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Expansively Framing Mathematics and Computer Science Teaching with Digital Technology in Elementary Classrooms

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Abstract

Expansive Framing (EF) is a theory and an instructional technique to facilitate connections between content and contexts. We employed EF as an approach to create a series of integrated mathematics and computer science (CS) lessons, using digital technology as a tool to leverage shared mathematical and computational ideas. We used deductive theoretical qualitative analysis of transcripts of classroom implementations to investigate how two fifth-grade teachers and one computer lab paraprofessional educator used EF during their teaching and what the EF approach looked like in practice. Findings suggested that educators engaged in EF principles when they were present in curricular materials, yet they also made additional impromptu (albeit school-based) expansive connections. The teachers in the study also regularly framed students as authors and owners of new knowledge. We recommend that mathematics-CS integrated curricular materials include language and other supports that make unambiguous, specific connections across learning contexts. We posit that EF theory can be a support to educators in the integration of mathematics and coding instruction with digital technology.

Introduction

Digital technology expands possibilities for mathematics learning in school settings. For example, digital technology makes possible the transformation of mathematics concepts with dynamic visualizations and multimodal interactions using digital software (e.g., Hoyles et al., 2004; Sinclair et al., 2017), reorganization of mathematical thinking through interaction with digital technologies like video and Internet (e.g., Borba & Villarreal, 2005), promotion of mathematics connections with digital math games (e.g., Moyer-Packenham et al., 2019), among many other new ways to learn and think about mathematics.

Most recently, the Covid-19 pandemic and Artificial Intelligence have changed (and are changing) the nature of humans' relationships with digital technology, which play important roles for the use of digital technologies in schools (Borba, 2021; Engelbrecht & Borba, 2023). Leveraging the use of digital technology in mathematics education is challenging because of the ever-changing nature of tools as well as the complexities of coordinating changes in mathematics curriculum, instruction, assessment, and professional development in school settings

(Roschelle et al., 2017).

Hence, the complexities of digital technology in mathematics education require an "infrastructural" approach (Roschelle et al., 2017, p. 872), and we think the use of theory to anchor design and implementation of instruction is one way to scale well-grounded pedagogical use of digital technology. Driven by this broad purpose, we chose a three-pronged approach in our study, specifically relevant to coding technology in mathematics education: 1) use a digital technology tool to leverage mathematical visualizations of important and difficult ideas in mathematics and computer science; 2) integrate computer science and mathematics concepts through cross-cutting disciplinary anchor ideas; and 3) use Expansive Framing theory to guide curriculum design and instructional implementation (e.g., framing statements in lesson plans to highlight connections).

Our approach is an important contribution to the question of how theory can support educators in using digital technology in meaningful and impactful ways to teach mathematics in elementary school. In our study, we used the theory of Expansive Framing (EF), which characterizes learning as a series of interrelated, overlapping ideas and provides a way to conceptualize transfer between contexts (Engle et al., 2012). Expansive Framing provided both a theoretical lens for our analysis and a pragmatic lens for the design of instruction. Hence, we used the design principles of EF to develop several integrated mathematics and CS lessons for elementary students (Beck et al., 2024). The aim of this approach was to support educators' enactment of expansively framed curriculum to foster students' connections across mathematics and CS, using a digital technology tool to anchor the content and contexts.

Our educational context is within a rural public school district in the western United States that aims to provide Computer Science (CS) education to all elementary students. However, the task of teaching CS is not within the classroom teachers' responsibilities. Rather, each school has a computer lab that is run by a paraprofessional educator. The paraprofessional educator in our study is called a Computer Lab Specialist (CLS). The CLS teaches a lesson to every class of students Grades 1-5 once a week, which typically involves digital technology for learning to code (e.g., Scratch coding which is a free and widely available online platform; CodeHS which is a curriculum purchased by the school district). In addition to the CLS, two fifth-grade classroom teachers, at the same school, participated in the study, and their students attended class once a week in the computer lab. In our context, the classroom teachers in the study had limited experience with the computer science digital technology while the CLS had limited experience with using mathematics as a context for coding. Hence, we found that the educators in our school district needed feasible methods and materials for using digital technology meaningfully and in ways that promote content connections and transfer for students.

Our inquiry was guided by the following research questions: 1) In what ways do educators use EF in their teaching of the integrated mathematics-CS lessons? and 2) In what ways are expansively framed content and context carried over from curriculum to instruction? The following relevant research areas guided the design and implementation of curriculum: integration of mathematics and CS instruction and Expansive Framing theory.

Mathematics and Computer Science Integration in Elementary School Instruction

Research on the integration of computer science (CS) and mathematics instruction in elementary school is growing, as seen in several recent literature reviews (e.g., Chan et al., 2023; Nordby et al., 2022; Ye et al., 2023). Research reveals that integration of CS in mathematics instruction can help lower the barrier to adoption of CS in elementary settings (Fofang et al., 2020) and lead to improved outcomes for students in understanding mathematical concepts and problem solving (Benton et al., 2018; Miller, 2019; Ng & Cui, 2021). In addition, the disciplines of mathematics and CS align intrinsically (Grover et al., 2015) and much prior work exists that has leveraged learning programming to support learning mathematics (Weintrop et al., 2016). Second, prior research has identified that students' existing mathematics literacy positively impacts CS learning (Lee et al., 2023). However, integrating CS into mathematics curricula is not without challenges. For example, Li et al. (2020) note that it is important to ensure good alignment between the topics in two disciplines. Also, when integrating CS and mathematics, the connections between the two need to be made explicit to students (Fofang et al., 2020; Israel & Lash, 2020) as it can help students transfer the knowledge to other contexts.

Expansive Framing

The theory of Expansive Framing (EF) is about learning through the interconnectedness of disciplines, such as the connections between CS and mathematics. As a theory of learning, EF is rooted in the historical debate about transfer of learning across contexts (for example, an individual's learning of addition in school can transfer to a real-life addition situation) and can guide principles of instruction. In the next sections, we discuss the emergence of EF theory from the challenges in studying transfer and then its use as an instructional approach.

Expansive Framing as a Theory of Learning: Historical Roots and Development of a Theory Based in Instruction

As a theory of learning, EF emerged from the field's attempts to understand the construct of transfer. Transfer is an individual's ability to abstract content learned in one context and apply it to another. Transfer has been extensively studied for over a century, yet much debate remains around how to define, facilitate, and measure transfer (Roberts et al., 2007). Barnett and Ceci (2002) acknowledged the thorny, and at times contentious, transfer debate and argued that operational definitions and a taxonomic framework were necessary for continued transfer-related research. Thus, they proposed a taxonomy to classify transfer and clearly defined the difference between transfer of content (what is transferred) and context (when and where content is transferred from and to). Engle et al. (2006, 2011, 2012) extended this research by centering their theoretical and instructional approach, Expansive Framing (EF), on contextual transfer. EF (Engle et al., 2006, 2011, 2012) reconceptualized research on transfer by focusing on how content is framed within and across contexts.

Expansive Framing as an Instructional Approach: From Transfer to Intercontextuality

As an instructional strategy, EF encourages educators to help students draw upon their existing knowledge and make distinct connections between the present learning environment and other times, places, groups of people, and topics (Lam et al., 2014). In this approach students are also framed as owners and creators of their own knowledge, placing the onus on the learner to create and own their ideas and to adapt existing knowledge to a new context more readily (Engle et al., 2012). Researchers such as Hickey et al. (2020) explained that EF is a pragmatic theory and used it as the basis for a 14-step practical approach in instructional design for online and hybrid courses. The theory of EF posits that these instructional approaches will create intercontextuality and foster transfer across settings by connecting settings and promoting student authorship. Intercontextuality occurs when multiple contextual frameworks become linked, thus signifying to learners that content will be relevant to a new (transfer) context (Engle, 2006). For example, Engle et al. (2012) found that EF instructional practices helped high school biology students more readily transfer knowledge across systems (i.e., from knowledge about the cardiovascular system to learning about the respiratory system). In contrast, bounded framing presents content devoid of contextual links and frames students as passive recipients of knowledge. Engle et al. theorized that bounded framing discourages transfer due to a lack of intercontextuality. Further, Engle et al. (2011) suggested that "the creation of intercontextuality is thought to give learners the message that they are allowed, encouraged, and even responsible for transferring what they know from one context to all others linked with it" (p. 605). Research suggests that creating intercontextuality through EF promotes transfer across contexts (Engle et al., 2011).

In the present study, we employed EF as an instructional approach to design integrated, cross-contextual mathematics-CS lessons. Further, we used digital technology to support this integrated, cross-context teaching. Specifically, Scratch programming served as a common digital technology tool between disciplines and classrooms in our study to help students make connections to a central big idea common to both mathematics and computer science. The technology tool in our study was an anchor in expansively framing content and context to enhance the connections between mathematics and CS.

Methods

This study used deductive theoretical analysis (Percy et al., 2015) of qualitative data to answer the research questions: 1) In what ways do educators use EF in their teaching of the integrated mathematics-CS lessons?, and 2) In what ways are expansively framed content and context carried over from curriculum to instruction?

The data in this study are part of a larger project aimed at supporting teachers and paraprofessional educators in rural schools in the United States to provide effective and equitable computer science (CS) education to all elementary students in the district (Shehzad et al., 2023).

Participants, Design Team, and Local Context

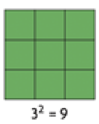
The participants in this study are two fifth-grade classroom teachers and one computer lab paraprofessional educator in the same school in a rural area of the western USA. This school is in a large district of 17 elementary schools in which there is a strong push for CS education at the elementary level. We focused on this particular school for this paper because it was considered a "cross-context" school, in which CS-integrated lessons were taught in the mathematics classes in the general elementary classroom and mathematics-integrated CS lessons were taught in the computer lab classroom (Shehzad et al., 2023). In other words, the lesson materials are focused on two different classroom spaces: the mathematics classroom, led by elementary teachers (pseudonyms Teacher Allen and Teacher West), and a computer lab, led by a Computer Lab Specialist (CLS) paraprofessional educator (pseudonym CLS Mathis). CLS Mathis taught Teacher Allen's and Teacher West's classes one time per week every week for 60 minutes, during the teachers' planning time.

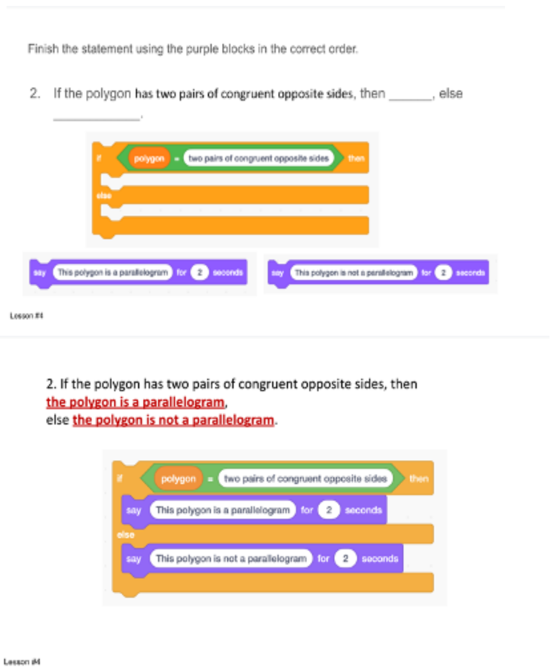
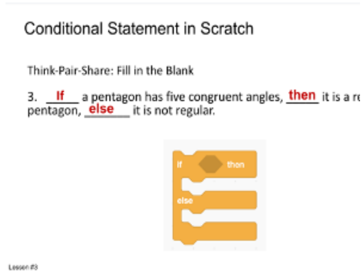
The CLS and teachers were part of a larger Design Team within a research-practice partnership (Beck et al., 2024; Shehzad et al., 2023). A Design Team is a group of people from various agencies who work together to conceptualize anchor ideas across mathematics and CS content and to design ways to make connections across the disciplinary content in the mathematics classroom and computer lab. The Design Team met 16 times over two years for one to two hours each to collaboratively plan, design, learn/practice, revise, and reflect on the lessons. During this time, the Design Team created two units linking mathematics concepts and computer coding ideas: 1) exponents unit with the anchor concept of repeats, and 2) geometry unit with the anchor concept of conditionals (see Table 2 in the next section). Within these two units, the lessons were specific to the teachers' and CLS's instructional routines and existing materials. The lessons for the mathematics classroom were designed as 15- to 20-minute "mini-lessons" that could be taught at the beginning of their regular mathematics lessons from existing district-adopted mathematics curriculum. The lessons for the CLS were 60-minute lessons that supplemented the existing CS curriculum created by resources the school used or purchased. The teachers and CLS coordinated the timing of the math mini-lessons with the CLS's lesson in order to leverage the connections across contexts and content.

Materials: Expansive Framing in the Lesson Plans

The Design Team identified topics that either had inherent cross-contextual features (e.g., conditionals in mathematics and Computer Science) or that students typically struggled with (e.g., conceptual understanding of exponents beyond their base 10 understanding). Using Expansive Framing (EF) as a lens for developing the lesson plans, the Design Team made clear connections across topics by creating computer coding visualizations to be used during mathematics lessons and using mathematics content as the basis of computer lab lessons on coding. The connections were based on core concepts across the two content disciplines, which we called the *anchor concepts* (Beck et al., 2024)—big ideas that are important in both mathematics and coding (see Table 1).

Table 1. Anchor Concepts to Expansively Frame Content

	Mathematics Anchor	CS Anchor
Exponents Unit	Repeats: repeated addition (multiplication) and repeated multiplication (exponents) 	Repeats: use the Repeat Loop Block in Scratch to repeat code 
Geometry Unit	Conditionals: use conditionals to compare polygons and to classify quadrilaterals	Conditionals: use the Conditional Block in Scratch to

Mathematics Anchor	CS Anchor
	provide specific criteria to instruct the computer's decisions 

In addition to anchor concepts for fostering connections across topics, we used EF in the mathematics lesson plans to support teachers in making connections across contexts (time and place) through Opening and Summary Statements (see Table 2). These kinds of statements were designed to prompt students' reflections on the interconnected learning experiences between the mathematics and CS classrooms, fostering an understanding of their intrinsic content connections and their ongoing relevance to students.

Table 2. Opening and Summary Statements in the Mathematics Lesson Plans

Type of Example	Lesson Plan Statements
Examples of opening teacher statement in an exponents lesson plan	*Now we are going to use a visualization from a Scratch activity that you will see in the computer lab this week. We'll watch the visualization and write what we see in mathematics notation. We'll use exponent form and word form to do it. I'll also ask you to write expanded form so we know what the exponent form is representing.*
Examples of summary teacher statement in an exponents lesson plan	*As you work in math and the computer lab, be thinking about how we find shortcuts when things repeat. Just like multiplication equations let us efficiently write repeated addition, using exponents is an efficient way to write repeated multiplication. Today, we saw how repeated addition looks much different than repeated factors or exponential growth. Like the loop block in the computer lab, exponent notations is a shortcut way to write repeated multiplication. Using the repeat loop block in Scratch and code.org*

Type of Example	Lesson Plan Statements
	is a shortcut way to instruct your program to repeat an action.
Example of opening teacher statement in a geometry lesson plan	*Take a closer look at the green triangle and the orange square in the center of the Venn diagram. These are regular polygons. I'm going to show you some statements that will help you learn more about regular polygons and connect it with something called conditionals, which you will use in the computer lab. Conditional statements use the words "if" and "then." You'll make a game in the computer lab and will need to use conditional statements.*
Example of summary statement in a geometry lesson plan	*You learned to define Triangle using My Blocks in the computer lab. Today, you learned more about the math behind doing this in Scratch.*

Finally, the construct of authorship in EF was used in the lessons for the purpose of framing students as owners and creators of their own knowledge. To encourage this role in the mathematics classrooms, the lesson plans included the routine of think-pair-share and questions for teachers to pose that prompted discussion among students. While think-pair-share is a common activity in constructivist classrooms, the purpose of the instructional approach in our lesson plans was not just students' construction of knowledge or a structure for collaboration. Rather its purpose was aligned with one of the EF principles, that of teachers' framing students as the authors or owners of knowledge. The lesson plans that were used in this study can be accessed at: digitalcommons.usu.edu/eled_support

These curriculum design decisions—use of anchor concepts, Opening/Summary statements, and authorship—evident in teacher lesson plans and student tasks, were intended to provide teachers and students with opportunities to see and experience the interconnectedness of these disciplines and leverage the Scratch digital technology to anchor concepts between the disciplines.

Data Source

The single data source for this study was a series of transcripts from educators' implementation of the two units of study. We video recorded the three educators' implementation of an exponents unit (4 math lessons by two teachers, 1 computer lab lesson) and a geometry unit (5 math lessons by two teachers, 1 computer lab lesson). In total, 7 hours of audio were transcribed for analysis.

Data Analysis

To analyze data, we enacted a multi-stage deductive theoretical analysis (Percy et al., 2015). Our initial phase of coding followed a Provisional Coding approach (Saldaña, 2021) based on Expansive Framing theory. A priori codes were based on Lam et al.'s (2014) contextual elements of time, place, role, participant, and topic.

After importing the cleaned transcripts into MAXQDA 2020 software (VERBI Software, 2021), we conducted an initial pass using *a priori* codes, marking transcripts line-by-line, and aligning specific text segments with the codes' definitions. Table 3 shows the initial code book with code descriptions and sample indicators for each.

Table 3. Initial Codebook of *A Priori* Codes with Sample Indicators

		Sample Indicators	
Code	Description	Expansive	Bounded
Time	When is the lesson happening? Includes all time-based framing. Expansive: connections to past/future Bounded: constrained to the present	On Friday, you will… We are continuing to learn about… Last year you learned how to… Present progressive verbs ("you're figuring out")	Today we are talking about… We are done talking about… Simple past with completion verbs ("we're finished with that now")
Place	Where is the lesson happening? Includes all location-based framing. Expansive: connections to locations outside of where the lesson is happening Bounded: constrained to the lesson location	In the computer lab, you will… At home, you will see this… Location framed as larger than the present context (references to the school, the city, other places)	Today in class… Location framed as the current place only with no reference to other locations
Participants	Who is participating in the lesson? Includes all framing pertaining to a person or group of people. Expansive: connections to people outside of those	Ask students how they might explain their ideas to someone outside of the current group Frame activity as one that involves other classmates, family members, friends	Limit conversations and explanations to current participants Frame lesson as private event, only involving current

		Sample Indicators	
Code	Description	Expansive	Bounded
	immediately participating in the lesson Bounded: constrained to the group of people in the lesson	outside of school, etc.	participants
Roles	How are the learners positioned intellectually? Includes all framing pertaining to how student roles are framed. Expansive: students are framed as owners and authors of their own knowledge Bounded: teachers/textbooks/materials, etc. are positioned as intellectual owners of content	Credit students for making their own discoveries (“Anna found that…”) Encourage students to go beyond what teacher/textbook/materials have presented Revoice student explanations and confirm that your explanation mirrors their thinking (“Anna told us that ____. Did I say that correctly, Anna?”)	Ask students to explain the teacher’s/ textbook’s/ materials’ ideas Compare student responses to what teacher/ textbook/materials say
Topics	What is the topical scope of the lesson? Includes all topic/content-based framing Expansive: connections to other topics and disciplines Bounded: constrained to the lesson topic only	Incorporation of computer science content into mathematics class, or vice versa This idea is found in [other content area] in this way…	Siloed content areas We learn ___ in [math/computer] class

Three coders met to discuss and refine the application of the codes and then conducted a second round of coding following similar procedures. This process yielded 1,556 instances of 10 codes (see Table 3; five *a priori* codes for expansive and five *a priori* codes for bounded). Next, we collapsed codes into broader categories and generated overarching themes around the context of these codes. This phase yielded 135 instances of three themes.

Results

Three overarching themes emerged from the data analysis. In this section we describe each theme and provide examples of each. The three themes are: first, purposeful planning supports Expansive Framing in practice; second, spontaneous contextual connections happen but are often school-based (as opposed to beyond-school-walls) connections; and third, promoting student authorship goes beyond lessons.

Purposeful Planning Supports Expansive Framing in Practice

Expansive Framing of topics was intentionally incorporated in the lesson plans by including mathematics topics as a basis for coding in the computer lab, and by referencing CS concepts and showing the Scratch visualizations throughout mathematics lessons. Educators' references to other content areas were primarily rooted in these materials. For example, Teacher West made the following statement about CS concepts:

> I'm going to show you some statements that will help you learn more about regular polygons and connect it to something called conditionals which you will use in the computer lab and Scratch. Conditional statements use the words *if* and *then*, and you'll make a game in the computer lab and you will use these conditional statements.

This statement was loosely based on the text from the lesson plan (but not verbatim), and it illustrates that Teacher West relied on the lesson plans to create a cross-contextual connection for her students. Further, the curricular materials supported additional mathematics thinking beyond the standard curriculum. For example, in Teacher West's fifth-grade classroom, the difference between angle measurements in Scratch and the mathematics curriculum led to a discussion on interior and exterior angles. She summarized the key ideas when she said:

> So, if I know my interior angle can I find my exterior by subtracting my interior angle from 180. In math [class], we use triangles' interior angles when we talk about our triangle measurement, but in Scratch, your sprites are rotated according to their exterior angles. So, to make Scratch work more like math, we have to use calculations, we have to use a variable for the exterior angles, so we are going to subtract the amount of degrees of the interior angle from 180 to get your exterior angle.

Because interior and exterior angles are not a standard part of the fifth-grade curriculum, this discussion illustrates that the lesson supported deeper thinking in Teacher West's classroom beyond the conventional materials.

Lesson supports such as pre-made coding visualizations for the classroom teachers and a math glossary for the computer lab educators also helped facilitate framing of topics across contexts. The lesson plans contained links to pre-made Scratch programs that showed the differences between repeated addition (multiplication) and repeated multiplication (exponents), for example this series of screenshots (to illustrate the dynamic aspects of the visualization) from one of the programs in Figure 1.

In reference to this visualization, Teacher Allen said to her students, "…we used Scratch to visualize how numbers grow, and a lot of you commented on how you liked being able to see each step in Scratch and how it grew." She then used the Closing Statement in the lesson to summarize the big idea: "In Scratch, we use the repeat block to help with that shortcut, just like this is a shortcut [exponents]…Using the repeat block in Scratch is a shortcut to doing the same thing over and over..." These quotes indicate that the visual supports included in the lesson materials were helpful for classroom teachers and students alike in illustrating the mathematical content (in this case, exponential growth) of the lesson.

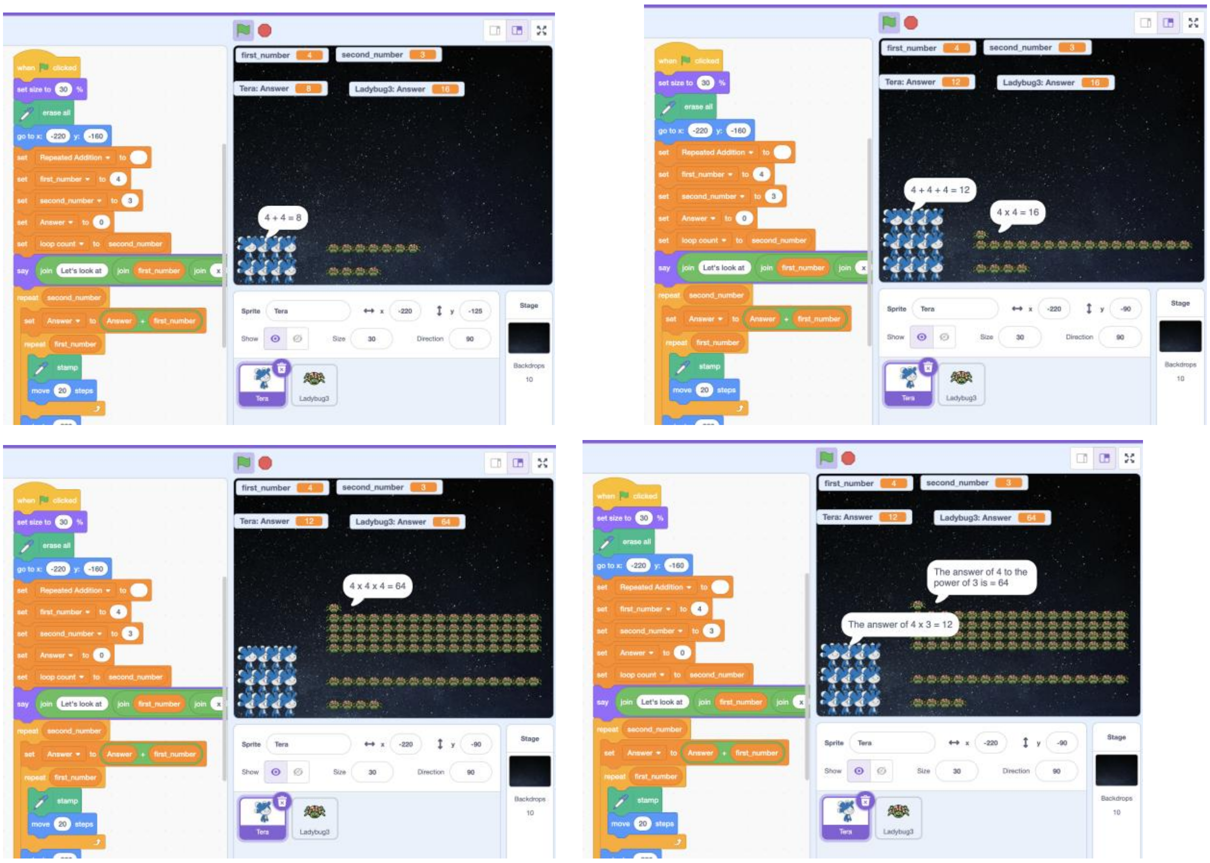


Figure 1. Series of Screenshots of Pre-made Coding Visualizations for the Classroom Teachers (images from Scratch, available for free at https://scratch.mit.edu and images are permitted by the Creative Commons Attribution-ShareAlike license).

For CLS Mathis, her transcript from the exponents unit showed regular use of the words "exponent," "repeats," "addend," "multiplier," "base," and "variables." She showed evidence of connecting the terms in her statements such as, "Okay, so we have two sprites, we have Tera and Ladybug, we're going to code her [Tera] to do repeats…we're going to take this code right here where it says repeat addend…" and in questions like "…we're going to make a math problem with coding, did you know that coding can solve math problems?" and "what do you think multiplier is going to do?"

These examples led to the theme of *purposeful planning supports EF in practice* because all three educators exhibited multiple examples of using the intentional EF instruction (i.e., the written lessons). What emerged in the educators' implementation was the emphasis on creating links between learning and contexts (Engle et al., 2012) and their statements were most often anchored in the big idea for the unit (i.e., repeats for the exponents lessons; conditionals for the geometry lessons) or in the visual/dynamic aspects of the digital technology (e.g., the "growth" of the sprites in showing exponential growth; the active drawing of the polygon by the sprite). Thus, the purposeful curricular materials supported the educators' EF in practice.

Spontaneous Contextual Connections Happen, but Often Remain School Based

Educators regularly made spontaneous contextual expansive connections beyond what was laid out in the lesson plans; however, these connections were often school based. CLS Mathis, for instance, even went as far as being specific to the data collections for this research project: "…when you were doing a math lesson, someone came in and was recording [Teacher Allen] while she was teaching – who can tell me what you were learning during that time?" While educators often referred to another location within the school, such as CLS Mathis did in this example referring to the students' fifth-grade classroom, locations or contexts outside of the school were not mentioned. This may set the expectation for students that the content learned will be useful in another school-based setting, but not necessarily outside of school.

This was also the case for other types of contextual framing such as framing across time. Activating background knowledge may be viewed as time-based expansive framing if the educator helps students make connections to what they already know. For example, CLS Mathis reminded her students of a coding concept that had been previously learned: "Nesting! Remember we learned that so long ago." Teacher West made temporal expansive framing to both past and future by calling back content previously learned and relating it to a future lesson in another domain when she said:

> You have been using conditionals in Scratch. In fact, you guys did a quiz similar to this, didn't you, where you had to build quadrilaterals using conditionals. You're going to continue to use conditionals in Scratch to help you with building your different shapes that we're working on.

The language "going to continue" implies that the content will continue to be relevant. Teacher Allen also set this expectation of continued relevance when she stated, "[Today's content] will help you with an upcoming lesson that we're going to have in just a couple of days in math." Though these connections are certainly expansive, they are arguably limited to school learning because they reference only the recent past and/or future at school.

Teachers also made unprompted topic-based connections between mathematics and coding outside of the lesson plan. Teacher West used the Scratch visualizations in her classroom to explore the mathematics more deeply:

> So, if we were to look back at this code, this isn't in the lesson but we're going to do it, let's go back to the code for a minute. And you guys will see, there's that repeat right? But the exponent, see that? There's that repeat. We're repeating that exponent. Right there. Yeah, coding magic happens. Because we're

repeating that base.

While not in the planned lesson, Teacher West used the materials improvisationally (in this case, a provided Scratch visualization) to reify mathematics content.

Teachers also encouraged students to examine excerpts of code in mathematics class. Teacher West said, "Here's the code you've seen on Scratch, take a look at it for a few minutes and think about what the code might do." This was again not part of the lesson plan, yet Teacher West took the opportunity to connect the topic to her mathematics lesson, thus solidifying her students' previously-learned coding knowledge in a different environment and making impromptu expansive connections between these topics.

Spontaneous topic-based expansive connections also happened in fifth-grade classrooms when teachers applied the content to topics other than mathematics or coding. For example, Teacher West was presenting the idea of a shape's attributes and noted its connection to science: "As we look at all these different shapes, we look at their attributes, which is a word we've been talking about in science too, same as properties." However, these connections do not span beyond the school setting, and further analysis of how educators followed, sorted, modified, created, or omitted lesson plan supports (Leufer et al., 2019) could provide more insight into these spontaneous topic-based expansive connections (Engle et al. 2012) and ways to encourage connections beyond the lesson plan.

Promoting Student Authorship Goes Beyond Lesson Plans

Engle et al. (2012) asserted that framing students as authors and owners of new knowledge is a vital component of EF theory because it encourages students to apply background knowledge more effectively and holds students accountable for knowing the content. In this view, the student is cast as the responsible party for creating and owning new knowledge (expansive framing), rather than the teacher or textbook being framed as the source of knowledge (bounded framing). Our instructional design included discussion questions and activities such as think-pair-share, which encouraged student authorship/ownership. The fifth-grade teachers used this type of expansive framing frequently during their implementation.

One way this was evident was educators crediting students for the creation of new knowledge during small group and whole-class discussions. For example, Teacher West credited student Betty as the creator of new knowledge when she said (in response to Betty's answer), "Four times three is 12. And I have four times four is 16. If I subtract four, what do I get? Oh, I haven't seen that before, Betty. Creative!" In another lesson, Teacher West presented a problem from the standard curriculum that she had used previously and remarked on a student's unique solution: "I have looked at this slide many times, and that is the first time I'm seeing what you're saying." And, in a discussion about the repeat block, Teacher West revoiced a student's key idea ,"So when I'm coding, like it would prevent me from having to write that code three times…because it will automatically repeat the code I have three times or however many time to tell, like you said" followed by crediting the student with, "I liked that. And I liked how you connected it to exponents." Rather than framing herself as the dispenser of new knowledge, Teacher West characterized and credited her students as knowledge creators, capable of innovative mathematical thought, which is a key component of EF theory. Teachers also supported this type of framing in their classrooms by setting an expectation that their students be responsible to learn new content and speak competently on the subject. In simple statements such as, “What do fifth graders know about these things?” the teacher conveyed an expectation that a fifth grader should come to the table with the appropriate prerequisite knowledge. This was evident in open-ended questions that encouraged discussion among students about their learning such as, “What did you learn today about the math behind this My Block here?” The teachers also asked questions that elicited their coding knowledge in the mathematics classroom: “…the repeat block…so how does that help us?”

Another regular activity in the fifth-grade classrooms was the use of individual whiteboards. Students were asked to write their solution to an open-ended problem on their whiteboard and then hold it up high, sharing their ideas with the class. This activity supported both student authorship (creating their own unique solution) and student ownership (as the student owns their solution and makes it public). The use of whiteboards was not always in the lesson plans but seemed to take off on lessons where this was not planned. Some activities were later adapted to be a think-write-pair-share discussion rather than think-pair-share.

Overall, when it came to framing students, the two fifth-grade classroom teachers primarily framed expansively as a classroom norm. However, there were also instances of bounded framing, such as when the teacher presented information and students repeated back what was said, thus framing the teacher as the expert. The teacher also used subtle language that alluded to the teacher being the source of knowledge in statements such as, “Wow, you cannot be tricked. Good job. Let’s give you another one and see if I can trick you this time,” implying that the teacher is the authority of this knowledge. Further, the paraprofessional educator in the computer lab almost exclusively used bounded framing of students by working through the lessons by modeling the next step in the coding process and asking the students to copy her work. This casts the instructor as the source of knowledge. This could be due to the short amount of time that students spend in her lab, the lack of framing opportunities built into the computer lab lesson plans, or the difference in professional development for teachers compared to paraprofessional educators. For example in the curriculum, pair-share activities and discussion prompts were included in the mathematics lessons, but not in the computer lab lessons because they were aligned to some typical practices in those classrooms. Hence, designing for authorship is an EF design consideration that merits further research. Despite these examples of bounded framing, overall, students were more frequently framed as responsible for their knowledge and creators of knowledge.

Discussion and Recommendations

Expansive Framing is an approach for integrating CS and mathematics. When content is framed expansively – across contexts, spaces, and times – learners may be better able to make broad connections to other ideas and ultimately transfer that content outside of the classroom, which are important pedagogical goals (Engle, 2006). We used EF theory to guide design and implementation of integrated CS and math lessons and leveraged a technology tool to anchor important ideas across the disciplines. The purpose of our study was to investigate in what ways educators carried over expansively framed content and context from curriculum to instruction. Overall, the educators in our study implemented EF principles in their instruction, and our analysis provided evidence that educators' implementations mirrored key EF elements in the lesson plans, such as integrated content, Opening/Summary statements, and authorship activities. We also saw examples of teachers applying their own instances of EF (Leufer et al., 2019), though much of the framing was school-based and rarely expanded beyond the classroom (Engle et al., 2012). The question of how deep the broad connections need to be to qualify as expansive remains. For example, while making a connection to the recent past while activating background knowledge could be considered time-based expansive framing, is it a deep enough connection to set the expectation that what students have learned continues to be important and relevant?

The lack of expansive connections made beyond the lesson plans combined with educators relying on lesson plans to make expansive connections supports the need for detail and clarity in integrated curricula. We recommend that mathematics-CS integrated curricular materials include teacher supports such as suggested language to illustrate broad framing (e.g., Opening Statements that frame connections) and classroom tools such as digital visualizations that overlap content areas. These purposeful supports were used by educators in this study.

Additionally, our analysis indicated that the two fifth-grade classroom teachers employed EF of roles (promoting student authorship/ownership) as a classroom norm. In the computer lab, however, the paraprofessional computer lab teacher almost exclusively used bounded framing when it came to framing students. This reveals a need for additional supports for paraprofessional educators. These supports may take the form of curricular improvements to more naturally implement framing of student roles in the lessons themselves, or opportunities for paraprofessional professional development.

More research is needed on the best ways to develop EF instructional techniques for in-practice educators. In particular, we recommend research on how teachers naturally engage with EF versus relying on curricular materials to support broad framing. Research is also necessary to further explore how curriculum can support educators when EF is unnatural or difficult due to content knowledge gaps or other factors.

Conclusion

We chose Expansive Framing to grapple with the challenges of cross-curricular teaching and leverage important concepts in mathematics and CS with digital technology, in this case, digital coding activities on the computer. These findings showed that this approach provided feasible methods and materials for using digital technology meaningfully in our local context. More broadly, this study contributes to the EF literature with a focus on elementary teachers' use of EF principles in cross-content and cross-context coding-mathematics lessons. Our study provides an example of research-practice and theory-practice connections, which are important contributions to scaling complex, evolving phenomena such as sound use of digital technologies in mathematics education in school settings (Roschelle et al., 2017). Further, digital technology played an important role in anchoring important ideas across the disciplines and enabling the study of difficult mathematics topics (exponents and classification of polygons), which contributes to the conversation about how to effectively study technology integration in mathematics education.

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Note

Scratch is a project of the Scratch Foundation, in collaboration with the Lifelong Kindergarten Group at the MIT Media Lab. It is available for free at https://scratch.mit.edu

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