceived the game features of Integers 24 and about one-third of students experienced Integer Product as sufficiently motivating to play outside of school strictly for enjoyment. This provides insight as to what degree students perceived these learning games as authentic games that motivate students to play within school. Go-High/Go-Low was a game that felt sufficiently authentic to possibly encourage out of school engagement with mathematics (see Figure 3A), because as we quoted in the results "*it's mostly luck and uh, you know, most people don't want to do math outside of school".* Perhaps this could inspire future research on family-based play as homework for upper elementary and middle school, whereas most such studies have been with young children (e.g., Sonnenschein et al., 2016).

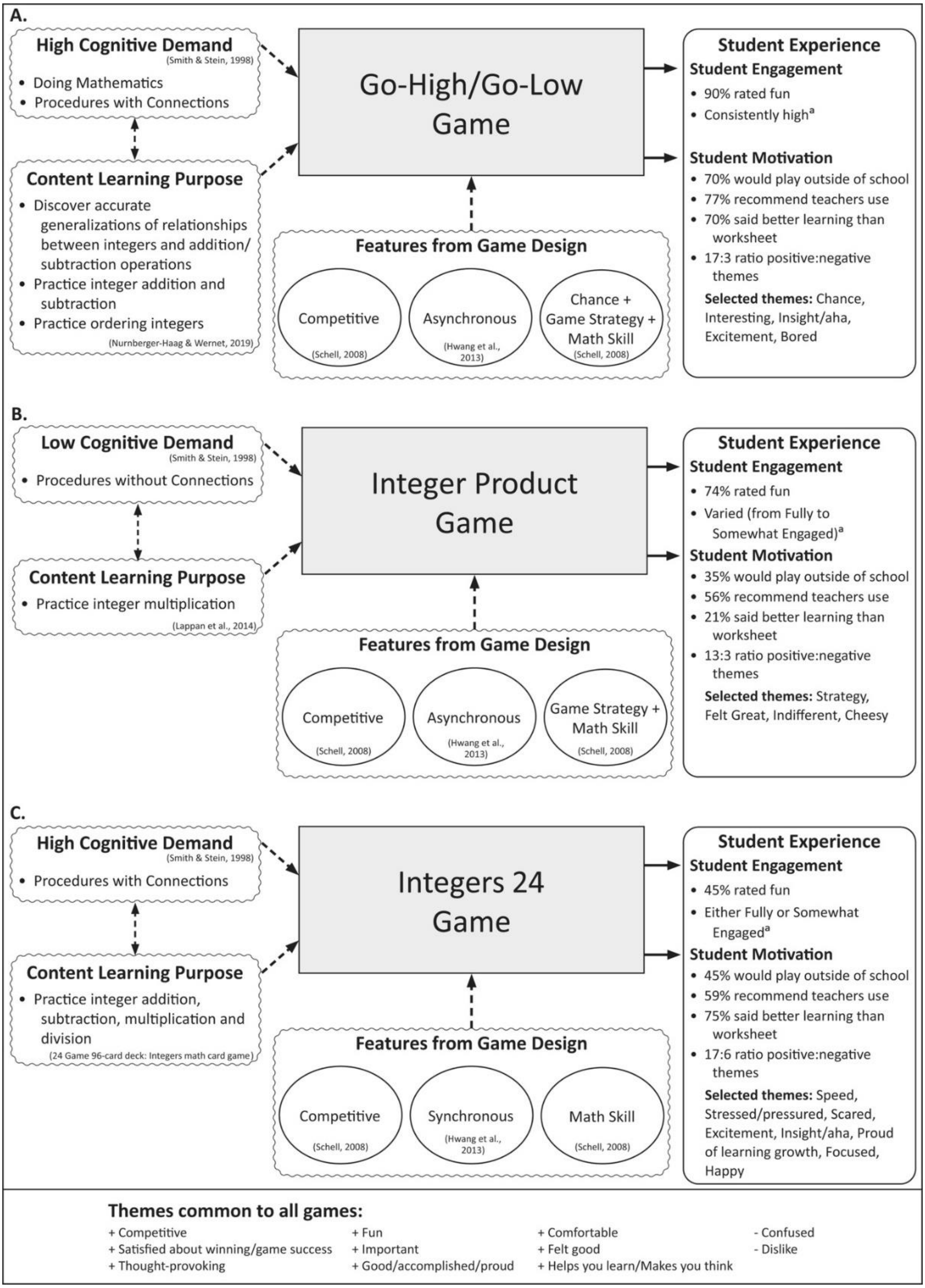

Figure 3. Summary of Reported Results Mixed and Overlaid on the Conceptual Framework to aid Inferences within and across Integer Games in relation to Game Features from Multiple Theoretical Perspectives. Note: [a]See Table 4 for the visual of this qualitative data.

Each integer game was compared to another game by some student, which according to game design theory (e.g., Schell, 2008) is a principle of effective game design. Integers 24, however, was compared to a mathematics competition game played by students who are high achieving. Rather than a school-based activity for the highest achievers, students compared the other two games of Integer Product and Go-High/Go-Low to more universal and authentic games played outside of school. Students either explicitly reported a structure similar to an authentic home game (i.e., Integer Product as 4-in-a-row or Bingo) or implicitly based on features (i.e., Go-High/Go-Low due to rolling dice as in other authentic games). For the two games in this study that failed to include luck as a feature to balance with students' mathematics skill (i.e., Integer Product and Integers 24), fewer than half of the students would engage or be motivated to play outside of school (see Figure 3B/C). The only high cognitive demand game that combined chance and game strategy along with math skill at a pace that allowed contemplation through turn-taking was more motivating for the majority of students to play outside of school (70% Go-High/Go-Low) and almost all students rated this game as fun (90%; see Figure 3A). This combination of game features was the only one that consistently met students' diverse achievement levels (see Table 4) and provided positive experiences as revealed by the student voices in the Findings (see Figure 3 and revisit interview responses quoted in the Findings).

Students Value Cognitively Demanding Games for Learning Integer Arithmetic

In terms of learning benefits, middle school students are savvy enough to recognize that the purpose of a game in school is to learn, which makes the activity more meaningful to them (Ares & Gorrell, 2002). RQ1b *How did students perceive the value of these integer games as effective learning tasks?* Although similar numbers of students recommended Integer Product (56%) and Integers 24 (59%) for classroom play, when interpreted in consideration of other data about learning, students perceived the value of these games differently (see Figure 3B/C). We take these data together with the student quotations in the cognitive demand section of the results to posit that this was due to the low cognitive demand of the Integer Product game-based task. The data suggested to us that students recognized that the higher cognitive demand games of Go-High/Go-Low and Integers 24 were better for learning and also better than the Integer Product game. Possibly due to the complexity of needing to bring together all operations ("*It's really hard because you have to really think because there's so many different,...you have to subtract and you have to add and you have to divide and all of that, you can't just do one* (4008*),"* more students said the high cognitive demand game Integers 24 provided better learning benefits than a worksheet (75%), whereas learning benefits of the low cognitive demand game Integer Product that was *"just multiplying so you don't really get the full experience (4020),*" just 21% claimed better than a worksheet (see Figure 3).

The purpose of this study was to honor each student's experience as they played each game for just a few minutes within a single class session. Thus, a limitation of this study that provides future research opportunities is to use these findings to design studies that could isolate integer learning gains due to games. To design pre-post-delayed post studies of integer learning, scholars could use the three forms of the Integer Test of Primary Operations, which was determined to be a valid and reliable measure of middle school students' integer addition, subtraction, multiplication and division knowledge (Nurnberger-Haag, Kratky, & Karpinski, 2022). A limitation of the study was that it was not meant to isolate learning due to these games from the other learning opportunities the classroom teacher provided during the unit. Future research could combine data collection of student experiences as this study did along with assessments of integer learning timed to isolate learning with games. The three forms of the Integer Test of Primary Operations (Nurnberger-Haag, et al., 2022) could be used for such future studies by administering for each student a different form as a pre-unit test, pre-game test, and post-game test.

Summary Recommendations about the Specific Integer Games Investigated

In terms of specific integer games, the results lead us to suggest that both integer games designed for classroom use that are available open source to teachers (Go-High/Go-Low and Integer Product) could continue to be used. Based on these analyses through lenses of game design theory, a modification that could improve student ratings of the Integer Product game in future studies might be to make it more like the classic game Bingo or four-in-a-row to which students compared it and incorporate the element of chance common to Bingo gameplay. This modification might be accomplished by using multiple randomized game cards rather than a single pre-determined board. Go-High/Go-Low was consistently perceived well for learning as well as entertainment so the only modifications might be a previously published recommendation that pairs rather than individuals compete against each other (Nurnberger-Haag & Wernet, 2019). For many students the proprietary game Integers 24 was engaging, motivating and provided a context for continued learning; however, because it marginalized other students, if Integers 24 is used in a classroom an alternative option must be offered. Such an alternative game should provide the same learning opportunities for all four operations in a high cognitive demand game but ensure turn-taking (sequential play) and includes elements of chance on each turn to foster equitable play among players. Integers 24 is just one of many of the proprietary 24 games that all have the same game features except for the content to be learned, so a broader implication would be to follow the same recommendations for any version of 24 used in a classroom.

General Game Design and Selection Recommendations for Mathematics Classroom Learning

With this study we intended to improve student experiences with mathematical gameplay. We were able to notice features of games and better understand student experiences because of the theoretical frameworks we synthesized into the Mathematics Classroom Games Features Framework. This is the purpose of a framework (Leatham, 2019; Spangler & Williams, 2019). Scholars could apply this framework to future investigations of game-based learning of mathematics topics other than integers (see Figure 1). Consistent with prior game design research that indicated people like to play games that are similar enough to known games but sufficiently different to warrant playing the new game (Schell, 2008), the findings suggest that games for learning mathematics might be engaging and motivating if reminiscent of out-of-school authentic gameplay experiences. Consequently, researchers, teachers, and game-designers might survey students about their favorite tabletop games to play outside of school and use these results as a basis for how to select, modify, or design new games with a specific mathematical learning purpose. The authentic games students name would, if viewed based on insights from this study, provide ways for teachers and researchers to recognize the features of those games. If such survey implementation is not feasible, we recommend the following key features of mathematics classroom games to ensure they meet the diverse needs of all students in ways that enhance motivation and engagement:

- High Cognitive Demand (Doing Mathematics or Procedures with Connections)
- Competitive
- Have all three features: Chance + (Game Strategy + Mathematics Skill)
- Asynchronous play (turn-taking or sequential play)
- Versatile purpose of learning as a game that teaches (introductory game) and also useful for building fluency (review game)

We discuss each of these recommended features next.

High Cognitive Demand Games Recommended for Learning Mathematics

An innovation of the current study was to view math games as special types of math tasks. Thus, we used the theoretical perspective of Cognitive Demand of Mathematical Tasks (Smith & Stein, 1998) to ensure that what matters for mathematics learning is considered in research on motivation, engagement, and learning with mathematical games. This is important especially because studies conducted in disciplines of psychology and educational psychology typically focus on arithmetic skills, whereas the study reported here included games that used high cognitive demand with the intention to develop conceptual understanding as well as accurate and efficient calculations. Overall, students preferred the higher cognitive demand games for learning. Given that only one low cognitive demand game was played and due to the variation of features within and across games in this exploratory study, an experimental study should be conducted to verify whether or in relation to which other features high cognitive demand games might be preferred.

Students like Competitive Asynchronous Games for Learning Mathematics

The qualitative results of this study consistently revealed competition as a positive game feature, so this may be a key feature of game design for mathematics learning. We should note here that although competition was a strong theme of student reasons for enjoying different integer games, a limitation of this study is that the authentic classroom gameplay we observed used only competitive games. It would be valuable for future studies to investigate collaborative as well as competitive integer games in authentic classroom mathematics learning. A collaborative game feature could be another option to increase motivation and reduce feelings of pressure during gameplay (Plass et al., 2013; Schell, 2008). This could be especially useful given that an experimental study of individual, competitive and collaborative video gameplay for developing fluency with arithmetic skills with middle school students found no differences of fluency achievement, but differences in motivation to play (Plass et al., 2013). Some mathematical games lend themselves to adapt for collaboration in pairs while still being a competition between pairs (Nurnberger-Haag & Wernet, 2019). We suggest that such an approach of collaborative play that encourages discussion amongst the pair might also raise the cognitive demand level of gameplay while making it easier for teachers to monitor student thinking, conjecturing, and argumentation (e.g., McFeetors et al., 2018; Stein et al., 2009).

A crucial implication of the current study is that many students felt a feature of synchronous play combined with speed negated competition as a positive feature. This is consistent with detrimental effects of timed tests on learning and motivation as well as inducing mathematics anxiety (Boaler, 2014). When features of time-based accuracy masquerade as a game, teachers might overlook that these features could be detrimental to student motivation. The results of this study demonstrated this was problematic from some students' perspectives. Thus, for example, if a game relies on mathematics skill and synchronous play, consider if there is a way to modify the game features to allow turn-taking (asynchronous play) and incorporate an element of chance using dice, spinners, cards, or random number generators to create chance-skill balance.

Balance Features of Mathematics Skill with Chance in Games for Learning Mathematics

As game design perspectives inform us, people have diverse perspectives as to what feels like a good balance of skill and chance when playing authentic games (Schell, 2008). Despite the numerous lenses of game design, Schell (2008) emphasized that the balance between chance and skill "will determine the character of your game," so even with authentic games they advised careful scrutiny in relation to "understanding how much skill and how much chance will be the right amount for the audience of your game” (p.183). In most situations of authentic gameplay, however, an audience can choose whether to play a game. Thus, in a mathematics classroom where the audience is required to play games, even greater care must be exercised to ensure there is a balance of the chance element that fosters equitable experiences. Especially when choosing or designing games for a content area in which a high percentage of students have anxiety about the content itself (Luttenberger et al., 2018), it behooves us to heed insights from game design that a simple modification to add elements of chance could foster greater comfort or “a more relaxed, casual” context of gameplay (Schell, 2008, p. 184). The themes and reported quotations about how students felt playing the game that included chance supported these recommendations, such as the student who stated that "*this game was a little more laid back*." Future studies might use measures of anxiety to further build on the recommendations and conceptual framework we provide here to test relationships with specific game features.

Interpretations and implications that we gleaned from these findings are that it is essential to consider game features when designing or selecting games for classroom instruction to ensure first and foremost, that instruction does no harm. The review context in which students experienced each game was intended to provide a variety of game features and varied learning purposes to help students enjoy their review of integer concepts and procedures. Yet, even after an entire unit to develop competence, just 10 minutes with the one synchronous time and skill-based game caused stress that some students expressed as a strong theme of their experiences.

The negative themes and other data reported in our findings and summarized in Figure 3C appear to be consistent with symptoms of math anxiety (Ashcraft, 2002; Luttenberger et al., 2018). Yet, even in a generally non-anxiety producing content area of literacy vocabulary, Hwang and colleagues (2013) reported that this same combination of competitive and synchronous features was found to evoke negative pressure. Moreover, in our study a student who said that they already have "bad anxiety" used the words “scared” and “paranoid” to describe their experience only of the game with these features, whereas they enjoyed the competition in games with the features of turn-taking and chance. Our findings that a game selected with the intention to increase student motivation induced the opposite effect for some students supports our claim that game features must be carefully analyzed and considered prior to classroom use. In contrast, we must be cognizant of the diverse ways students experience game-based learning. Consider again the engagement in Figure 3C along with the fact that a little less than half the students would play this speed-based skill-based game at home, and that the word “happy” was used by at least one student, math games with the combination of speed, competition, and math-skill may be ideal for some students who crave “excitement.” Principles of game design are consistent with our results and reinforce the need to draw on these diverse theoretical perspectives (Plass et al., 2015). Prior to use teachers and researchers must strike a balance between game demands of mathematical skill and chance (Schell, 2008) for every student, not just the majority of students.

Thus, to first do no harm and advocate for every individual for whom we are responsible in a mathematics classroom, based on these findings we make an emphatic recommendation. We take the strong stance that such a game that invokes counterproductive stress and fear should never be required of all students in a mathematics classroom. Especially in middle school where students have a higher rate of self-evaluation in comparison to their peers (Scarpello, 2007), the risk to some students is greater than the potential reward for any student If a game with the combination of features (competitive, synchronous, math-skill based) that has been shown here to cause unproductive motivation and engagement is offered to meet the needs of some learners, then a game with the same math learning purpose, level of cognitive demand, and asynchronous play feature that incorporates chance must also be offered for students to choose. This approach we recommend should better meet the preferences of all players in terms of game design (Schell, 2008; Whitehill, 2008), expectancy value-theory and motivation (Brophy, 2004; Wigfield & Eccles, 2000; Yurt, 2015), and their needs as learners of mathematics (Henningsen & Stein, 1997). We encourage research on outcomes of teacher education and teacher professional development that informs teachers and prospective teachers of these principles of game selection and design.

Future Research and Implications for Game-Based Learning of STEM Content

Future research should confirm the finding for other mathematics topics that turn-taking and chance are recommended features to design into classroom games to support and avoid marginalizing learners. With the Mathematics Classroom Games Features Framework delineated here (see Figure 1), we intended to improve theorizing about game-based mathematics learning. The purpose of a theoretical framework in mathematics education is to “lend both focus and structure to our effort to explain phenomena” (Spangler & Williams, 2019, p.6). Our framework drew focus to the features of games and provided specific theoretical structures by which to notice, analyze and interpret these features, which scholars can apply to future investigations of other mathematics topics that will lead to improved student experiences.

Like an effective non-game mathematical task (Zager, 2017), an effective game is challenging (Schell, 2008) and a learning game should be sufficiently engaging to feel like an authentic out of school game (Habgood & Ainsworth, 2011). We agree with Plass and colleagues (2015) that research on game-based learning requires synthesis of multiple theoretical perspectives. Game-based learning within classrooms should attend to game design features and bring many other perspectives to interpret game play. We contend, however, that extant frameworks from game-based learning research intended to apply broadly to any content area have thus far been too broad to be useful for mathematics classroom game analyses. In contrast, with this study we focused attention and design innovation on games for mathematics learning in classrooms by synthesizing an established framework for learning mathematics with relevant perspectives from game design. An implication of the conceptual framework we developed is that this need not be limited to the Cognitive Demand Framework of math tasks or indeed mathematics.

We offer the STEM Classroom Game Features Framework so that scholars investigating game-based learning of other content such as chemistry, geology, physics, biology, computer science, or engineering could use the conceptual framework by replacing the Cognitive Demand Framework with a content-specific framework useful for advancing understanding of their respective disciplines (See Figure 4). Future research could further enhance understanding of game-based STEM learning by incorporating additional perspectives. For instance, the degree of social interaction of a game (Schell, 2008) in combination with theoretical perspectives of status and power in classrooms (e.g., Civil & Planas, 2004) would be important additions to the STEM Classroom Game Features Framework to better understand student experiences with game-based STEM learning.

Games as a context for learning within content areas have been overgeneralized as universally valuable for increasing engagement and motivation of mathematics learning (e.g., Ernest, 1986). This is similar to the ways overgeneralizations of the value of using children’s picture books for mathematics have been insufficiently critiqued with theoretical perspectives that have been too far removed from the phenomenon (Nurnberger-Haag, 2017; Nurnberger-Haag, 2018a; Nurnberger-Haag et al., 2021), as manipulatives once were (Ball, 1992; Nurnberger-Haag, 2018b; Nurnberger-Haag, 2018c). This study contradicted claims about the universal value of classroom games to offer a more measured and nuanced focus based on game features. Thus, by demonstrating that games are one more resource that needs this critical attention, we contribute to pushing the field to critically investigate and design features of all resource types (games, children’s books, manipulatives, tasks, etc.) used in mathematics classrooms. Furthermore, we offered the STEM Classroom Game Features Framework to support broader implications to advance research and practice of other P-16+ STEM content learning.

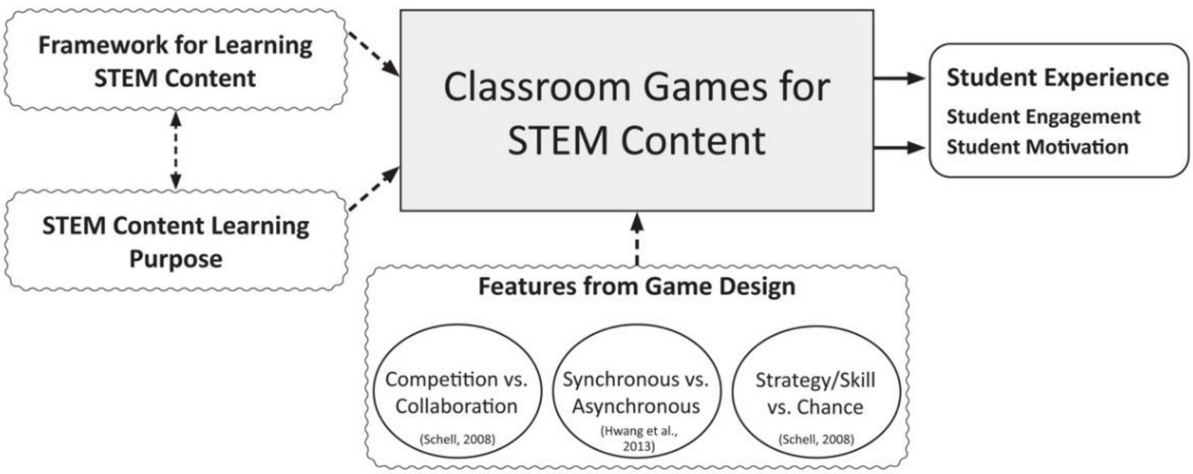

Figure 4. STEM Classroom Game Features Framework

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