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The Social Outcomes of Learning Mathematics: Standard, Unintended or Visionary?

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Abstract

Mathematics is a fundamental part of human knowledge and one of the central planks of the modern technological revolution. But in our enthusiasm to promote its benefits too rarely do we stop to question our intended aims of teaching mathematics and the outcomes of learning mathematics in school. In this paper the standard aims of school mathematics are acknowledged, but so too are the unexpected and unintended outcomes for some or all students. These are primarily the unplanned and unintended values, attitudes and beliefs that students develop during their years of schooling. Many of these are negative. To counter them and more generally to aim higher in school mathematics teaching I propose four additional visionary aims for mathematics that are empowering and broadening. These comprise developing higher level orientations and capabilities including mathematical confidence; mathematical creativity through problem posing and solving; social empowerment through mathematics contributing to critical citizenship; and the broader appreciation of mathematics, its nature and its key ideas.

Key words: Social outcomes, Learning mathematics.

Introduction

Mathematics is a fundamental part of human knowledge and one of the central planks of the modern technological revolution. But in our enthusiasm to promote the benefits of the applications of mathematics too rarely do we stop to question what its role in education is or what it should be. So I want to ask the following questions. Why do we teach mathematics? Why should students in school learn mathematics? What are our intended aims and the outcomes of teaching and learning mathematics in school? To offer new answers to these questions I wish to distinguish three groups of aims and outcomes.

1. *Standard aims of school mathematics* – What are generally agreed to be the basic or standard reasons for teaching the subject?
2. *Unintended outcomes of school mathematics* – Are there unexpected and unintended outcomes of the process for some or all students?
3. *Visionary aims for school mathematics* – What do we as mathematics educators wish to see as both aims and outcomes of school mathematics teaching/learning? What new emphases would enhance our students and indeed society beyond what we do now?

The Standard Aims of School Mathematics

The standard aims of school mathematics are the generally agreed basic and functional goals intended to develop the three capabilities of functional numeracy, practical and work-related knowledge and skills, and advanced specialist knowledge of mathematics. These are capabilities that are mathematics-related powers that we want students to acquire as a result of their years in school, so that they can be utilised in life beyond school.

1. **Functional numeracy.** This involves being able to deploy mathematical and numeracy skills adequate for successful general employment and functioning in society. This is a basic and minimal requirement for all at the end of schooling, excluding only those relatively few with some preventative disability.

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2. **Practical, work-related knowledge and skills.** This is the capability to solve practical problems with mathematics, especially industry and work centred problems. This is not necessary for all, for the depth and type of problems vary across employment types, and most occupations requiring specialist mathematics also provide specialist training. However, a strong case can be made for school providing the basic understanding and capabilities upon which further specialist knowledge and skills can be built.

3. **Advanced specialist knowledge.** This knowledge, learned in high school or university, is not a necessary goal for all adults, but advanced study does lead to a highly numerate professional class, as they have in France, Hungary, etc., where all study mathematics to around 18 years of age minimum. Advanced specialist knowledge is essential for a minority of students as a foundation for a broad range of further studies at university, including science, technology, engineering and mathematics (STEM) subjects as well as medical and social science studies. Clearly this option must be available in an advanced technological society, and indeed more students should be encouraged to pursue it, but it should not dominate or distort the school mathematics curriculum for all.

These three categories constitute useful or necessary mathematics for all or some, primarily for the benefit of employment and society from an economic perspective, as well as sustaining mathematics and mathematical interests themselves. They also benefit the recipient students in terms of functioning in society, work and further study.

Unintended Outcomes of School Mathematics

In a discussion of what we want from the teaching and learning of mathematics in society it might be unusual to discuss the collateral or by-products of the processes. What could the unintended outcomes of school mathematics be? What I have in mind are the values, attitudes and beliefs that students develop during their years of schooling that are not planned or intended, outcomes of what is known as the 'hidden' curriculum of schooling. These concern beliefs about the nature of mathematics, about what is valuable in mathematics, and about who can be successful in mathematics. Some of these beliefs are as follows:

a) Mathematics is intrinsically difficult and inaccessible to all but a few.
b) Success in mathematics is due to fixed inherited talent rather than to effort.
c) Mathematics is a male domain, and is incompatible with femininity.
d) Mathematics is a European science, to which other cultures have contributed little.
e) Mathematics is an abstract theoretical subject disconnected from society and day-to-day life.
f) Mathematics is abstract and timeless, completely objective and absolutely certain.
g) Mathematics is universal, value-free and culture-free.

I want to claim that every one of these beliefs is wrong, or at least in the case of e), f) and g) is controversial and rejected by many. Many of my writings over the past 30 years have been devoted to critiquing these beliefs and the associated claims (e.g., Ernest 1991, 1998). On the positive side, a growing number of researchers and teachers have come to reject these beliefs. Furthermore, their acceptance has always varied greatly by country and culture, so for example the populations of Far Eastern countries typically subscribe to the belief that mathematical success is due to effort rather than intrinsic ability, in contradiction to belief b). However, such beliefs are still held by many students and parents. Such beliefs are still communicated through popular images of mathematics widespread in society and the media, and in the image of mathematics presented in some classrooms.

One widespread outcome, although far from universal, is that many students develop negative attitudes about mathematics and about their own mathematical capabilities. Of course the major influence on students' attitudes and perceptions of mathematics and their own abilities is what happens in the classroom. This includes most notably the success at mathematical tasks experienced, provided that students do not regard these tasks as too easy. However, there is a surprisingly large influence from other factors too: the way students are viewed and treated in the classroom and elsewhere, and the beliefs and opinions about mathematics espoused by teachers, peers, parents, and portrayed in the public media. Thus attitudes are susceptible to powerful influences in addition to a student's own perceived competence and performance in mathematics. As we have learnt from sport, attitudes are vital to success, and for students a lack of confidence in their own mathematical abilities becomes a self-fulfilling prophecy – a failure cycle (see Fig. 1).

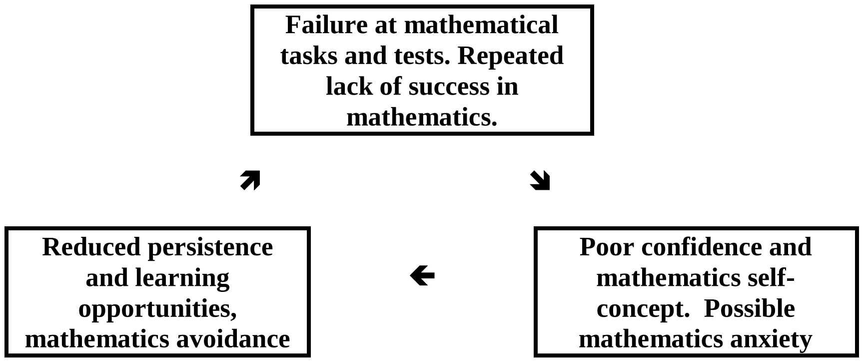

Figure 1. The failure cycle (adapted from Ernest, 2013)

This illustrates that students can experience repeated failure at mathematical tasks and tests, and an overall lack of success in mathematics. This leads to and reinforces poor confidence and poor mathematics self-concepts. Too much failure perhaps with a sense of humiliation can in a minority of cases leads to mathematics anxiety. Following Maslow's (1987) Hierarchy of Needs theory, persons will do a great deal to avoid risks including threats to personal self-esteem. So this outcome leads to reduced persistence and learning opportunities, and some degree of mathematics avoidance. The consequence of this is further failure and lack of success in mathematics, completing this self-reinforcing cycle and leading to a downward spiral.

Another example of a mistaken belief concerns gender. Despite progress, mathematics is still widely seen as a male domain, and although girls now equal boys in mathematical achievement at 16 years of age or so, too many women still doubt their own abilities and choose not to pursue mathematics related studies or careers after this age. In my view, values, images, beliefs and attitudes about mathematics underlie many of the differences in learning outcomes observed across different groups of students defined in terms of sex, socio-economic status, and ethnicity. For example, in Australia, indigenous mathematics performance can lag over two years behind that of non-Indigenous students (Queensland Studies Authority, 2004). But a full account of such inequalities requires more complex explanations involving such notions as Bourdieu's cultural capital and structural inequalities present in society, as well as the mathematics related misconceptions discussed here (Bourdieu and Passeron 1977).

I have only addressed samples of the range of the unintended outcomes of schooling in mathematics here. My point is that we need to address more research and attention to the values, attitudes and beliefs that students develop during their years of schooling. Although neither planned nor intended, these outcomes of the 'hidden' curriculum of schooling stand as obstacles in the way of achieving the basic aims 1, 2 and 3 above. One way to address these obstacles is to adopt broader and more visionary goals for school mathematics.

Visionary Goals for School Mathematics

The traditional way of presenting the mathematics curriculum is defined in terms of mathematical content and its use. Instead I want to propose new more visionary aims for mathematics that are empowering and broadening for students. For greater achievement and success students need to develop the following higher level orientations and capabilities concerning mathematics:

1. Mathematical confidence,
2. Mathematical creativity through problem posing and solving,
3. Social empowerment through mathematics (critical citizenship),
4. Broader appreciation of mathematics.

These four aims are to do with personal, cultural and social relevance, although ultimately I believe they have powerful incidental benefits for society, as well as for individual students, leading to increased achievement and success in mathematics.

Aim 1. Mathematical Confidence

Elevating this to an aim should come as no surprise given the importance I attach to attitudes as part of the incidental outcomes of school mathematics. Mathematical confidence includes being confident in one's personal knowledge of mathematics, feeling able to use and apply it, and being confident in the acquisition of new knowledge and skills when needed. This is the most directly personal outcome of learning mathematics. It uniquely involves the development of the whole person in a rounded way encompassing both intellect and feelings. Effective knowledge and capabilities rest on freedom from negative attitudes to mathematics, and build on feelings of enablement, empowerment as well as enjoyment in learning and using mathematics. These latter lead to persistence in solving difficult mathematical problems, as well as in willingness to accept difficult and challenging tasks. Corresponding to, but inverting the failure cycle discussed above (Fig. 1) is the virtuous, upwardly spiralling success cycle (Fig. 2)

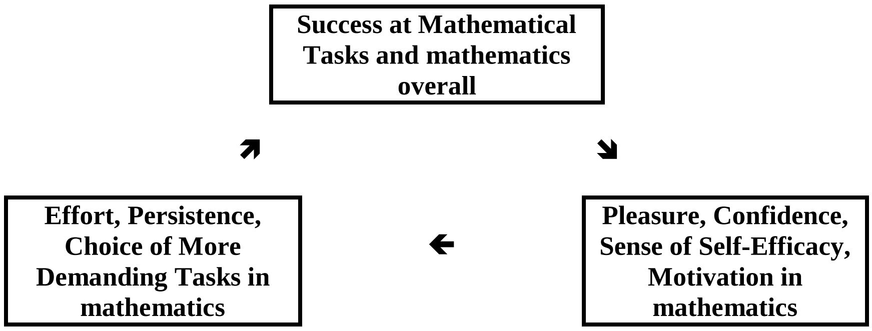

Figure 2. The success cycle (adapted from Ernest 2013)

The figure illustrates how success at mathematical tasks and mathematics overall leads to pleasure, confidence, an increased sense of self-efficacy, and motivation towards mathematics for students. Such intrinsic rewards lead to increased effort, persistence, and the choice of more demanding tasks in mathematics. Consequently students experience continued growth in success at mathematical tasks and mathematics overall. This completes the positive success cycle, leading to further achievements in mathematics and growth in positive attitudes, in a positive upward spiral. Indeed psychologists including Howe (1990) have shown that a mechanism like that shown in Fig. 1 is an important factor in the development of exceptional abilities among gifted and talented students. Students who demonstrate some giftedness and talent at around the age of 10 are very significantly further ahead of their peers at the age of 20 precisely because of the factors shown in the figure. Early success and the attitudes it breeds lead to much greater effort, persistence, and choice of more demanding tasks which lead to the flowering of the later manifested exceptional abilities. Howe found that the exceptionally talented invested an extra 5,000 hours in practice of their skills and abilities. This was double the time spent by their capable but less outstanding peers. This finding has been popularized as the '10,000 hour rule' by Gladwell (2008).

This cycle illustrates the intrinsic mechanism which draws us to the pleasures of success and self-enhancement. Indeed we can potentially turn a failure cycle into a success cycle by subtracting risk and making success achievable. In school this means reducing the importance of examinations and paying more attention to the quality of student learning experiences. Like an engine, once the cycle has been started in the right direction it keeps going under its own steam.

Aim 2. Mathematical Problem Posing and Solving

Mathematics is too often seen as a non-creative and mechanical subject. However, deploying mathematical knowledge and powers in both posing and solving problems is the area of greatest potential for creativity in school mathematics. In this pedagogical style, students choose what models and approaches to use in their solutions, some of the time. Although problem solving is widely endorsed, too often it is focussed on routine problems. True problem solving, the creative use of mathematics, requires non-routine problems, in which new methods and approaches must be created. Problem posing, the articulation and formulation of questions and problems to be solved, has been even more neglected in school mathematics. But it enables the seeing of mathematical connections between superficially diverse questions and topics, and the framing of questions by analogy. It involves seeking models for different aspects of life or mathematical patterns as discovered or chosen by students themselves. This is where full creativity flowers through student choices at every stage: problem or model formulation, the choice of methods to apply, and the construction of solutions. This capability builds on mathematical confidence and provides the springboard for increased student ability to use and apply mathematics, in both the pure and applied domains, which both students and society need to flourish.

Aim 3. Social Empowerment through Mathematics

Contrary to popular belief, mathematics *is* a political subject. Government and commercial policies are largely about financial inputs and planned social outputs, and these can only be evaluated if the underlying mathematics is understood. Economics is applied mathematics and this is the main language of politics, power and personal functioning in society. Mathematics should be taught so as to socially and politically empower students as citizens in society. It should enable learners to function as numerate critical citizens, able to use their knowledge in social and political realms of activity, for the betterment of both their own selves and for democratic society as a whole. This involves critically understanding the uses of mathematics in society: to identify, interpret, evaluate and critique the mathematics embedded in social, commercial and political systems and claims, from advertisements, such as in the financial sector, to government and interest-group pronouncements. Every citizen needs to understand the limits of validity of such uses of mathematics, what decisions it may conceal, and where necessary reject spurious or misleading claims. Ultimately, such a capability is a vital bulwark in protecting democracy and the values of a humanistic and civilised society. (See literature on Critical Mathematics Education, e.g., Skovsmose 1994, and the special issue of *The Philosophy of Mathematics Education Journal* 2010.)

In practice, this means building the critique of answers, methods, graphs, arguments, models, etc. into the mathematics classroom from an early age. The aim is not to politicize students, but to empower them to think for themselves. This should be reflected in improved mathematical capabilities, because too often students do not look critically at solutions or check their own or others' work. However, there should be a further outcome in students being able to apply their mathematical knowledge critically outside of the classroom as they mature.

Aim 4. Appreciation of Mathematics

The last of my seven proposed aims is the development of mathematical appreciation, as opposed to the inculcation of simple skills and capabilities. The latter dominate much of the study of mathematics in school and beyond, as perhaps they must. But learners also need a sense of the bigger picture to understand why mathematics is so highly prized.

There is an analogy between the study of language versus that of literature on the one hand, and, mathematical capability versus the appreciation of mathematics, on the other. Mathematical capability is like being able to use language effectively for oral and written communication, whereas mathematical appreciation resembles the study of literature, in that it concerns the significance of mathematics as an element of culture and history, with its own stories and cultural pinnacles, and the objects of mathematics need to be understood in that context, just as great books are in literature.

The appreciation of mathematics concerns its role in history, culture and society in general, as well as some philosophical understanding of mathematics and having a sense of its big ideas. These aspects of appreciation include the aspects of knowledge in the following list, which makes no claims to completeness.

a) *Students should have a sense of mathematics as a central element of culture, art and life, present and past, which permeates and underpins science, technology and all aspects of human culture.* This overview extends from symmetry in appreciating elements of art and religious symbolism, to understanding how modern physics and cosmology depend on algebraic equations such as Einstein's $E=mc^2$. It must include understanding how mathematics is increasingly central to all aspects of daily life and experience, through its import in commerce, economics (e.g., the stock market), telecommunications, ICT, and the role it plays in representing, coding and displaying information. However it must also recognised that mathematics is becoming invisible as it is built-in to the social systems that both control and empower us in our increasingly complex societies and lives.

b) *Students should have some awareness of the historical development of mathematics, the social contexts of the origins of mathematical concepts, its symbolism, theories and problems.* The evolution of mathematics is inseparable from the most important developments in history, from ancient societies in Mesopotamia, Egypt, India and Greece (number and tax and accounting, geometry and surveying) via medieval Europe and the Middle East (algorithms and commerce, trigonometry and navigation, mechanics and ballistics) to the modern era (statistics and agriculture-biology-medicine-insurance, logic and digital computing-media-telecommunications). This should also include being aware of ethnomathematics, which studies informal culturally embedded mathematical concepts and skills from cultures around the globe, both rural and urban, past and present (Ascher 1991).

c) *Students should have a sense of mathematics as a unique discipline, with its central branches and concepts as well as their interconnections, interdependencies, and the overall unity of mathematics.* This includes the central role of mathematics in many other disciplines such as in applied mathematics. After many years spent studying mathematics learners should have some conception of mathematics as a discipline, including understanding that there is much more to mathematics than number and what is taught in school.

d) *Student understanding of the ways that mathematical knowledge is established and validated through proof is also important, as well the limitations of proof.* Mathematics should include an introduction to the philosophy of mathematics: understanding that there are big questions and controversies about whether mathematics is discovered or invented, about the certainty of mathematical knowledge and about what type of things mathematical objects are. Being aware of such controversies supports a more critical attitude to the social uses of mathematics, as well as withstanding attributions of certainty to anything mathematical.

e) *Learners should gain a qualitative and intuitive understanding many of the big ideas of mathematics*. These include: pattern, symmetry, structure, proof, paradox, recursion, randomness, chaos, infinity, and so on. Mathematics contains many of the deepest, most powerful and exciting ideas created by humankind. These extend our thinking and imagination, as well as providing the scientific equivalent of poetry, offering noble, aesthetic, and even spiritual experiences.

Is this aim concerning appreciation feasible for school? Even big ideas like infinity can be appreciated by schoolchildren. Many an interested 9 year old will happily discuss the infinite size of space, or the never-ending supply of natural numbers, based on their intuitions and enquiring minds. We need to foster such enthusiasm and fascination, alongside the developing the more humdrum but necessary skills of mathematics. In mathematics we are privileged to have up to 2000 hours of compulsory school time over the years to 16. We can surely afford to spend some time on all of these visionary aims without threatening the development of skills and understanding. These aims have the potential to help build more confident, knowledgeable and successful students and citizens, and dare I say it, a better society?

Overall my argument is that we need to be more ambitious in our goals for school mathematics. We need to dare to aim at these visionary aims, not least to overcome of the negative if unintended outcomes that follow from an unenriched diet of traditional mathematics teaching.

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