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To cite this article:

Çaylı, S. & Akoğlu, K. (2025). The impact of an undergraduate course on teaching statistics and probability on preservice mathematics teachers' statistical knowledge. *International Journal of Education in Mathematics, Science, and Technology (IJEMST), 13*(4), 914-929. https://doi.org/10.46328/ijemst.4917

IJEMST

International Journal of Education in Mathematics, Science and Technology

2025, Vol. 13, No. 4, 914-929 https://doi.org/10.46328/ijemst.4917

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Article Info

Article History

Received:
29 January 2025
Accepted:
20 May 2025

Keywords

Statistics education
Teaching statistics
Preservice mathematics teachers
Statistical knowledge

Abstract

This study aims to investigate the influence of an undergraduate course on teaching statistics and probability on the statistical knowledge of preservice mathematics teachers. Statistics Concept Inventory (SCI) was used to measure the statistical understanding of the participants. It was implemented at both the beginning and the end of the course. A paired sample t-test was used in the analysis, and the pre-and post-test scores were compared in order to investigate the influence of the course on prospective teachers' statistical knowledge. The results indicated that the preservice mathematics teachers' level of knowledge in SCI increased after taking the course. In both pre-tests and post-tests, preservice mathematics teachers (PSMTs) lacked an understanding of statistical inference and sampling.

Introduction

Improving technology and the growing need to make sense of the world require students to have a statistical understanding. Not only to pass the courses but also to become critical thinkers and responsible citizens in decision-making, statistical thinking is significant for young learners (Burrill & Biehler, 2011). Educators who realized this need for statistics education at the K-12 level included statistics as one of the five main strands of mathematics education (NCTM, 2000). The principles and standards for statistics education were announced by the National Council of Mathematics Teachers (NCTM) in 2000. The emphasis on statistics education continued with the Guidelines for Assessment and Instruction on Statistics Education (GAISE) Report (Franklin et al., 2007). This report provided a framework for mathematics teachers about vital elements in statistics education. Lee and Tran (2015) introduced the Students' Approaches to Statistical Investigations (SASI) Framework based on Franklin and her colleagues' report (2007). In this framework, the authors outlined how to teach statistical reasoning to the students through four phases: posing a question, collecting data, analyzing data, and interpreting results. While teachers have underlined the importance of teaching statistics, the education of these teachers about teaching statistics has increased the need for research studies about it. Accordingly, Bargagliotti and Franklin (2015) published the Statistical Education of Teachers (SET) Report. In this report, they highlighted the differences between statistics education and mathematics education and the need to prepare teachers for it. The recommendations in the report focused on how to teach statistics and prepare preservice teachers to teach statistics.

This study, presented at the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), aims to investigate the influence of an undergraduate course on teaching statistics and probability on preservice mathematics teachers' statistical knowledge (Akoğlu & Çaylı, 2023). The study examines how preservice mathematics teachers' statistical knowledge changes after taking an undergraduate course on teaching probability and statistics. Digital tools and the SASI framework were used to frame the Teaching Probability and Statistics course. Statistics Concept Inventory (SCI) was used to assess the knowledge of statistics and was implemented both before and after taking the course. The results were examined based on the differences between pre and post-test scores.

Literature Review

Statistical Reasoning

Teaching statistics has been conducted under the mathematics curriculum. However, statistical reasoning differs from mathematical reasoning, according to many researchers and educators (Moore & Cobb, 2000). Therefore, what statistics and statistical thinking require became a popular topic for researchers. Gal and Garfield (1997) asserted that while mathematics is more precise and explicit, statistics can be viewed as numbers presented in a context. Therefore, the expected correct solution in mathematics education does not reflect the nature of statistics. Statistical problems require making inferences based on data in context. Shaughnessy (2010) supports the idea by claiming that certainty and proof in mathematics are against the nature of statistics. For instance, when a judge decides on a crime, analyzing existing data is inevitable. On the other hand, the judgment can be different for the same type of crime in different conditions. Hence, the nature of statistics is outlined by ambiguity and inferences. According to Pfannkuch and Wild (2005), statistical thinking is not a station to reach; it is a process where people interpret the data by considering its antecedents and beyond to make sense of the world. Therefore, where the data comes from and how it is collected are essential points during the statistical thinking process. Even though statistical thinking and reasoning have been used interchangeably in literature, Del Mas (2004) asserted that they are two different but related terms. According to the author, statistical thinking works when people know when and how to use statistical knowledge in given situations. However, statistical reasoning is a more complicated system that operates while making judgments and justifying a solution or idea. For example, while evaluating whether the model selected for the data fits the existing context, people use their statistical reasoning skills. Improving students' statistical reasoning and thinking should be highlighted in statistics education. Franklin et al. (2007) stated a framework for statistics education in the K-12 mathematics curriculum. According to this framework, the statistical process includes four components: formulating a question, collecting appropriate data for the question, analyzing data by using appropriate methods, and interpreting results by considering variability and context. These components are presented with three cognitive levels: A, B, and C. While Level A is the first step as cognitive load, Level C represents the most complicated and higher level of understanding. Therefore, it is emphasized that the aim of statistics education should be to improve students' statistical reasoning process from level A to C (Franklin et al., 2007). In 2015, Lee and Tran proposed a framework to support Students' Approaches to Statistical Investigations (SASI). The SASI Framework emphasizes the importance of allowing students to explore and investigate data in statistics education to acquire statistical literacy. Lee and Tran (2015) presented statistical habits of mind as a merger of the statistical investigation process with components such as variability, sampling, context, and uncertainty. In this way, students can improve their statistical reasoning, and later, it can turn into a statistical habit of mind. Considering the effectiveness of using the SASI framework in improving statistical reasoning, the course that the participants of this study participated in was designed with the SASI Framework and statistical investigation process in mind. Hence, it aimed to help the preservice teachers improve their statistical reasoning and understand how to build students' knowledge of reasoning based on the SASI Framework. The course is named Teaching Probability and Statistics. It was a mandatory course in a high-ranked Turkish university's mathematics education program. Pre-service teachers took this course in their sixth semester. It is important to note that there were no other mandatory course about probability or statistics before this; thus, even though the course was intended to present teaching methods about probability and statistics, at least 4 weeks of the course was spent with the introduction of basic probability and statistics content, instead of the pedagogy of the content.

Preservice Mathematics Teachers' Statistical Knowledge

While the mathematical knowledge of teachers in mathematics education is crucial, teachers' statistical knowledge is also necessary to have specific knowledge of the content. Mathematics teachers should be capable of guiding students in the learning process of statistics and statistical reasoning (Batanero et al., 2016). Lovett and Lee (2017) assert that mathematics teachers should be able to differentiate mathematics and statistics because, in contrast to mathematics, as Franklin et al. (2007) highlighted, the nature of statistics consists of ambiguity.

Groth (2013) advocates that mathematical knowledge for teaching and statistical knowledge for teaching are not exactly the same, but they share some common grounds. In addition to understanding basic conceptual differences, comprehending statistical concepts such as variability, measures of center, distribution, etc., is necessary to teach statistics. According to González et al. (2011), teachers require two types of knowledge to be competent in teaching statistics: statistical knowledge and pedagogical content knowledge. Considering the subject of graphical representations in statistics, the authors state that statistical knowledge highlights the graphical competency in which teachers need to know how to interpret the data from the given graphs, create graphs for the data, and choose a suitable graph for the given situation. On the other hand, pedagogical content knowledge refers to teachers' knowledge about how they will teach the graphs to the students. Groth (2007) elaborated statistical knowledge as a combination of the teachers' common knowledge, which might be thought to be similar to statistical knowledge mentioned by González et al. (2011), and specialized knowledge named statistical content knowledge by González et al. (2011). In his framework, Groth (2007) includes mathematical and non-mathematical knowledge as subcategories of the types of knowledge described above. In specialized knowledge, dealing with students' misconceptions and other challenges while learning statistical concepts is also covered. According to Groth's (2013) framework, as seen in Figure 1, teachers need to have different types of knowledge to be capable of teaching statistics, as advocated by other researchers.

The author highlights the importance of pedagogical knowledge and pedagogically powerful ideas in addition to content knowledge that Hill et al. (2008) identified. Pedagogically powerful ideas cover teachers' ability to direct students through stochastic reasoning and their ability to convey mathematical ideas to students, which requires elimination to the required level.

In her dissertation study, Lovett (2016) found that preservice mathematics teachers do not have sufficient statistical knowledge to teach statistics. She asserts that their statistical understanding decreases when the level of sophistication of the question increases. In the study, the sophistication of the questions was determined according to the GAISE level (GAISE levels A, B, and C) described by Franklin et al. (2007). From Level A to Level C, the concepts become more abstract. Therefore, more practice in collecting and analyzing data and interpreting results is needed to support preservice teachers' understanding.

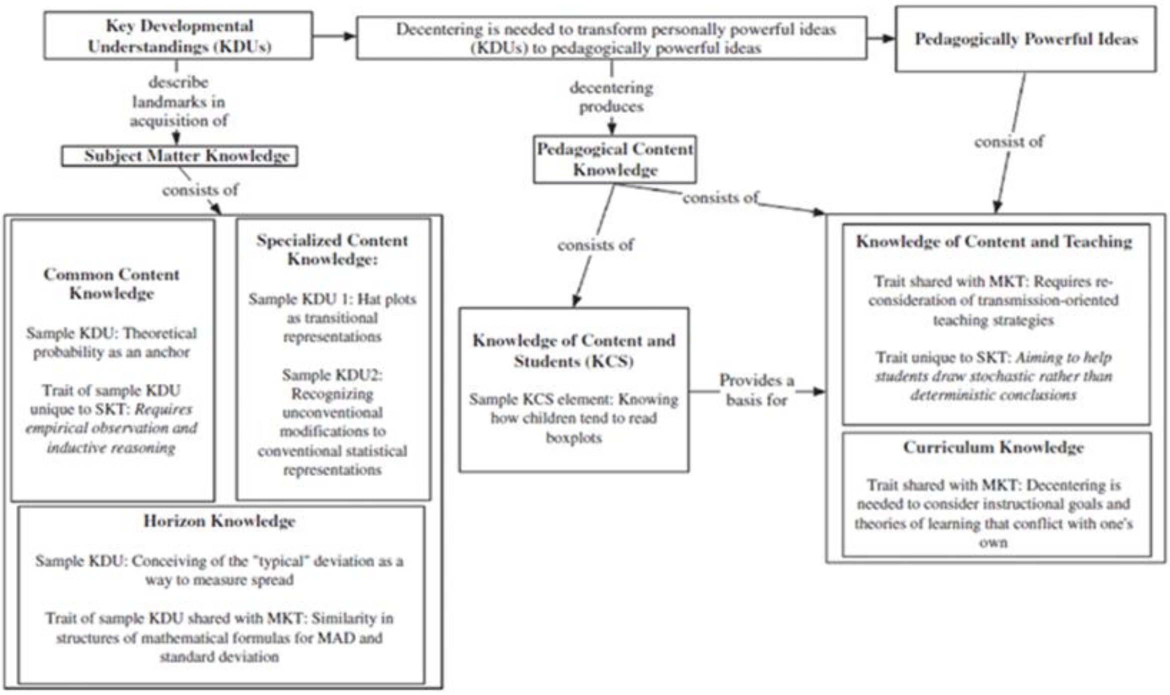

Figure 1.Groth's (2013) Framework

Statistics Education of Teachers

Teacher education has a huge significance in improving teachers' content knowledge. Lee and Hollebrands (2011) adapted Technological Pedagogical Content Knowledge and identified Technological Pedagogical Statistical Knowledge (TPSK) for teachers in statistics education. In this framework, they identified that mathematics teachers with sufficient TPSK should be able to monitor students' learning processes in statistical thinking, use technology to improve students' statistical understanding, and comprehend how the students' conceptual understanding improves with technology. The TPSK is essential for a mathematics teacher to teach statistics properly. However, Bargagliotti and Franklin (2015) assert that an introductory statistics course is insufficient for mathematics teachers to help students acquire statistical reasoning. To develop prospective middle school mathematics teachers' statistical knowledge, a course should emphasize using technological tools while making inferences and creating a discussion environment on statistical issues. Akoğlu (2018) designed a study in which teachers participated in Massive Open Online Courses (MOOC) and Professional Learning Teams to strengthen their statistical understanding and teaching. The results demonstrated that the participants improved their understanding after learning new strategies with the help of the SASI Framework and digital tools. In addition to sharing ideas and working collaboratively during professional development, integrating digital tools into this process seems crucial to positively impact teachers' development. Besides the improvement in statistical knowledge, when Akoğlu and Lee (2022) examined the teachers' confidence in teaching statistics after taking the Teaching Statistics through Data Investigations (TSDI) Massive Open Online Course (MOOC), they found that the confidence of the teachers to teach statistics increased significantly. Accordingly, Pfannkuch and Ben-Zvi (2011) stated that there are five components that educators need to consider while designing a course for preservice mathematics teachers: (1) "Developing the understanding of key statistical concepts, (2) Developing the ability to explore and learn from data, (3) Developing statistical argumentation, (4) Using formative assessment, and (5) Understanding students' reasoning".

The Role of Digital Tools in Statistics Education

The use of digital technologies is now an unavoidable aspect of research on statistics education. Teachers' role in applying technological tools to statistics education is crucial. Chance et al. (2007) claimed that technology triggered too many changes in teaching practices in statistics. Those changes in statistical practice directly affected the way statistics is taught. The proper use of technology also helps students better learn statistical concepts. Technology allows teachers to apply diverse ways of representing data sets, making it possible for students to manipulate representations while exploring data sets (Garfield & Ben-Zvi, 2007). Technology also helps students to understand abstract ideas under uncertainty. For instance, by integrating instructional software into a statistics course, students gain the opportunity to develop an understanding of specific statistical theorems, such as the central limit theorem, by constructing populations and investigating the distributions that are derived from them (Ben-Zvi, 2000).

Teachers need comprehensive knowledge about technological tools in order to benefit from them. They also need to have a depth of knowledge of pedagogical issues related to teaching and learning statistics with technology (Akoğlu, 2014; Lee & Hollebrands, 2011). According to Pfannkuch and Ben-Zvi (2011), there are many technological and pedagogical developments in the research about statistics education, and thus, the courses aimed to improve teaching statistics should be well-informed about those developments.

Methods

Course Design

As stated earlier, the Teaching Probability and Statistics course is mandatory for the undergraduate mathematics education program in Türkiye. Specifically, this course is mandatory for the Mathematics Education department at a high-ranked public university in Türkiye. The course aims to be able to develop tasks, questions, and lessons that can be used to assess as well as further develop grades 6-14 students' understanding of statistics and probability and to understand how to teach statistics and probability. Therefore, this course is not a general content course offered by the statistics or mathematics department. While the course provides brief content knowledge (around 4 weeks), the actual goal of this course is to prepare the preservice teachers to teach statistics and probability in the class. Therefore, the course relies heavily on teaching methods and the learning of probability and statistics. Students usually take this course in their 6th semester if the prerequisite conditions are met. The course is designed in two sections: the first half of the course presents concepts and rules about probability and statistics, and the second half is about teaching methods for those subjects. The course lasts 14 weeks (about 3 months), and around 55 students take the course on average. The course focused on teaching probability and statistics through data investigations and the use of digital tools and simulations. Students were engaged in probability problems and statistical investigations with various real-world contexts. The course started by explaining the historical background of probability and basic probabilistic terms. Then, it continued with conditional probability, Bayes' theorem, and discrete random variables. This part of the lesson includes different situations to discuss and many examples to internalize the topic. The probability section of the course ends with covariations and joint distributions.

The second part of the course was designed to cover the teaching of statistics and statistical reasoning. It started with understanding the difference between mathematics and statistics and continued with the Statistical Investigation Cycle and Statistical Habits of Mind, which focuses on understanding the statistical thinking process. As a next step, teaching statistics, its importance, and how to implement it in the class were discussed during the allocated lesson hours. In these discussions, the SASI Framework, Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report (Franklin et al, 2007), and technology integration were at the center while thinking about task design. The statistics part of the course was finalized with inferential reasoning by focusing on how to grow it in students' minds.

Participants

The participants in this study were chosen as junior students enrolled in the undergraduate program in mathematics education. The participants were selected among the preservice mathematics teachers who took a formal course named "Teaching Probability and Statistics," and they participated in the study voluntarily. In Türkiye, every student takes the national exam at the end of the senior year to be able to enroll in a program in a university, and each program in each university has different requirements for the percentile of students in the national exam. After students order their selection of programs and universities, the results are announced by the Higher Education Institution. Therefore, the exam is not designed by the universities; it is nationwide. The university where the study is implemented requires high mathematics scores. Thus, these participants had higher math scores in the national exam to be selected as students for this program, and they enrolled in one of the top five universities in mathematics education in Türkiye. Before taking this course, these preservice teachers had completed their mandatory math courses, which are about calculus, differential equations, mathematical structures/proofs, and matrices. However, in mathematics education programs, probability or statistics courses are not required as one of the mathematics courses. Therefore, the knowledge of the participants about statistical concepts comes from what they learned during high school. While 45 preservice teachers participated in the pre-test, 44 preservice teachers took the post-test. Then, the results of the pre-test and post-test were matched. The number of participants in the study is less than the number of test-takers in the pre/post-period since it was the add-drop period of the registration at the university, and some of the preservice teachers who took the pre-test did not enroll in the post-test or vice versa. As a result, the study participants were determined to be 40 preservice teachers who took both pre- and post-tests. When the attendance of these participants to the lessons was examined, it can be reported that 60% of the participants attended 90% to 100% of the classes, 30% of the participants attended 80% to 90% of the classes, and 10% of the participants' attendance was noted as below 80%.

Instrument

The statistical concept inventory (SCI) developed by Jacobs, Lee, Tran, and Doerr (2015) was used in the study as an instrument. Twenty multiple-choice questions were included in total. The SCI mainly includes five statistical topics based on the types of questions: distribution, probability, sampling, statistical inference, and formal inference. Table 1 shows the distribution of the questions based on the topics.

Table 1.Question Numbers matched with Statistical Concepts

Statistical Concept	Question Number
Distribution	Q1, Q2, Q3, Q5, Q6
Sampling	Q8, Q9, Q10, Q11, Q12, Q16, Q17
Informal Inference	Q13, Q14, Q15
Statistical Inference	Q18, Q19, Q20
Probability	Q4, Q7

While Q1, Q2, Q3, Q5, and Q6 belong to "Distribution", "Probability" are included in Q4 and Q7. "Sampling" was taken into consideration in Q8, Q9, Q10, Q11, Q12, Q16, Q17 whereas "Informal inference" in Q13, Q14, Q15 and "Statistical inference" in Q18, Q19, Q20 are involved. See the appendix for a detailed explanation of the questions.

Data Analysis

PSMTs' answers to each question were scored out of 1. If the answer is correct, 1 point is given; if not, it is scored as 0. Since questions 9,15, and 17 included two sub-questions, each one scored 0.5. The same scoring method was applied for the Q19 and Q16. There were four sub-questions in each, and each sub-question was scored as 0.25 to avoid inconsistency between the questions. To implement a paired sample t-test, PSMTs' scores on each question were summed, and a total score was gained for both pre- and post-SCI. Then, these scores were compared accordingly. The maximum score that a PSMT could get was 20 since there were twenty questions in total. On the other hand, the total scores were computed for each question to determine the weaknesses and strengths of PSMTs in the test. The maximum score for a question was 40 because of the number of participants. After dividing each total score by the number of participants, the mean scores were gained, and they were used to make a comparison between their knowledge of different topics in both the pre- and post-period.

In this study, a repeated-measure design was used. Therefore, one group of people was assessed more than once before and after the treatment (Privitera, 2017). The pre-test and post-test scores were compared using a paired sample t-test. R Statistical Computing Software was used to analyze preservice mathematics teachers' scores on the pre-test and post-test. The assumptions of the paired sample t-test needed to be checked before applying the test (Pallant, 2020). The first assumption was a normal distribution of the scores. The R program does not require the normality test since the number of participants exceeds 30. However, differences in the scores were checked for normality by using the Shapiro-Wilk Normality test. The results indicated that data is normally distributed (p>o.o1). Therefore, the normality assumption was not violated. The second assumption is that each participant in the group is independent. In this study, the score of one participant did not affect the other one as the participants were different from each other. Since all assumptions were met, a paired sample t-test was run to compare the scores before and after taking the course.

Results

Pre & Post- Test Comparison

The scores in the pre-test implementation ranged from 3.75 to 14.25 (M= 9.6, SD= 2.7), while in the post-test, the lowest score was 5.75 and the highest score on the test was 17 (M= 11.41, SD= 3.1), illustrated in Figure 2. and Figure 3.

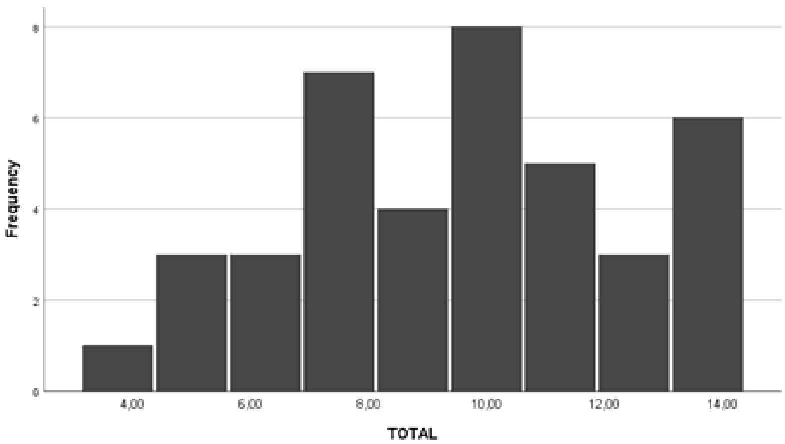

Figure 2. Distribution of the Pre-test Scores

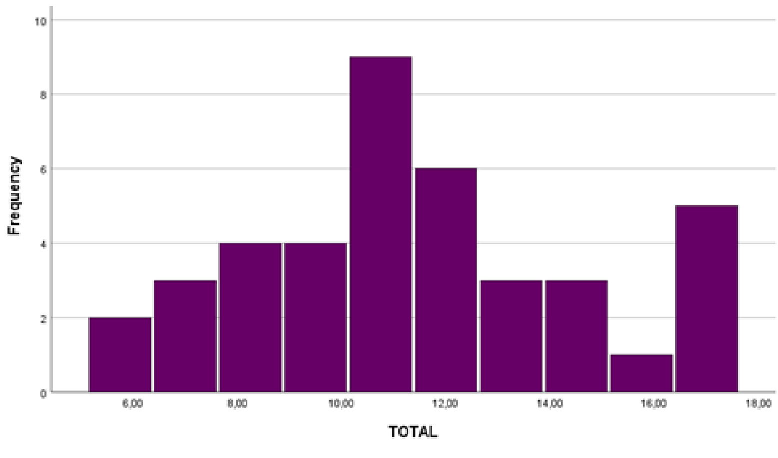

Figure 3. Distribution of the Post-test Scores

A paired sample t-test was applied to see the differences in the scores of SCI before and after taking the course Teaching Probability and Statistics in the 2020-2021 academic year using R Statistical Computing Software. There was a statistically significant increase in the scores of SCI from the beginning of the term (M=9.6, SD= 2.7) to the end of the term (M= 11.4, SD= 3.3), t(39)=3.073, p<.005 (two-tailed). The mean increase in SCI scores was 1.82, with a 95% confidence interval ranging from 0.62 to 3.03. The effect size based on Cohen's d (d=0.49) can be accepted as a moderate effect of the treatment since Cohen's d =0.5 is accepted as a moderate effect.

The mean difference between the scores in different statistics components was examined after checking the normality for each group. Sampling and distribution components were normally distributed based on the Shapiro-Wilk Normality test (p>0.05), while distribution, informal inference, and probability components did not have a normal distribution (p<0.05). Therefore, pre- and post-tests were compared using a paired sample t-test for sampling and distribution components. At the same time, the other three groups were examined by applying the Wilcoxon signed-rank test as an alternative to the t-test.

The results indicated that the increase in the scores of preservice mathematics teachers related to sampling after the course is statistically significant, p<.005 (two-tailed), t(39)=2.04. While the mean value of the scores at the beginning of the term was 3.55 (SD= 1.4), it was 4.10 points at the end of the term (SD= 1.5). The distribution of each component is shown in Figure 4 and Figure 5. The component distribution illustrated significant differences between pre- and post-tests in the non-parametric test implementations. In the pretest, the percentage of correct answers was 41% in distribution (M=2.05, SD=1.0) and 54% (M=2.70, SD=1.1) in the post-test. The Wilcoxon Signed-Rank Test indicated that post-test scores (Mdn = 2.00) were significantly higher than pre-test scores (Mdn = 3.00). This suggests that the intervention had a meaningful impact on the distribution scores.

The pretest implementation showed that 42.5% of the answers in informal inference (M=1.36, SD=0.9) were correct, while this ratio was 50.8% (M=1.59, SD=0.8) in post-test implementation. The percentage of correct answers in the pretest was 26.7% in statistical inference (M=1.20, SD= 0.8), while 35% in the post-test (M=1.43, SD= 0.8). The percentage of correct answers in the probability component differed between the pre and post-test. It was 70% (M=1.40, SD=0.6) in the pretest implementation as it was 80% (M=1.60, SD=0.5) in the post-test.

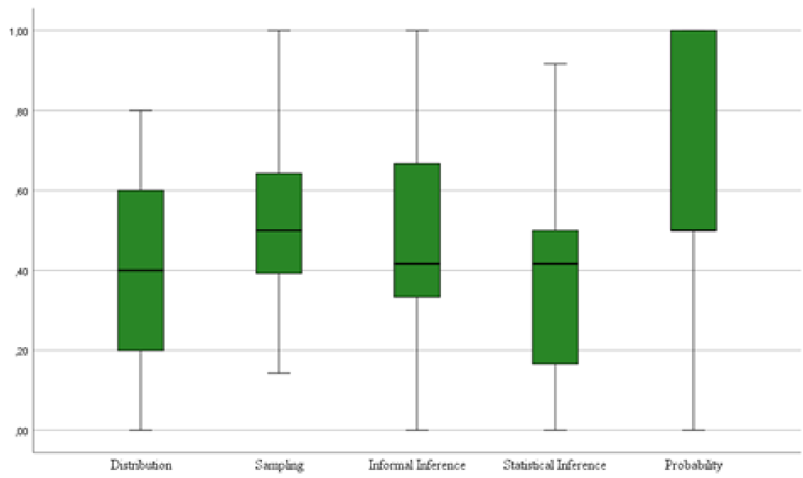

Figure 4. The Distribution of Average Pre-test Scores for the Statistical Concepts

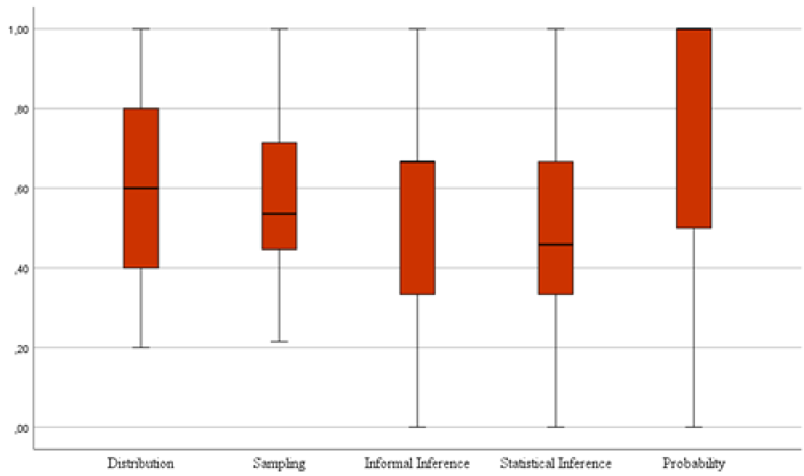

Figure 5. The Distribution of Average Post-test Scores for the Statistical Concepts

In addition, the scores for each question were examined. The number of correct answers was considered while evaluating the lowest and highest scores because some questions had partial credit. In the pretest implementation, Q19, under the statistical inference concept, achieved the lowest score with no full correct answer. While the number of answers to Q19 that were partially correct (2 correct answers out of 4 sub-questions) was 11, 15 of the PSTMs answered one-quarter of Q19 correctly (M=0.4, SD=0.2). In the post-test implementation, Q19 still had the least number of correct answers, too, with only one (M=0.4, SD=0.3). 16 out of 40 PSMTs answered half of the Q19 correctly, while ten prospective teachers reached one-quarter points in the Q19.

Q2, which is in the distribution category, had the second-lowest number of correct answers in the pretest implementation: only two of the answers were correct (M=0.05, SD= 0.2). Since Q2 does not include sub-questions, there were no partial credits. However, in the post-test implementation, the second-lowest score was found in Q17, under the sampling category (M=0.4, SD= 0.3). Only 12.5% of the PSMTs answered this question correctly, while 32.5% gave wrong answers.

The questions with the highest scores before taking the course were Q4 (M=0.88, SD= 0.3), which is in the probability concept, with 35 full correct answers, and Q10 (M=0.75, SD= 0.4), which is in the sampling concept, with 30 correct answers. On the other hand, the highest scores were found in Q5 in the distribution concept (M=0.88, SD= 0.3) and Q4 (M=0.9, SD= 0.3) in the probability concept. 90% of the PSMTs got the full score in the Q5, while 80% of them answered Q4 correctly.

Discussion

The scores were examined according to different statistical concepts assessed in the SCI. The results demonstrated that even though PSMTs increase their scores from pre- to post-test, they have a lack of understanding about statistical inference, which has the least number of correct answers in both trials. In this topic, PSMTs had difficulty in making interpretations of the p-value, and the improvement of the scores was almost stable in both of implementations. Batanero et al. (2004) stated that to be able to develop statistical inference, enhancing the understanding of distributions is essential. Even though prospective teachers' knowledge of distributions increased, their statistical inferencing skills did not demonstrate the same improvement despite a special inclination toward the concept. On the other hand, Lovett and Lee (2018) emphasized the weaknesses of the PSMTs in understanding p-values and confidence intervals. As they highlighted, many research studies have shown the same pattern for this lack of understanding. Although interpreting the null hypothesis was another concept with a low understanding of statistical inference, PSMTs improved their understanding after the course.

In both implementations of the test, it is found that PSMTs have a better understanding of interpreting and estimating an event's probability than any other statistical concept. Statistics is more about inference, estimation, and interpretation, and it is against the nature of mathematics (Shaughnessy, 2010). However, most teachers are unfamiliar with this type of reasoning process (Franklin et al., 2015). Probability includes a part that requires applying procedures like finding ratios to find a solution, like mathematics (Kvatinsky & Even, 2002). That can be associated with the prospective teachers' strength on the topic.

When the scores were examined question by question, PSMTs' strengths and weaknesses changed before and after taking the course. PSMTs lacked understanding of estimating the mean and median of a distribution and displaying distributions in the pre-test. However, they did not have a problem locating the mean and median of a distribution after taking the course. With this course, PSMTs improved their level of understanding of distributions significantly. The findings of this study align with Frischemeier and Biehler's (2018) assertion that using digital tools like TinkerPlots facilitates a deeper understanding of statistical concepts, particularly in comparing and interpreting distributions. Reading and Canada (2011) asserted that a conceptual understanding of the distributions was significant for teachers to have well-developed statistical knowledge. They claimed that it relies highly on variation, center, and probability knowledge. During the course, prospective teachers were encouraged to solve problems related to these concepts, which might be the reason for improving their understanding.

This study investigated the impact of a formal course, "Teaching probability and statistics," on preservice mathematics teachers' statistical knowledge. The formal course was enriched by presenting digital simulation tools, videos, and other materials for improving statistical reasoning, as Bargagliotti and Franklin (2015) stated. Also, the SASI Framework and the activities based on it were used during the course. Lee and Tran (2015) highlighted the importance of the SASI Framework in having a deep conceptual understanding of statistical reasoning. To assess preservice teachers' statistical knowledge, the Statistics Concept Inventory (SCI) was used at the beginning and at the end of the academic term. The scores were compared using a paired sample t-test in R Statistical Computing Software, and it was shown that preservice mathematics teachers' scores increased compared to their scores at the beginning of the term. Similar findings were indicated in Akoğlu's (2018) study, where teachers participated in a MOOC based on similar content, which was prepared based on statistical investigation and reasoning with digital tools. Therefore, it can be noted that rather than professional development, a formal course structured for improving statistical understanding and teaching statistics with inquiry can positively impact preservice mathematics teachers' statistical knowledge. In further studies, SCI can be implemented with self-efficacy in teaching statistics instruments before and after taking a formal course to see the relationship.

However, the mean scores for both pretest and post-test scores were found to be low, and the difference was not as high as expected after the intervention. The reason for this might be the background of the preservice teachers. The participants of this study did not have other statistics or probability courses in the university like other mass mathematics courses. Because of this, the first four weeks of the methods course had to be an introduction to the basic concepts of probability and statistics. If preservice mathematics teachers take another course on probability and statistics concepts before the course of interest in this research, with the help of this prior course, our method course would only focus on teaching methods of probability and statistics and be more productive in terms of its content and timing. Therefore, what they did in the test mostly relied on knowledge from high school. Since this course was mainly designed for teaching statistics and probability, the concepts were mentioned roughly and focused on the pedagogy of teaching these concepts. Therefore, adding statistics and probability courses might efficiently improve these teachers' knowledge, especially considering the new curriculum of this program in Türkiye. After this cohort, the higher education institution in Türkiye added statistics and probability as a compulsory course for the curriculum of this cohort. This means that teachers can grasp the idea of the concepts first before thinking about teaching it.

Conclusion and Implications

Statistical knowledge is crucial for teachers since it conveys key concepts to students. Our study shows that preservice mathematics teachers are not fully qualified to teach statistics, especially considering the concept of statistical inference. Therefore, teaching programs for prospective teachers are significant for acquiring the necessary knowledge (Bargagliotti and Franklin, 2015). In the study, the improvement of PSMTs' understanding of statistical concepts was shown. To be able to enlarge this enhancement to the understanding of other concepts that couldn't be developed, like statistical inference, the course would be designed more comprehensively. Further studies might consider this low level of understanding while assessing the knowledge and designing the course. It is important to note that having a formal course specifically about teaching probability and statistics is not common in Türkiye. The course is offered only in two highly ranked universities' programs. Instead of a formal course about teaching statistics and probability, the preservice teachers mostly take regular probability or statistics courses from the Mathematics departments, which only provide the content knowledge about the subject.

Limitations include the number of participants. Since this study was designed for a specific course, the number of preservice mathematics teachers was restricted. In further studies, more participants might provide the chance for more comprehensive findings. In addition, since it was the only course for a limited number of students by considering the total number of students in this cohort in the department, adding a control group was not an option for this study. In further studies, a control group might enable comparing the scores before and after the course. Also, future studies might be implemented and enlarged by having multiple universities to compare pre-service teachers who take a formal course on teaching statistics and probability, and those who do not.

Acknowledgements

The data was collected as part of Akoğlu's Start-up Project, which was funded by Boğaziçi University with a

Research Fund Grant Number 18721.

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Appendix

The concepts assessed by SCI (Jacob et al., 2015).

Question Number	Concepts Assessed
1	Distribution: Locate the Mean and Median given graph of distribution
2	Distribution: Displaying; Choosing, and correctly displaying a variable
3	Distribution: Coordinating mean and standard deviation to consider data value as unusual
4	Probability: Interpret Probability
5	Distribution: (describing) Standard deviation
6	Distribution: Displaying; choose likely histogram if the median > the mean note: reverse of #2
7	Probability: Estimate probability from experiment
8	Sampling: Sample size effect on variability (note: More likely than extreme)
9	Sampling: Estimate mean visually and expect interval for one sam le mean
10	Sampling: Expected variability in sample means on dot plots
11	Sampling: Sample size effect on variability (note: less likely than extreme)
12	SampIing : Expected variability given in proportions
13	Informal Inference: Boxplot comparisons attending to center and spread
14	Informal Inference: Compare dot plots attending to center and spread
15	Informal Inference: Boxplot comparisons attending to center and spread
16	Sampling : Expected Variability in sample proportions on dot plots
17	Sampling: Distinguishing between sample and distribution of sample means: i- variability in 1000 samples; ii- variability in 1000 samples means
18	Statistical inference: meaning/interpretation of confidence intervals in context
19	Statistical inference: meaning/interpretation of p-value (in context),
20	Statistical inference: interpreting the meaning of rejecting the null hypothesis in a context